Localised Collective Excitations in Doped Graphene in a Strong Magnetic Field

[arXiv:0902.4176]

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What is Graphene?

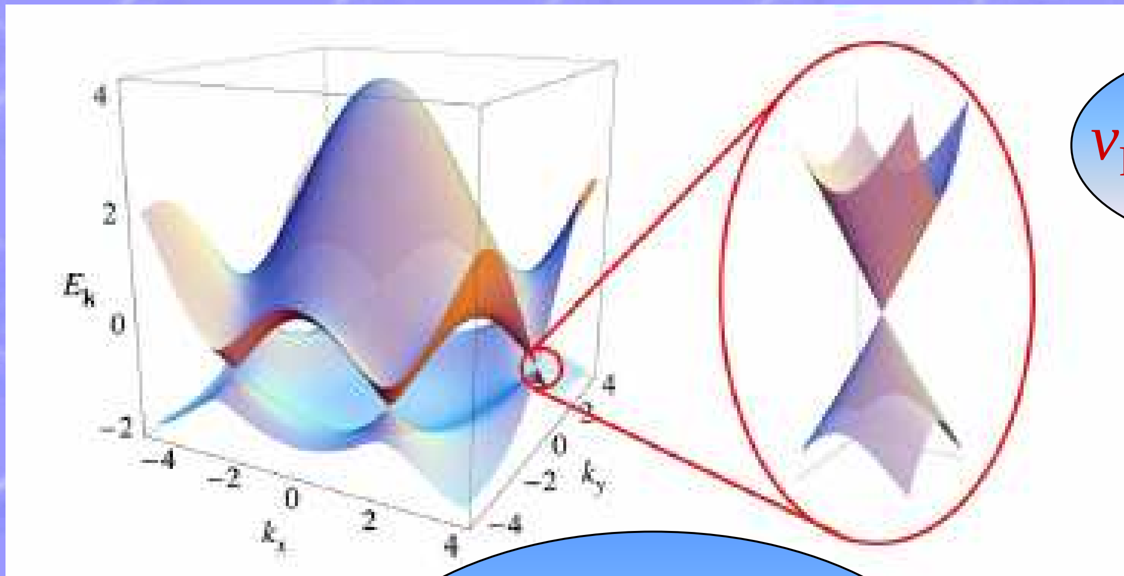
- *"Graphene is the name given to a single layer of carbon atoms densely packed into a benzene-ring structure"*
- First isolated in 2004 by physicists at the University of Manchester, UK and the Institute for Microelectronics Technology, Chernogolovka, Russia

Andre Geim

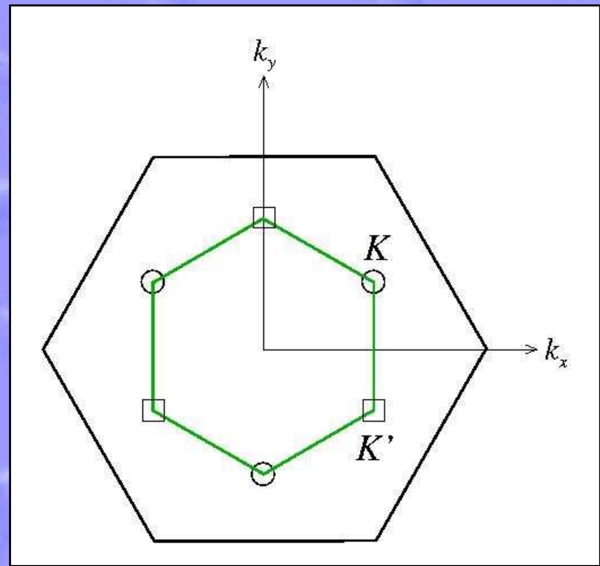
K.S. Novoselov *et al.*, *Science* **306**, 666 (2004)

Dispersion Relation

A. H. Castro Neto *et al.*, Rev. Mod. Phys **81**, 109 (2009)



$v_F \sim 10^6\ \mathrm{ms}^{-1}$



linear dispersion relation
at low energies

P.R. Wallace, Phys. Rev., **71**, 622 (1947)

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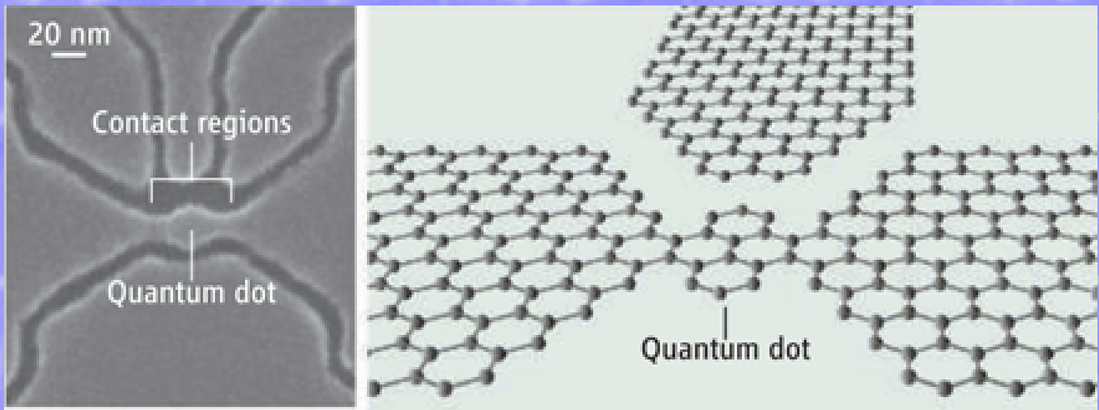
What's all the fuss?

