

Cracking me softly:

The mechanics of hyper elastic Kirigami structures

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School of Engineering *and* Materials Science

Division of Engineering Science

Fracture @QMUL

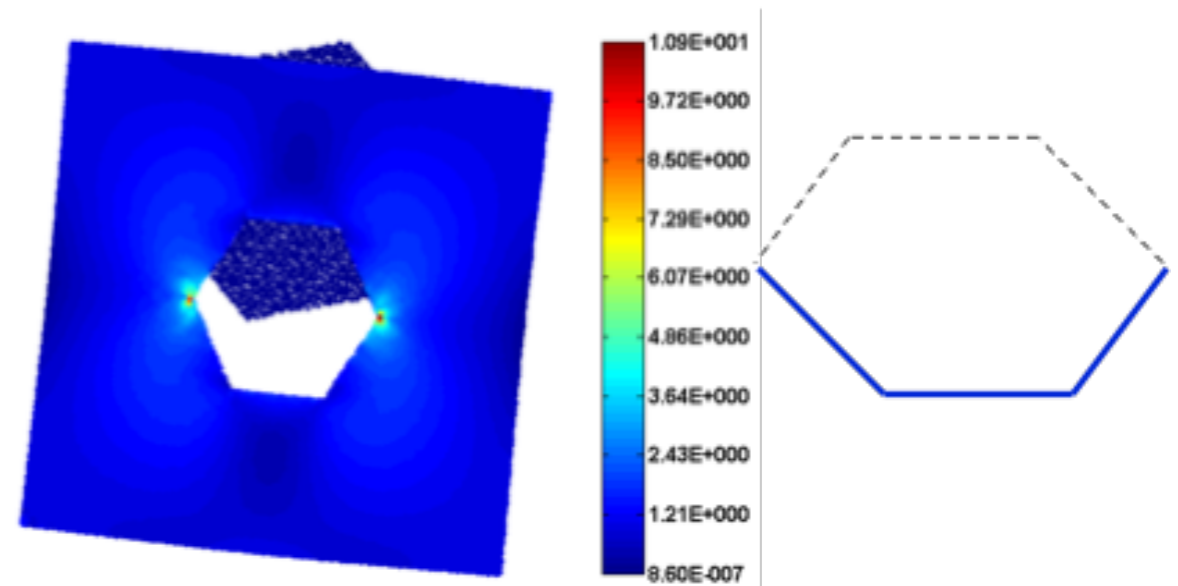
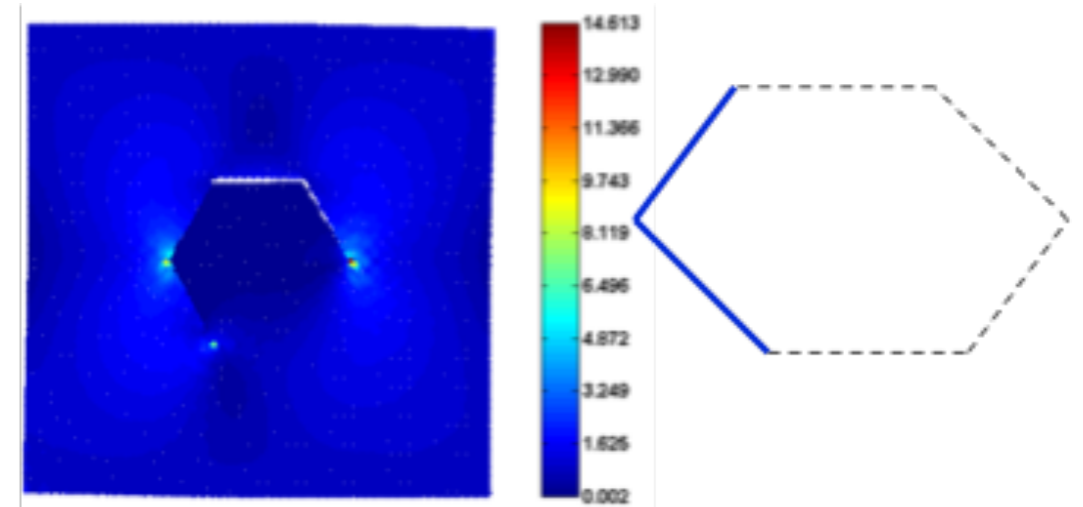
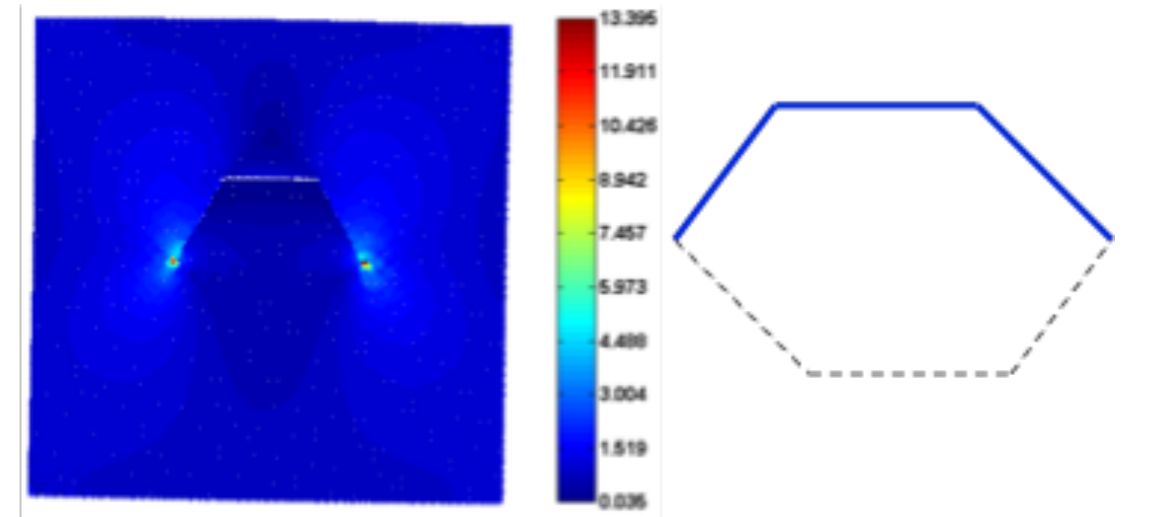


