## mathcentre

# A Numeracy Refresher 

V2. January 2005

This material was developed and trialled by staff of the University of Birmingham Careers Centre and subsequently used widely throughout the HE Sector. The contributions of Tom Frank, Eric Williams and Clare Wright are particularly acknowledged.


## C

## IMPROVE



## YOUR

## 7

## NUMERA( 3 )



THE UNIVERSITY OF BIRMINGHAM


University of Bristol Careers Advisory Service

## INTRODUCTION

Many students worry about anything to do with numbers, having done little since their GCSEs. This booklet has been designed to offer practice and explanation in basic processes, particularly to anyone facing employers' selection tests. It uses material developed by Clare Wright, Tom Frank, and Eric Williams of Birmingham University Careers Centre, where it has been successfully used for some time.

Regaining numerical confidence can take a little while but, if you once had the basic skills, practice should bring them back. If, however, you encounter real problems perhaps you should question your motives. Employers don't test you just to make life difficult. They do it because their jobs demand a particular level of proficiency. If you're struggling to meet that level, or not enjoying it, is their job right for you?

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## SECTION 1 - DECIMALS

The most common use of decimals is probably in the cost of items. If you've worked in a shop or pub, you're probably already familiar with working with decimals.

## Addition and Subtraction

The key point with addition and subtraction is to line up the decimal points!

## Example 1

$$
2.67+11.2=\begin{array}{r}
2.67 \\
+\frac{11.20}{13.87}
\end{array} \rightarrow \text { in this case, it helps to write } 11.2 \text { as } 11.20
$$

## Example 2

$14.73-12.157=14.730 \rightarrow$ again adding this 0 helps

$$
\underline{12.157}
$$

$$
2.573
$$

## Example 3

$127.5+0.127=127.500$
$+\quad 0.127$
127.627

## Multiplication

When multiplying decimals, do the sum as if the decimal points were not there, and then calculate how many numbers were to the right of the decimal point in both the original numbers - next, place the decimal point in your answer so that there are this number of digits to the right of your decimal point.

## Example 4

$2.1 \times 1.2$.
Calculate $21 \times 12=252$. There is one number to the right of the decimal in each of the original numbers, making a total of two. We therefore place our decimal so that there are two digits to the right of the decimal point in our answer.
Hence $2.1 \times 1.2=2.52$.
Always look at your answer to see if it is sensible. $2 \times 1=2$, so our answer should be close to 2 rather than 20 or 0.2 which could be the answers obtained by putting the decimal in the wrong place.

## Example 5

$1.4 \times 6$
Calculate $14 \times 6=84$. There is one digit to the right of the decimal in our original numbers so our answer is 8.4
Check $1 \times 6=6$ so our answer should be closer to 6 than 60 or 0.6

## Division

When dividing decimals, the first step is to write your numbers as a fraction. Note that the symbol / is used to denote division in these notes.
Hence $2.14 / 1.2=\frac{2.14}{12}$ 1.2

Next, move the decimal point to the right until both numbers are no longer decimals. Do this the same number of places on the top and bottom, putting in zeros as required.
Hence 2.14 becomes 214
1.2120

This can then be calculated as a normal division.

Always check your answer from the original to make sure that things haven't gone wrong along the way. You would expect 2.14/1.2 to be somewhere between 1 and 2. In fact, the answer is 1.78 .

If this method seems strange, try using a calculator to calculate 2.14/1.2, 21.4/12, $214 / 120$ and $2140 / 1200$. The answer should always be the same.

## Example 6

$4.36 / 0.14=\frac{4.36}{0.14}=\frac{436}{14}=31.14$

## Example 7

$27.93 / 1.2=\frac{27.93}{1.2}=\frac{2793}{120}=23.28$

## Rounding Up

Some decimal numbers go on for ever! To simplify their use, we decide on a cut off point and "round" them up or down.
If we want to round 2.734216 to two decimal places, we look at the number in the third place after the decimal, in this case, 4. If the number is $0,1,2,3$ or 4 , we leave the last figure before the cut off as it is. If the number is $5,6,7,8$ or 9 we "round up" the last figure before the cut off by one. 2.734216 therefore becomes 2.73 when rounded to 2 decimal places.