- Fundamentally interesting physics and useful physical properties
- QED in a mesoscopic system – e.g. room temperature anomalous integer quantum Hall effect

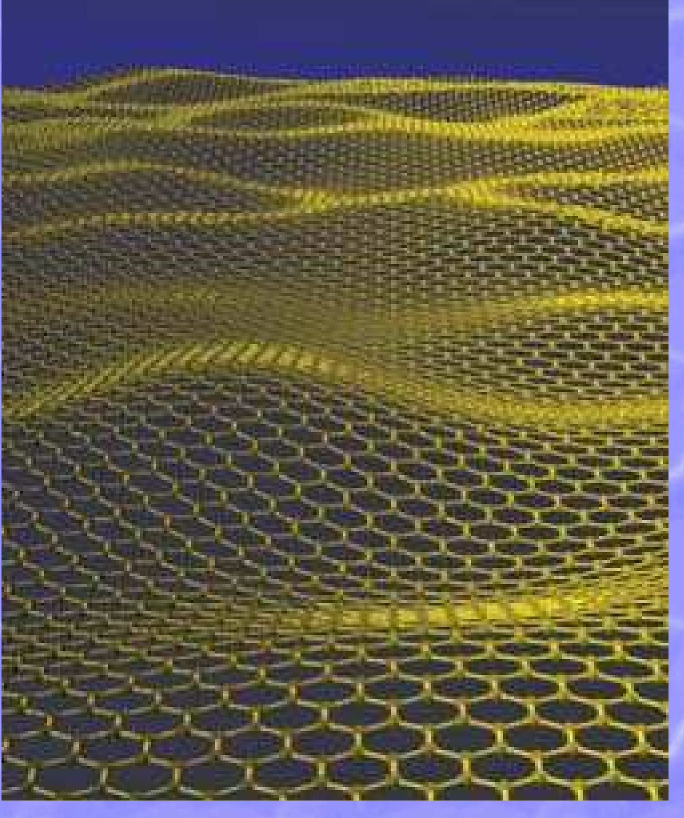
K.S. Novoselov et al, Nature **438**, 197

- Important applications i.e. nanoelectronics – is graphene the new silicon?



Credit: Andre Geim

Unanswered Questions



Credit: Andre Geim

- How does substrate affect graphene properties?

- What is the nature of disorder in graphene?

- How could an energy gap be opened at the K points?
  - constriction i.e. nanoribbons
  - application of an electric field

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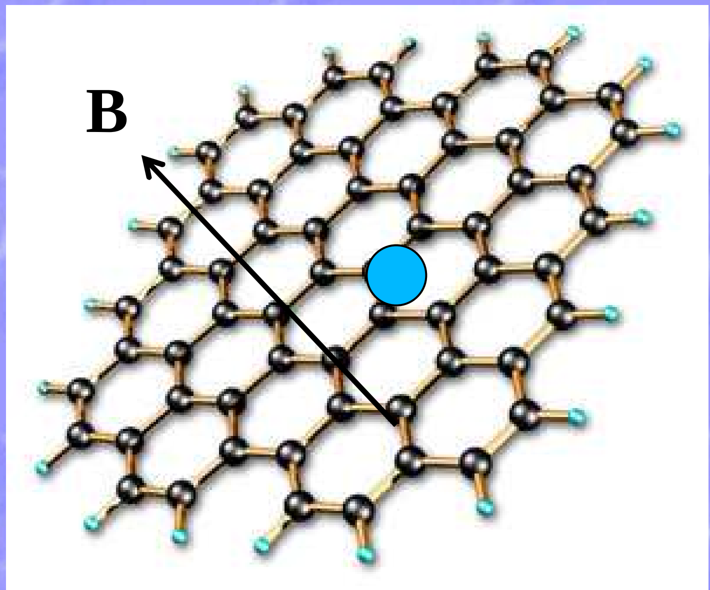
Aim of Project

• Single graphene sheet in a strong perpendicular magnetic field and single axially symmetric charged impurity
- corresponds to a low impurity density



High magnetic field approximation – reduce disorder broadening, so LLs well defined

• Want to find collective excitations localised on impurity

Single Particle Picture

- Single impurity at origin – symmetric gauge: **A**=1/2 **B** x **r**
- No intervalley scattering

$$\mathrm{H_D} = v_\mathrm{F}\sigma.\mathrm{p} = \hbar\frac{\sqrt{2}\mathrm{v_F}}{\ell_\mathrm{B}}\begin{pmatrix} 0 & -\frac{i}{\sqrt{2}}\left(\frac{z^\star}{2\ell_B} + 2\ell_B\partial_Z\right) \\ \frac{i}{\sqrt{2}}\left(\frac{z}{2\ell_B} - 2\ell_B\partial_{Z^\star}\right) & 0 \end{pmatrix} = \hbar\omega_c\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

- Corresponding eigensystem:

$$\Phi_{ns\Uparrow m}(\mathbf{r}) = 2^{\frac{1}{2}(\delta_{n,0}-1)}\begin{pmatrix} s_n\phi_{|n|-1\,m}(\mathbf{r}) \\ \phi_{|n|\,m}(\mathbf{r}) \end{pmatrix}\chi_s, \qquad \epsilon_n = \mathrm{sign}(n)\hbar\omega_c\sqrt{|n|}$$