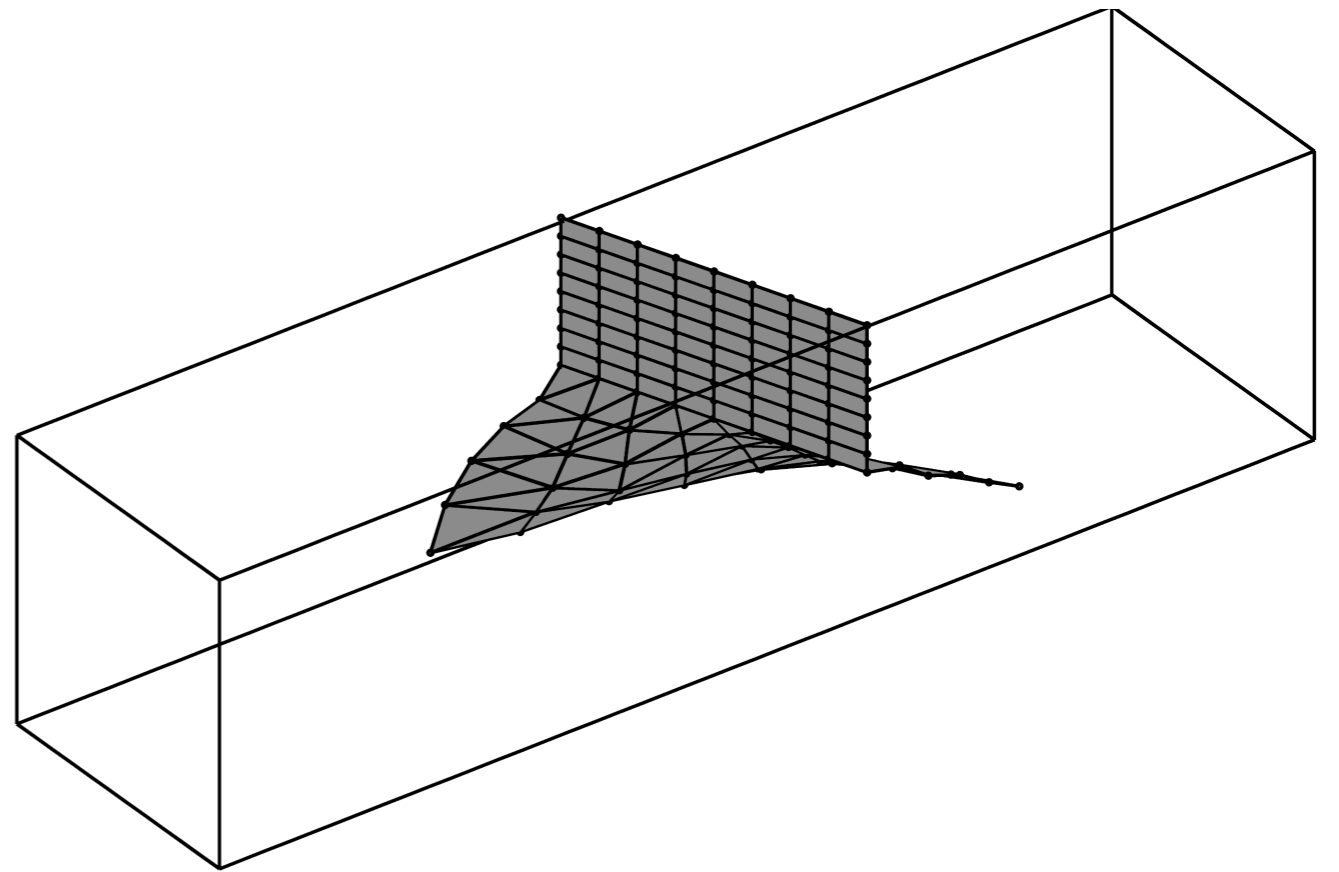
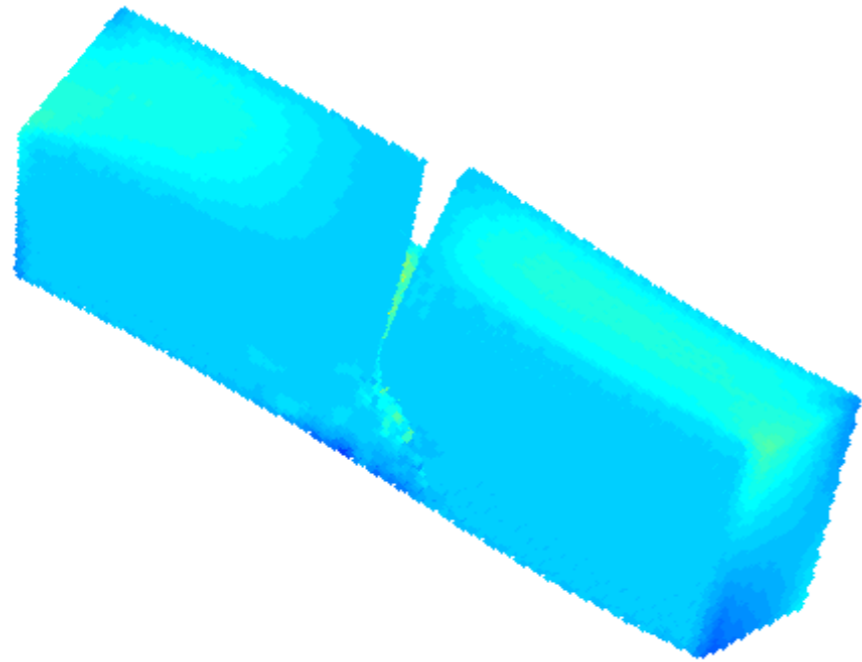