If we are rounding to 2 decimal places, we leave 2 numbers to the right of the decimal. If we are rounding to 2 significant figures, we leave two numbers, whether they are decimals or not.

## Example 8

$243.7684=243.77$ ( 2 decimal places)
$=240$ ( 2 significant figures)
$1973.285=1973.29$ ( 2 decimal places)
= 2000 (2 significant figures)

$$
\begin{aligned}
2.4689 & =2.47 \text { ( } 2 \text { decimal places }) \\
& =2.5 \text { ( } 2 \text { significant figures }) \\
0.99879 & =1.00 \text { ( } 2 \text { decimal places }) \\
& =1.0 \text { ( } 2 \text { significant figures })
\end{aligned}
$$

Try these examples. Give all your answers to 2 decimal places and 2 significant figures

1. $2.45+7.68$
2. $3.17+12.15$
3. $2.421+13.1$
4. $162.5+2.173$
5. 12.5-3.7
6. $9.6-7.8$
7. $163.5-2.173$
8. 2.416-1.4
9. $26.95-1.273$
10. $1.5 \times 7.2$
11. $2.73 \times 8.14$
12. $6.25 \times 17 \times 3$
13. $2.96 \times 17.3$
14. 4.2 / 1.7
15. 53.9 / 2.76
16. 14.2 / 6.1
17. 2.5 / 0.03
18. 250/2.35

Answers on page 19

## SECTION 2 - FRACTIONS

## Cancelling Down

When we use a fraction, we usually give it in its simplest form. To do this we look at the top (the numerator) and the bottom (denominator) and see if there is a number by which both can be divided an exact number of times.

Hence $\frac{2}{8}=\frac{1 \times 2}{4 \times 2}=\frac{1}{4}$ since the twos "cancel out"
E.G. $\frac{6}{8}=\frac{3 \times 2}{4 \times 2}=\frac{3}{4}$

$$
\begin{aligned}
& \frac{15}{35}=\frac{3 \times 5}{7 \times 5}=\frac{3}{7} \\
& \frac{16}{24}=\frac{8 \times 2}{8 \times 3}=\frac{2}{3} \quad \text { OR } \quad \frac{16}{24}=\frac{4 \times 4}{6 \times 4}=\frac{4}{6}=\frac{2 \times 2}{3 \times 2}=\frac{2}{3}
\end{aligned}
$$

Use as many steps as you need to reach the answer.

## Adding Fractions

When the denominators (the bottom lines) are all the same, you simply add the top line (numerators)

Eg: $\quad \frac{2}{6}+\frac{3}{6}=\frac{2+3}{6}=\frac{5}{6} \quad \frac{5}{9}+\frac{3}{9}=\frac{5+3}{9}=\frac{8}{9}$

Remember to cancel down if necessary.

When the denominators are different, we need to change the fractions so that the denominators are the same then we can add the top line as above.

Suppose we wish to calculate $\underline{1}+\underline{1}$
24

From the cancelling down process, we know that $\frac{1}{2}=\frac{1 \times 2}{2 \times 2}=\frac{2}{4}$

The denominators of both fractions are now the same so we can calculate

$$
\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}
$$

Sometimes the denominators are not multiples of each other
Eg: $\quad \frac{1}{4}+\frac{2}{3}$
In this case we can make 12 the common denominator using

$$
\frac{1}{4}=\frac{1 \times 3}{4 \times 3}=\frac{3}{12} \quad \frac{2}{3}=\frac{2 \times 4}{3 \times 4}=\frac{8}{12}
$$