Quantum numbers: *n* = …,-1,0,1,…, *m*=0,1,…, σ = ⇑,⇓, *s* = ↑,↓

- C.f. 2DEG

$$\epsilon_n = \hbar\frac{eB}{mc}\left(n+\frac{1}{2}\right)$$

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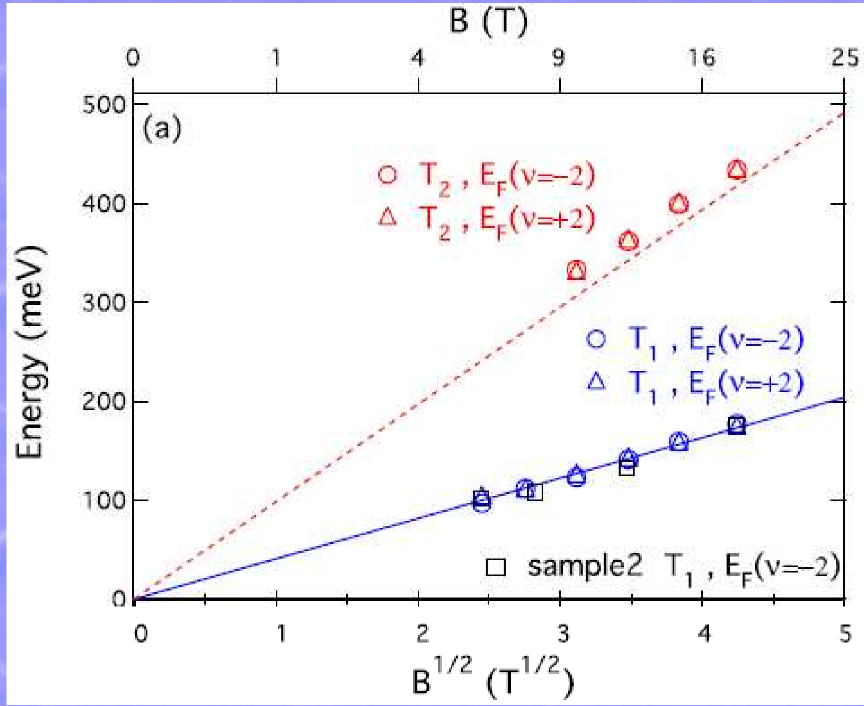
Self Energy Corrections to Cyclotron Resonance

- In 2DEG Kohn's theorem applies -CR independent of e-e interactions
- Doesn't hold for graphene – e-e interactions very important
- LL energy renormalised by self energy corrections due to exchange with electrons below Fermi level
- Corrections to CR diverge logarithmically with the cut off



Seen experimentally: Z. Jiang *et al.*, PRL **98**, 197403 (2007)

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Collective Excitations

$$Q^{\dagger}_{\mathcal{N}_1\mathcal{N}_2M_z} = \sum_{m_1,m_2=0}^{\infty} A_{\mathcal{N}_1\mathcal{N}_2M_z}(m_1,m_2)\, c^{\dagger}_{\mathcal{N}_1 m_1} d^{\dagger}_{\mathcal{N}_2 m_2}, \qquad A_{\mathcal{N}_1\mathcal{N}_2M_z}(m_1,m_2) \sim \delta_{M_z,|n_1|-m_1-|n_2|+m_2}$$

$c^{\dagger}$ creates electron

$d^{\dagger}$ creates hole

Renormalised single particle energies

Impurity interaction

$$\hat{H}^{\mathcal{N}_1'\mathcal{N}_2'}_{\mathcal{N}_1\mathcal{N}_2} = \sum_{m=0}^{\infty}\left(\tilde{\epsilon}_{\mathcal{N}_1} + \mathcal{V}_{\mathcal{N}_1 m}\right) c^{\dagger}_{\mathcal{N}_1 m} c_{\mathcal{N}_1 m} - \sum_{m=0}^{\infty}\left(\tilde{\epsilon}_{\mathcal{N}_2} + \mathcal{V}_{\mathcal{N}_2 m}\right) d^{\dagger}_{\mathcal{N}_2 m} d_{\mathcal{N}_2 m} + \sum_{\substack{m_1,m_2\\ m_1',m_2'}} \left(\mathcal{W}^{\mathcal{N}_2 m_2 \mathcal{N}_1' m_1'}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'} - \mathcal{W}^{\mathcal{N}_1' m_1' \mathcal{N}_2 m_2}_{\mathcal{N}_1 m_1 \mathcal{N}_2' m_2'}\right) c^{\dagger}_{\mathcal{N}_1' m_1'} d^{\dagger}_{\mathcal{N}_2' m_2'} d_{\mathcal{N}_2 m_2} c_{\mathcal{N}_1 m_1}$$

Dynamical exchange interaction

Direct electron-hole attraction

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Connection to 2DEG

$$\mathcal{W}^{\mathcal{N}_1' m_1' \mathcal{N}_2' m_2'}_{\mathcal{N}_1 m_1 \mathcal{N}_2 m_2} \equiv \langle \Phi_{\mathcal{N}_1' m_1'} \Phi_{\mathcal{N}_2' m_2'} | U_{ee} | \Phi_{\mathcal{N}_1 m_1} \Phi_{\mathcal{N}_2 m_2} \rangle = \delta_{s_1,s_1'} \delta_{\sigma_1,\sigma_1'} \delta_{s_2,s_2'} \delta_{\sigma_2,\sigma_2'} \mathcal{U}^{n_1' m_1'\, n_2' m_2'}_{n_1 m_1\, n_2 m_2}$$

$$\mathcal{U}^{n_1' m_1'\, n_2' m_2'}_{n_1 m_1\, n_2 m_2} = 2^{\frac{1}{2}\left(\delta_{n_1 0} + \delta_{n_1' 0} + \delta_{n_2 0} + \delta_{n_2' 0} - 4\right)} \Big[U^{|n_1'|\, m_1'\;\; |n_2'|\, m_2'}_{|n_1|\, m_1\;\; |n_2|\, m_2} + s_{n_1} s_{n_1'} U^{|n_1'|-1\, m_1'\;\; |n_2'|\, m_2'}_{|n_1|-1\, m_1\;\; |n_2|\, m_2}$$
$$+ s_{n_2} s_{n_2'} U^{|n_1'|\, m_1'\;\; |n_2'|-1 m_2'}_{|n_1|\, m_1\;\; |n_2|-1\, m_2} + s_{n_1} s_{n_2} s_{n_1'} s_{n_2'} U^{|n_1'|-1\, m_1'\;\; |n_2'|-1\, m_2'}_{|n_1|-1\, m_1\;\; |n_2|-1\, m_2} \Big]$$

$$\mathcal{V}_{nm} = 2^{\frac{1}{2}(\delta_{n0}-1)} \left(V_{|n| m} + s_n V_{|n|-1 m} \right)$$

Examples

$$\mathcal{V}_{0m} = V_{0m} = -(2m-1)!!/2^m m!$$

$$\mathcal{V}_{1m} = \frac{1}{2}(V_{0m} + V_{1m}) = (8m-3)/(8m-4) V_{0m}$$

Energy units

$$E_0 = \sqrt{\frac{\pi}{2}} \frac{e^2}{\varepsilon \ell_B}$$

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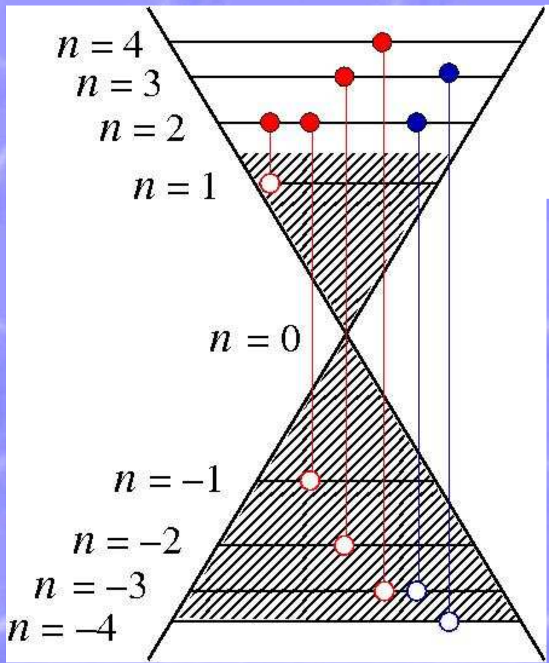
Which transitions are allowed?

In the dipole approximation, contribution to Hamiltonian from incoming circularly polarised light is

$$\delta H_{\pm} = \frac{ev_F\mathcal{E}}{i\omega c} e^{-i\omega t} \begin{pmatrix} \sigma_{\pm} & 0 \\ 0 & \sigma_{\mp} \end{pmatrix}$$

$\sigma_{\pm} = \sigma_x \pm i\sigma_y$ corresponds to left and right circularly polarised light