We can then add these two fractions directly:

$$
\frac{1}{4}+\frac{2}{3}=\frac{3}{12}+\frac{8}{12}=\frac{3+8}{12}=\frac{11}{12}
$$

Eg: $\frac{2}{5}+\frac{1}{6}=?$

$$
\begin{aligned}
& \frac{2}{5}=\frac{2 \times 6}{5 \times 6}=\frac{12}{30} \quad \frac{1}{6}=\frac{1 \times 5}{6 \times 5}=\frac{5}{30} \\
& \frac{2}{5}+\frac{1}{6}=\frac{12}{30}+\frac{5}{30}=\frac{12+5}{30}=\frac{17}{30}
\end{aligned}
$$

## Subtracting Fractions

This works in the same way as addition. If the denominators are the same, simply subtract along the top line:

$$
\begin{aligned}
& \frac{5}{6}-\frac{3}{6}=\frac{5-3}{6}=\frac{2}{6}=\frac{1 \times 2}{3 \times 2}=\frac{1}{3} \\
& \frac{12}{15}-\frac{1}{5}=\frac{12}{15}-\frac{3 \times 1}{3 \times 5}=\frac{12}{15}-\frac{3}{15}=\frac{12-3}{15}=\frac{9}{15}
\end{aligned}
$$

Cancelling down gives $\frac{9}{15}=\frac{3 \times 3}{3 \times 5}=\frac{3}{5}$

NB: an alternative method would have been to cancel down $12 / 15$ to $4 / 5$ initially leaving an easier sum of $4 / 5-1 / 5=3 / 5$

## Multiplication of Fractions

It may help to understand multiplication if you interpret the ' $x$ ' sign as 'of'.
Hence:

$$
\frac{1}{2} \times \frac{2}{5} \text { means } \frac{1}{2} \text { of } \frac{2}{5}=\frac{1}{5}
$$

The calculation involves multiplying both numerators and both denominators then cancelling down:

$$
\text { Eg: } \begin{aligned}
\frac{1}{2} \times \frac{2}{5}=\frac{1 \times 2}{2 \times 5}=\frac{2^{1}}{10_{5}}=\frac{1}{5} \\
\frac{4}{5} \times \frac{2}{3}=\frac{4 \times 2}{5 \times 3}=\frac{8}{15} \\
\frac{7}{11} \times \frac{3}{8}=\frac{7 \times 3}{11 \times 8}=\frac{21}{88}
\end{aligned}
$$

Note that when multiplying, you can cancel down during the sum as well as at the final stage - it often makes the calculation easier:

Eg: $\quad \frac{3^{1}}{13} \times \frac{2}{3}=\frac{2}{13}$

$$
\begin{aligned}
& \frac{4^{1}}{9} \times \frac{3}{8_{2}}=\frac{1}{9_{3}} \times \frac{3^{1}}{2}=\frac{1}{3} \times \frac{1}{2}=\frac{1}{6} \\
& \frac{5}{24_{4}} \times \frac{6^{1}}{7}=\frac{5 \times 1}{4 \times 7}=\frac{5}{28}
\end{aligned}
$$

## Dividing Fractions

The trick with division of fractions is to turn the second fraction upside down and then to multiply:

Eg. $\frac{2}{3} \div \frac{1}{5}=\frac{2}{3} \times \frac{5}{1}=\frac{10}{3}$

Note that this answer can also be written as $31 / 3$

$$
\begin{aligned}
& \frac{2}{7} \div \frac{4}{5}=\frac{2^{1}}{7} \times \frac{5}{4}=\frac{5}{14} \\
& \frac{2}{3} \div \frac{7}{8}=\frac{2}{3} \times \frac{8}{7}=\frac{16}{21}
\end{aligned}
$$