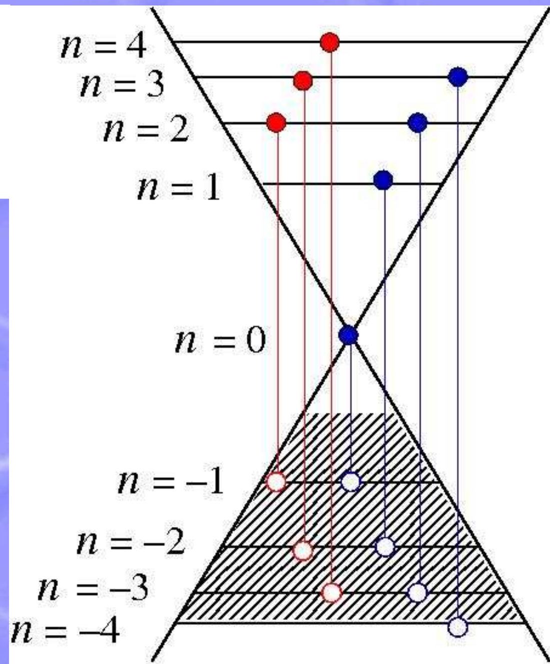
σ^+ σ^-

$|n_{\text{final}}| - |n_{\text{initial}}| = \pm 1$

$m_{\text{final}} = m_{\text{initial}}$

$\sigma_{\text{final}} = \sigma_{\text{initial}}$

$s_{\text{final}} = s_{\text{initial}}$



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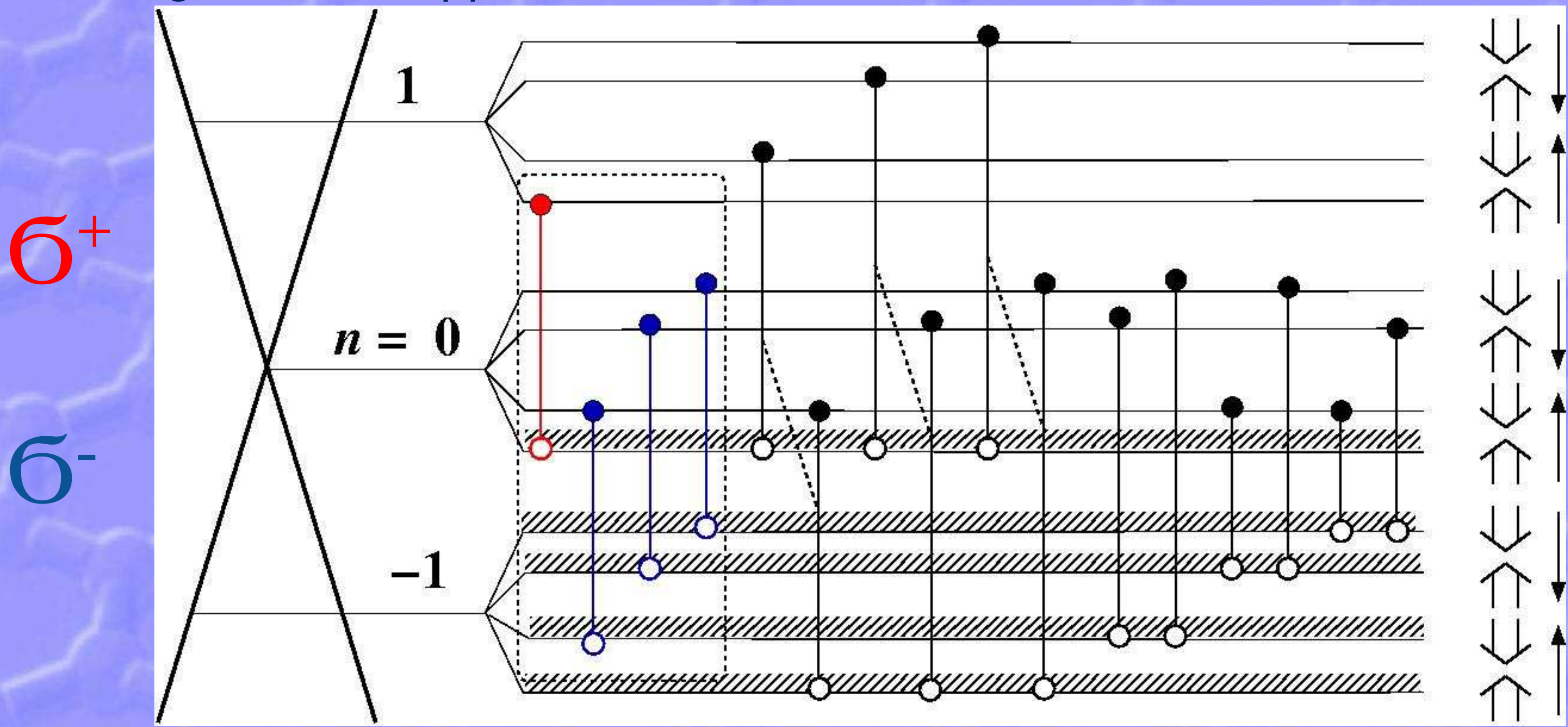
Which transitions should we take into account?

- Do calculations for sublevel filling factors of the zeroth LL.
- Infinitely many transitions with same M_z are mixed by Coulomb interactions
- Only consider mixing for those with the same energies – strong magnetic field approx



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Numerical Considerations

- Matrix comes in 4x4 blocks
- Need to truncate at finite m – ok, because only seek localised states
- Size limit imposed by calculating matrix elements with high order Laguerre polynomials, not by diagonalisation
- Use 4x50=200 basis elements
- Results stable wrt changing matrix size