## Improper Fractions

An improper fraction is one where the numerator is larger than the denominator, eg. 10/3 To convert this to a mixed number (one combining a whole number and a fraction), think of 10/3 in the following way:

| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$3 \times 1 / 3=1$ so out of the ten $1 / 3^{\prime}$ s, nine can be grouped into three whole units, leaving only $1 / 3$ left over. Hence, $10 / 3=31 / 3$

Similarly, $12 / 5=22 / 5$

$$
\begin{aligned}
& 16 / 11=15 / 11 \\
& 21 / 15=16 / 15=12 / 5 \text { (by cancelling) }
\end{aligned}
$$

We can also go from mixed numbers to improper fractions:
$2^{1} / 8=\frac{(2 \times 8)}{8}+\frac{1}{8}=\frac{16}{8}+\frac{1}{8}=\frac{17}{8}$
$3^{1 / 4}=\frac{(3 \times 4)}{4}+\frac{1}{4}=\frac{12}{4}+\frac{1}{4}=\frac{13}{4}$

## Addition of mixed numbers

Eg. $\quad 2 \frac{1}{2}+31 / 5=2+3+\frac{1}{2}+1 / 5$

$$
\begin{aligned}
& =5+5 / 10+2 / 10 \\
& =5+7 / 10 \\
& =57 / 10
\end{aligned}
$$

$$
\begin{aligned}
17 / 8+52 / 3 & =1+5+7 / 8+2 / 3 \\
& =6+21 / 24+16 / 24 \\
& =6+37 / 24 \\
& =6+13 / 24 \\
& =713 / 24
\end{aligned}
$$

## Multiplication and Division of mixed numbers

Here, it is usually easiest to convert to improper fractions and multiply or divide as normal:
Eg: $\quad 25 / 6 \times 3^{1 / 8}=17 / 6 \times 25 / 8=425 / 48=841 / 48$

$$
\begin{aligned}
& 12 / 3 \div 2 \frac{3}{4}=5 / 3 \div 11 / 4=5 / 3 \times 4 / 11=20 / 33 \\
& 2 \frac{1}{2} \times 41 / 3 \div 1 \frac{1}{4}=5 / 2 \times 13 / 3 \div 5 / 4=5 / 2 \times 13 / 3 \times 4 / 5=260 / 30=26 / 3=82 / 3
\end{aligned}
$$

Now try these examples:
Note that in these questions $2 / 5$ is the same as $\frac{?}{5}$

| $1.2 / 5+1 / 3$ | $2.7 / 8-1 / 5$ | $3.5 / 8+2 / 3$ | $4.2 / 9+4 / 11$ |
| :--- | :--- | :--- | :--- |
| $5.6 / 7-4 / 9$ | $6.7 / 8-1 / 3$ | $7.2 \frac{1}{2}-15 / 8$ | $8.7 \frac{1}{4}-52 / 3$ |
| $9.24 / 5+67 / 8$ | $10.2 / 3 \times 4 / 5$ | $11.8 / 9 \times 1 / 3$ | $12.7 / 12 \times 4 / 5$ |
| $13.8 / 9 \div 2 / 3$ | $14.2 / 11 \div 4 / 5$ | $15.6 / 7 \div 9 / 8$ | $16.2 \frac{1}{4} \times 3 \frac{1}{2}$ |
| $17.51 / 7 \times 21 / 5$ | $18.32 / 3 \times 44 / 9$ | $19.72 / 3 \div 34 / 5$ | $20.2 \frac{1}{2} \div 61 / 5$ |
| $21.31 / 3 \div 41 / 3$ | $22.2 \frac{1}{4} \div 1 / 3$ | $23.52 / 7 \times 11 / 3$ | $24.6 \times 21 / 3$ |
| Answers on page 19 |  |  |  |

## SECTION 3 - APPROXIMATING \& ESTIMATING

Estimates and approximations bring 2 major benefits:

1. They enable you to check that the final answer is near to the figure that your estimate led you to expect. This is particularly valuable if you are unsure in the use of the keys on the calculator.
2. They can provide you with a 'short cut', by showing plainly that you do not need to do all the calculations (eg. when comparing figures in tables).