In the Absence of an Impurity

A. Iyengar *et al*. PRB **75**, 125430 (2007); Yu. A. Bychkov and G. Martinez, PRB **77**, 125417 (2008)

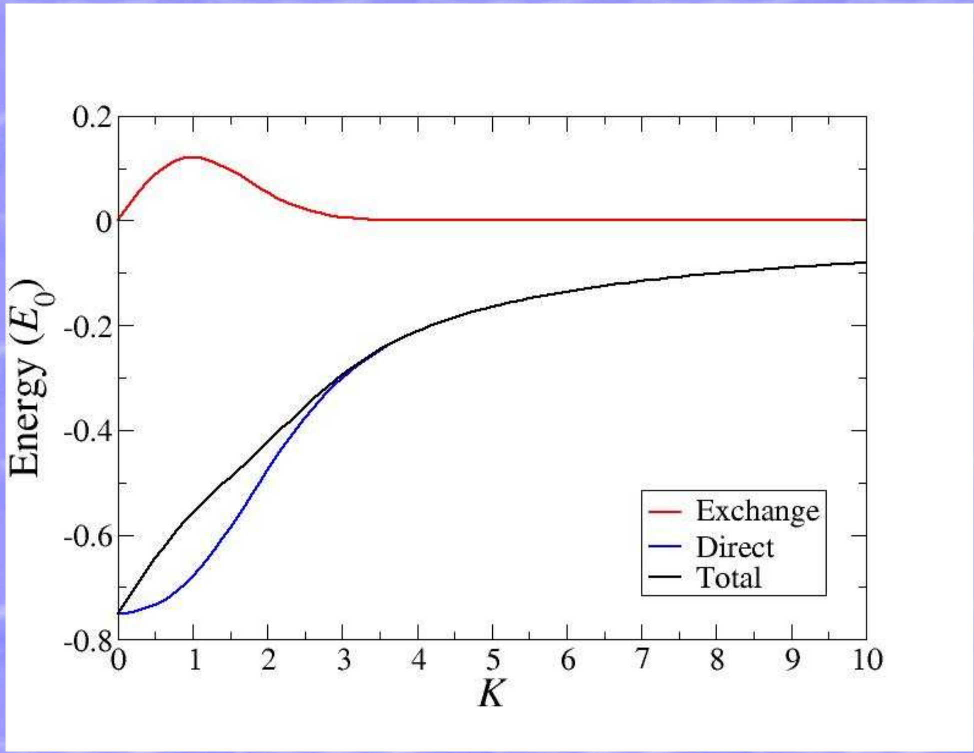
- Use Landau gauge

$$\mathbf{A} = -\mathbf{B}y\hat{\mathbf{x}}$$

- $m \rightarrow k_y$

- Thickness of band is $0.75E_0$ with

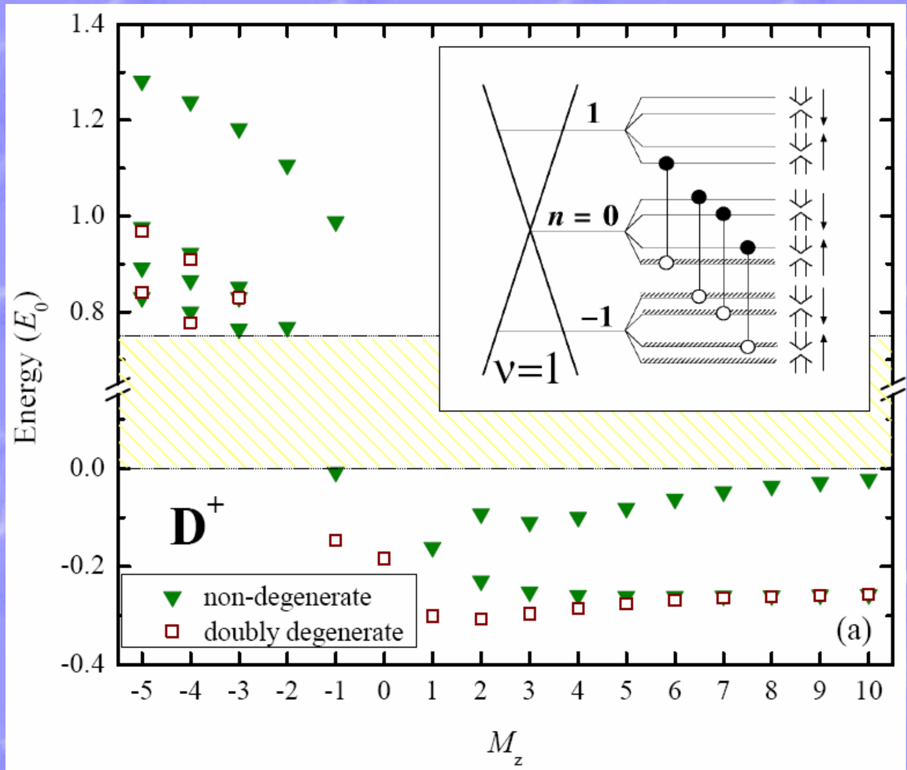
$$E_0 = \sqrt{\frac{\pi}{2}}\frac{e^2}{\varepsilon \ell_B}$$