## Example 1

Calculate $51 / 73$ of 300
steps
i. Approximate 51/73 to 50/75 (move to figures that make easily handled fractions)
ii. simplify $50 / 75$ to $2 / 3$ (by dividing top and bottom by 25)
iii. Approximate answer is $2 / 3 \times 300$ i.e 200
iv. Calculator answer 209.59 (in fairly close range to 200, your approx. answer)

## Example 2

The following table shows the number of items sold in one year by the Beans Co. in each of its operating districts. Column $Y$ shows the number of salespeople employed in each district.

| Column X | Column Y | Column Z |
| :--- | :---: | :---: |
| District | Number of <br> salespeople | Number of <br> items sold |
| SW | 57 | 53,971 |
| NW | 38 | 64,562 |
| Mid | 31 | 21,123 |
| SE | 27 | 47,051 |
| NE | 23 | 42,510 |

Task 1. In which district was the highest number of items sold per salesperson?

Task 2. In which district was the lowest number of items sold per salesperson?

## Steps:

i. To make handling of figures more manageable, make approximations of the items sold figures.

Approximation table

| District | Salespersons | Items sold (in thousands) |
| :---: | :---: | :---: |
| SW | 57 | 54 |
| NW | 38 | 64.5 |
| Mid | 31 | 21 |
| SE | 27 | 47 |
| NE | 23 | 42.5 |
|  |  |  |

Approximation and comparison can eliminate certain districts from the need to calculate accurately.

Task 1. The highest average sales
i. Mid and SE have similar numbers of salespeople, but Mid sold far fewer items. Hence Mid can be eliminated.
ii. SW sold fewer items than NW, but had far more salespeople. Hence SW can be eliminated.
iii. The districts ' left in ' for consideration are SE, NW, NE
iv. Use a calculator to do the division, USING THE ACCURATE FIGURES, to find the averages of the 3 districts.

| SE | $47,051 / 27=1742.6$ |
| :--- | :--- |
| NW | $64,562 / 38=1699$ |
| NE | $42,510 / 23=1848$ |

Hence the highest average sales figures per salesperson was in the NE district.
Task 2. The lowest average sales per person per district
From your approximation table it is evident that Mid sold far fewer items than each of NW,SE,NE, despite having similar numbers of salespeople.

It is not so clear as to how Mid compares with SW, so two calculations are desirable.

| SW | $53,971 / 57=946.86$ |
| :--- | :--- |
| Mid | $21,123 / 31=681$ |

Hence Mid district had the lowest average number of items sold per salesperson

## SECTION 4 - AVERAGES

There are several ways of expressing an average. The most common way is called a mean. This is one value which is representative of all the numbers in a group. The mean is calculated by adding up all the numbers in a group, then dividing by how many numbers there are.

Example 1 Find the average, or mean, of the following numbers:
Mean $=\frac{2+6+4+8+5+5}{6}=\frac{30}{6}=5$

Check your answer - is an answer of 5 representative of the numbers?
Example 2 Find the average, or mean, of the following numbers:

$$
\begin{gathered}
1 \begin{array}{ccccc}
3 & 4 & 2 & 1
\end{array} \\
\text { Mean }=\frac{1+3+4+2+1+20}{6}=\frac{31}{6}=5.17
\end{gathered}
$$

Check your answer - in this case, most of the numbers are less than 5 but the very high value of 20 pulls the mean upwards, so the answer is sensible.

Example 3 In a test of numerical ability, students are tested in two groups. Which group has the higher average score?

| Group A | Group B |
| :---: | :---: |
| 3 | 7 |
| 15 | 6 |
| 8 | 10 |
| 9 | 11 |
| 12 | 19 |
| 13 | 20 |

$\frac{3+15+8+9+12+13}{6}$
$=\frac{60}{6}$
$\frac{7+6+10+11+19+20}{6}$
$=\underline{73}$
$=10$

6
$=12.17$

Group B has the higher average score

| TRY THESE QUESTIONS: |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Find the mean of each of these sets of data: |  |  |  |  |  |  |  |  |  |  |  |
| 1. | 2 | 4 | 6 | 7 | 4 | 3 | 5 |  |  |  |  |
| 2. | 49 | 50 | 63 | 61 | 67 | 59 | 73 | 68 | 42 |  |  |
| 3. | 6 | 7 | 8 | 7 | 8 | 6 |  |  |  |  |  |
| 4. | 15 | 16 | 12 | 19 | 14 | 20 | 18 | 17 | 18 | 16 |  |
| 5. | 107 | 109 | 117 | 119 | 112 |  |  |  |  |  |  |
| 6. Which group of students scored best in this numerical reasoning test? |  |  |  |  |  |  |  |  |  |  |  |
|  | A: | 11 | 12 | 13 | 6 | 18 | 17 | 4 | 12 | 10 | 15 |
|  | B : | 10 | 11 | 9 | 12 | 14 | 13 | 12 | 10 | 11 | 14 |
| 7. A lecturer decided to find out how punctual students really are. He recorded how late his seminar group arrived on three occasions. The table shows how many minutes late each of the ten students were on these days. What was the overall average lateness? Which day was worst? On which day were most students more than 5 minutes late? |  |  |  |  |  |  |  |  |  |  |  |
|  |  | Day |  | Day |  | Day |  |  |  |  |  |
|  |  | 2 |  | 5 |  | 4 |  |  |  |  |  |
|  |  | 0 |  | 10 |  | 1 |  |  |  |  |  |
|  |  | 4 |  | 8 |  | 3 |  |  |  |  |  |
|  |  | 15 |  | 6 |  | 0 |  |  |  |  |  |
|  |  | 10 |  | 0 |  | 0 |  |  |  |  |  |
|  |  | 4 |  | 0 |  | 5 |  |  |  |  |  |
|  |  | 5 |  | 10 |  | 4 |  |  |  |  |  |
|  |  | 0 |  | 9 |  | 1 |  |  |  |  |  |
|  |  | 0 |  | 0 |  | 0 |  |  |  |  |  |
|  |  | 1 |  | 0 |  | 15 |  |  |  |  |  |
| Answers on page 20 |  |  |  |  |  |  |  |  |  |  |  |

## SECTION 5 - PERCENTAGES

Percentage is one of three ways of expressing a value
e.g. $\quad 17 \%$ has the same value as
$17 / 100$ or 0.17

It is recommended that, before using the calculator, you first understand, through mental calculation, the principles of layout of expressions and the functions of addition, subtraction, multiplication and division in fractions and decimals.

## Percentage and decimal - the relationship

Examine the 'meaning' of the figure 425.631. The position of each digit gives it its value. Regard each digit as being a member of a column
i.e.

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 4 | 2 | 5 | 6 | 3 | 1 |

Column a expresses 'whole HUNDREDS'. b expresses 'whole TENS..
c expresses UNITS (ONES). d. expresses $1 / 10$ ths $e$. expresses $1 / 100$ ths F. expresses $1 / 1000$ ths.
$1 \%$ is expressed as a fraction as $1 / 100$. Hence, from the 'column rule', $1 \%$ is 1 in columne i.e 0.01

Try to memorise the following table. It shows how identical values are expressed in different forms - percentages, fractions and decimals

| $100 \%$ | $\equiv$ | $100 / 100$ | $\equiv$ | 1.00 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $10 \%$ | $\equiv$ | $10 / 100$ | $\equiv$ | $1 / 10$ | $\equiv$ | 0.1 |
| $1 \%$ | $\equiv$ | $1 / 100$ | $\equiv$ | 0.01 |  |  |
| $0.1 \%$ | $\equiv$ | $0.1 / 100$ | $\equiv$ | $1 / 1000$ | $\equiv$ | 0.001 |