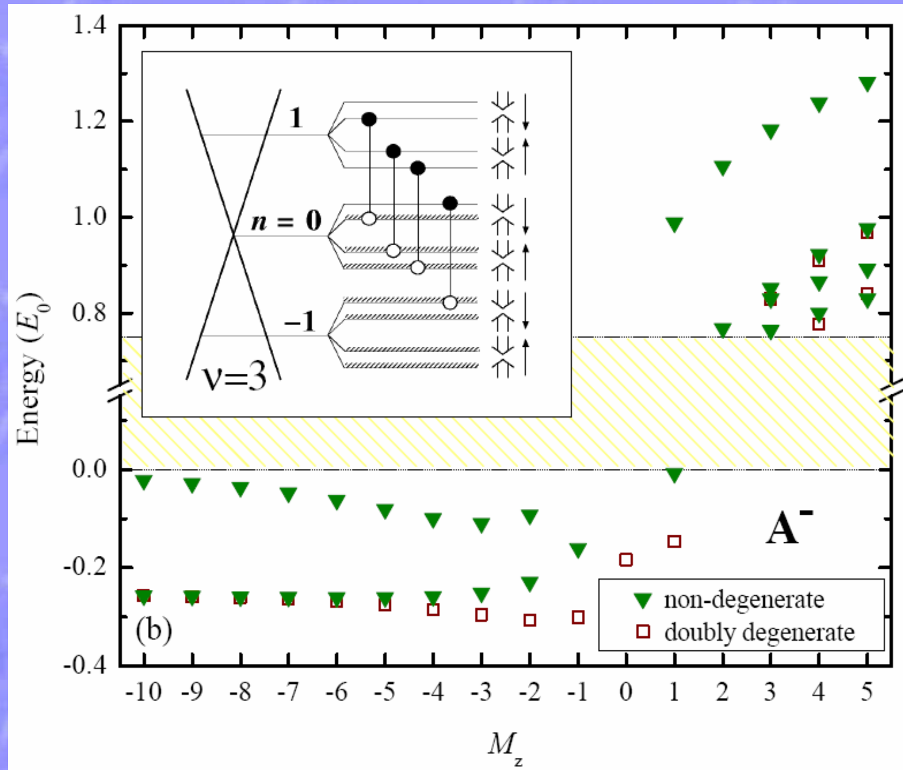


Results

Symmetry relation: Results same under

$M_z \rightarrow -M_z$, $\nu \rightarrow \nu$-4, $D^+ \rightarrow A^-$





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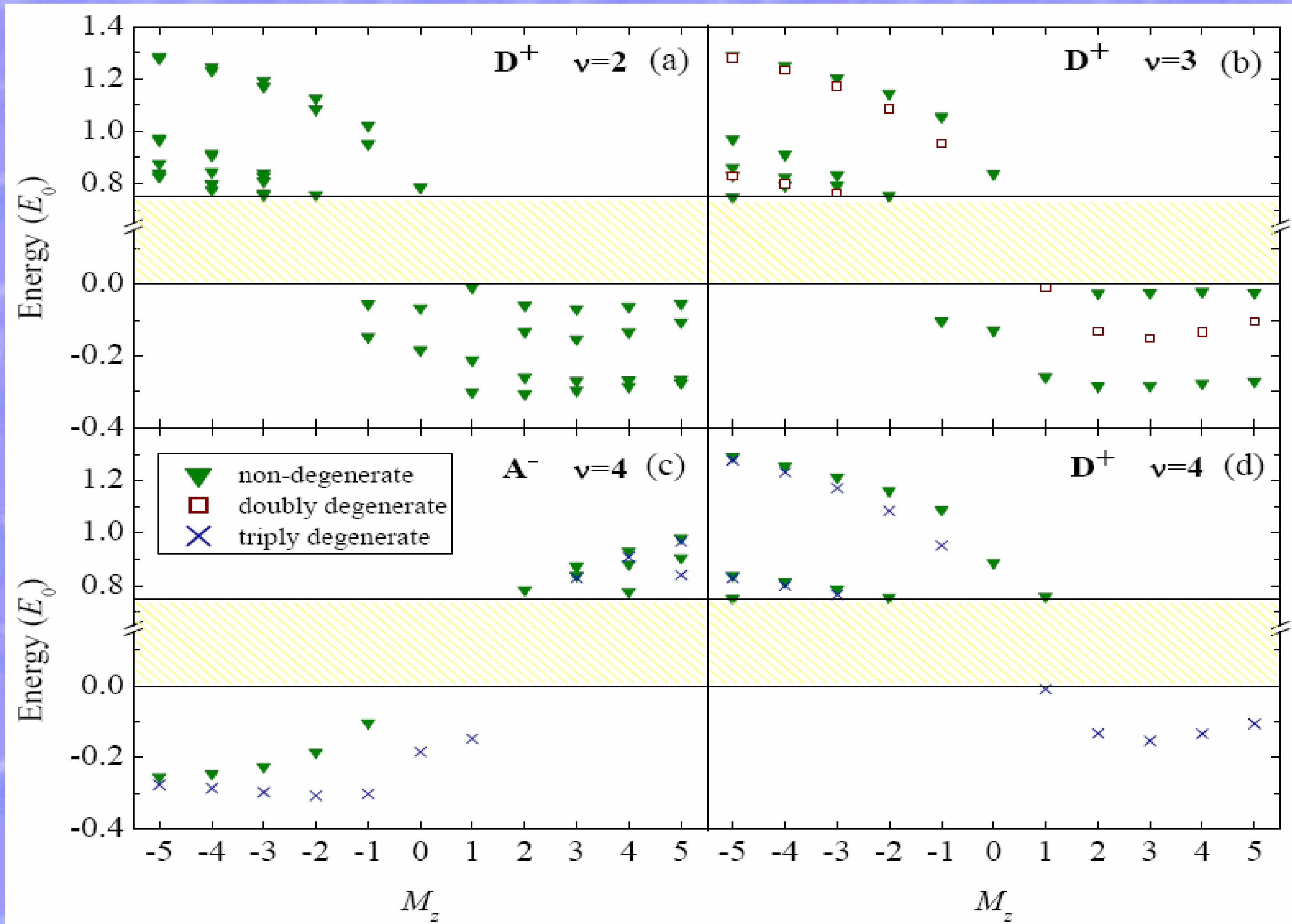
Evolution with Filling Factor