## Example

Calculate $17 \%$ of $3 / 4$ Give the answer as a percentage.
i. Express $17 \%$ as $17 / 100$
ii. $17 \%$ as $17 / 100 \times 3 / 4$ (note that 'of' becomes ' $\times$ ' )
iii. The rule of multiplication of fractions applies. Multiply the top values (numerators), and multiply the bottom values (denominators)
i.e $\quad(17 \times 3) /(100 \times 4)=51 / 400$
iv. Approximate $51 / 400$ to $50 / 400$
v. Simplify $50 / 400$ to $1 / 8$
vi. Convert $1 / 8$ to a percentage. The principle is that the 8 (denominator) has to be converted to 100 (in this case by multiplying by 12.5). You treat the top figure in the same way (i.e 1 multiplied by 12.5 )
vii. You thus produce a fraction 12.5/100 (NB. you would not normally leave a fraction in this format). This indicates $12.5 \%$ as an APPROXIMATE ANSWER to the question.
viii. Use the calculator to find the ACCURATE answer Answer 12.75\%

Now try these questions:
i. What is $20 \%$ of $£ 80$ ?
ii. What percentage of $£ 8$ is 20 p ?
iii. What is $50 \%$ of $5 / 8$ ? Express the answer as a fraction.
iv. By how much is $43 \%$ of 64 more than $35 \%$ of 64 ? Answer to the nearest whole number.
v. Calculate $70 \%$ of $1.5+60 \%$ of 3.5
vi. How much (to the nearest pound) would three dozen bottles cost @ $£ 5.75$ each plus $17.5 \%$ V.A.T?
vii. Express 30 p as a percentage of $£ 50$
viii. Calculate $50 \%$ of 0.005
ix. Express 7/12 as a percentage
x. In a constituency of 120,680 voters the outcome of an election was as follows.

Moderate Party $43 \%$ of valid votes cast
Radical Party $22 \%$ of valid votes cast
Extremist Party 9\% of valid votes cast
Countryside Party $8 \%$ of valid votes cast
$15 \%$ did not cast a vote
3620 voting slips were spoiled
NB. In answering the following questions, give the whole number, but regard 0.5 or more as 1. (e.g 1.5 adjusted upwards to 2, 1.499 adjusted downwards to 1 )
x.a What was the number of the majority of votes of the Moderates over the Radicals?
x.b What was the number of the majority of votes of the Moderates over all parties?
x.c What percentage of the constituents spoiled their voting papers?

Answers on page 20

## SECTION 6 - RATIO

Ratio is crudely explained as the proportion in which something or some things are shared. i.e. an expression of how the shares of the total compare with each other.

## Example 1

Ann, Betty and Chris win a total of $£ 1200$ in a lottery. The money is shared according to how much each staked in the lottery.
Ann is to receive 2 times as much as Chris. Betty is to receive 3 times as much as Chris.

How much should each receive?

You need to begin by establishing the total number of shares.
Begin by allocating 1 share to Chris
Ann will then have 2 shares ( 2 times Chris's share)
Betty will receive 3 shares ( 3 times Chris's share)

Thus the total prize is to be shared 6 ways $(1+2+3)$

So one share will be $£ 1200$ divided by 6 i.e $£ 200$
Chris will receive $£ 200$, Ann $£ 400$ (2 times Chris's share), and Betty $£ 600$ (3 times Chris's share)

Check that the shares ( $£ 200+£ 400+£ 600)$ total $£ 1200$

Different phraseology of this question could be.....
$£ 1200$ is to be shared out between Chris, Ann and Betty, in the RATIO of 1:2:3
respectively.
How much will each receive?

## Example 2

A school collected money for a new laboratory, using three methods, raffle, sponsored walk and fete. The total collected was $£ 5200$ in the ratio of 5:2:3 for each method in the order - raffle, walk, fete.