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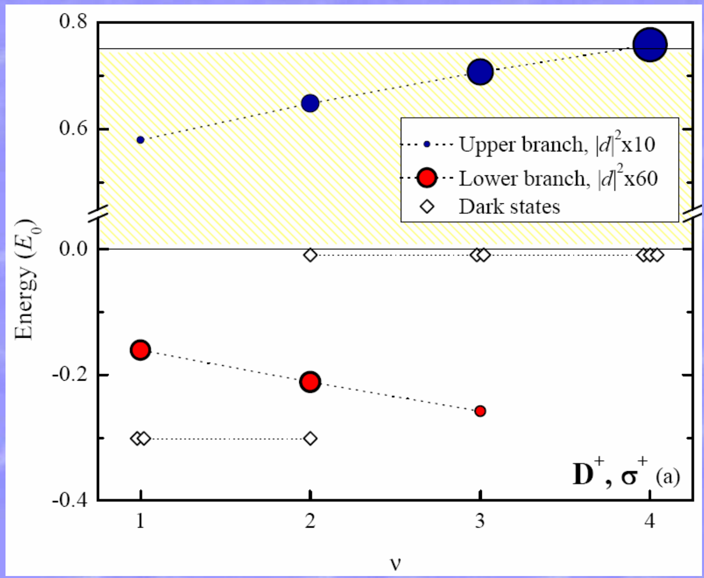
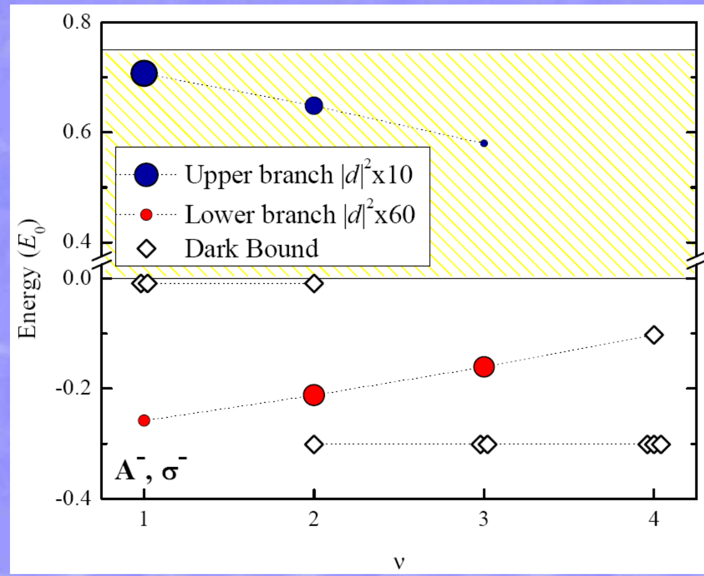
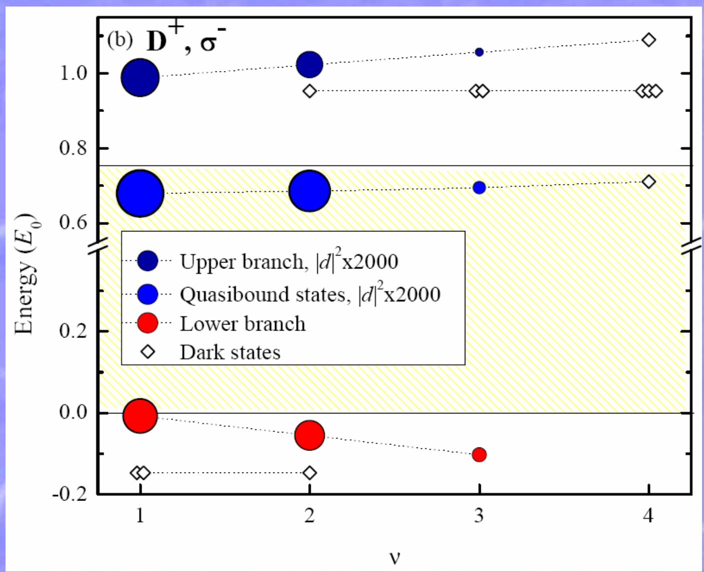
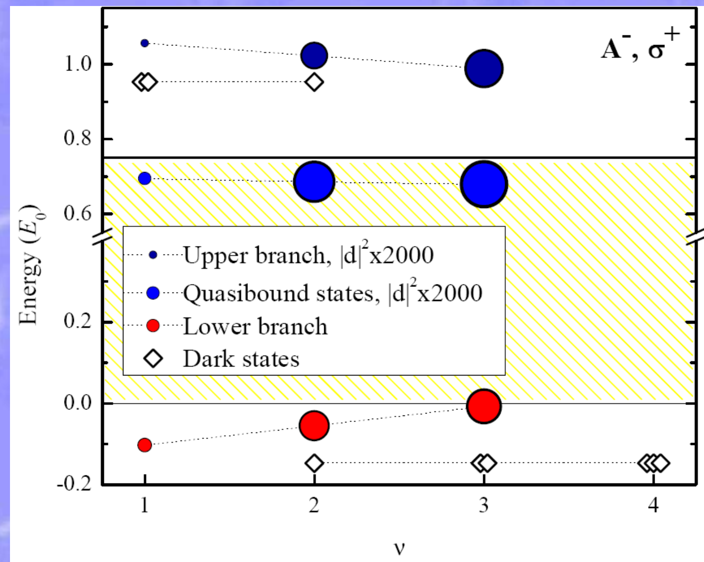


OPTICS







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Summary/Outlook

- Used a secondary quantised approach to determine collective excitations bound on an impurity in a single sheet of graphene in the presence of a strong perpendicular magnetic field **[arXiv:0902.4176]**
- Could our results be experimentally detected?

  Only know relative strengths

  Could think about:

  - rippling in graphene layer

  - intervalley scattering

  - nature of impurity
- First step towards combining work on magnetoplasma in pristine graphene and disordered graphene
- Could they enhance understanding of the nature of disorder in graphene?
- Further work:

  Non integer filling factors: consider opposite limit of low electron density, so D- states are formed.

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Thank you for listening

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Other work:

- Anderson transition in BCC and FCC lattices using transfer-matrix method [PRB **77**, 245117-8 (2008)]
- Excitonic storage in an Aharonov-Bohm nanoring with applied in-plane electric field [PRL **102**, 096405 (2009)]

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