How much did each method raise?
i. 5:2:3 expresses the proportions ('shares')
ii. The total number of shares is $5+2+3$ i.e. 10
iii. The total collected was $£ 5200$
iv. Each share was $£ 5200 / 10$ i.e. $£ 520$
v. The raffle raised 5 'shares' of $£ 520$ i.e. $£ 2600$

The walk raised 2 'shares' of $£ 520$ i.e. $£ 1040$
The fete raised 3 'shares' of $£ 520$
vi. $\quad$ CHECK the total by addition
i.e. $£ 1560$
i.e. $£ 5200$

Try these questions:
i. $\quad 5$ brothers inherit a total of $£ 70000$ in the proportions 1:1:2:3:3

How much did each receive?
ii. In a darts game Ben scored twice as many points as Alan. Chris scored three times as many as Ben. Doug scored half as many as Chris.
The total number of points of the 4 players added together was 480.
How many points did each player score?

Answers on page 20

## ANSWERS

## SECTION 1 - DECIMALS

| Question | 2 decimal places | 2 significant figures |
| :---: | :---: | :---: |
| 1 | 10.13 | 10 |
| 2 | 15.32 | 15 |
| 3 | 15.52 | 16 |
| 4 | 164.67 | 160 |
| 5 | 8.80 | 8.8 |
| 6 | 1.80 | 1.8 |
| 7 | 161.33 | 160 |
| 8 | 1.02 | 1.0 |
| 9 | 25.68 | 26 |
| 10 | 10.80 | 11 |
| 11 | 22.22 | 22 |
| 12 | 318.75 | 320 |
| 13 | 51.21 | 51 |
| 14 | 2.47 | 2.5 |
| 15 | 19.53 | 20 |
| 16 | 2.33 | 2.3 |
| 17 | 83.33 | 83 |
| 18 | 106.38 | 110 |

## SECTION 2 - FRACTIONS

1. $11 / 15$
2. $27 / 40$
3. $17 / 24=31 / 24$
4. $58 / 99$
5. $26 / 63$
6. $13 / 24$
7. $7 / 8$
8. $17 / 12=19 / 12$
9. $927 / 40=387 / 40$
10. $8 / 15$
11. $8 / 27$
12 7/15
12. $4 / 3$
13. $5 / 22$
14. $16 / 21$
15. $77 / 8=63 / 8$
16. $1111 / 35=396 / 35$
17. $168 / 27=440 / 27$
18. $21 / 57=115 / 57$
19. $6 \frac{3}{4}=27 / 4$
20. $25 / 62$
21. $10 / 13$
22. 14

## SECTION 4 - AVERAGES

1. 4.43
2. 59.11
$3.7 \quad 4.16 .5$
3. 112.8
4. Group $A=11.9$, Group $B=11.6$ so the answer is Group $A$ (just!)
5. Overall average $=122 / 30=4.067$. Day 1 average $=41 / 10=4.1$. Day 2 average $=48 / 10=4.8$ Day 3 average $=33 / 10=3.3$, so Day 2 has the worst overall average.
On Day1, 2 students were more that 5 minutes late, Day2 $=5$ students and Day $3=1$
student so Day 2 is also the day on which most students were more that 5 minutes late.

## SECTION 5 - PERCENTAGES

| i. | $£ 16$ | v. | 3.15 | ix. | $58 \%$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| ii. | $2.5 \%$ | vi. | $£ 243$ | x.a | 20,781 |
| iii. | $5 / 16$ | vii. | $0.6 \%$ | x.b | 3958 |
| iv. | 5 | viii. | 0.0025 | x.c | $3 \%$ |

## SECTION 6 - RATIOS

i. $7,000,7,000,14,000,21,000,21,000$
ii. Alan 40, Ben 80, Chris 240 and Doug 120