Cutting

Automatic intersection of crack paths leading to fragment generation

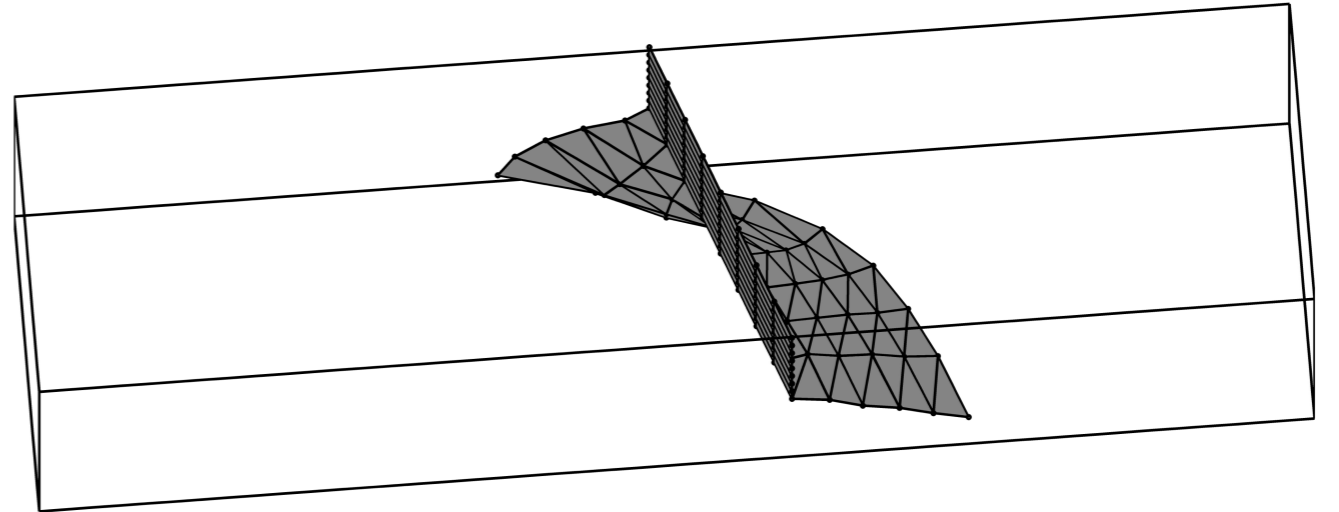
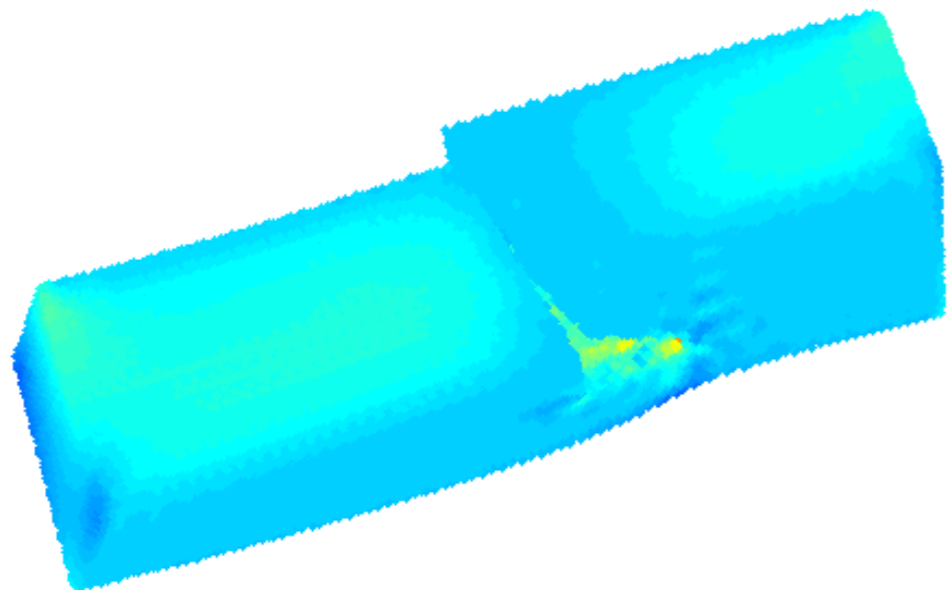
Multiple intersecting cracks

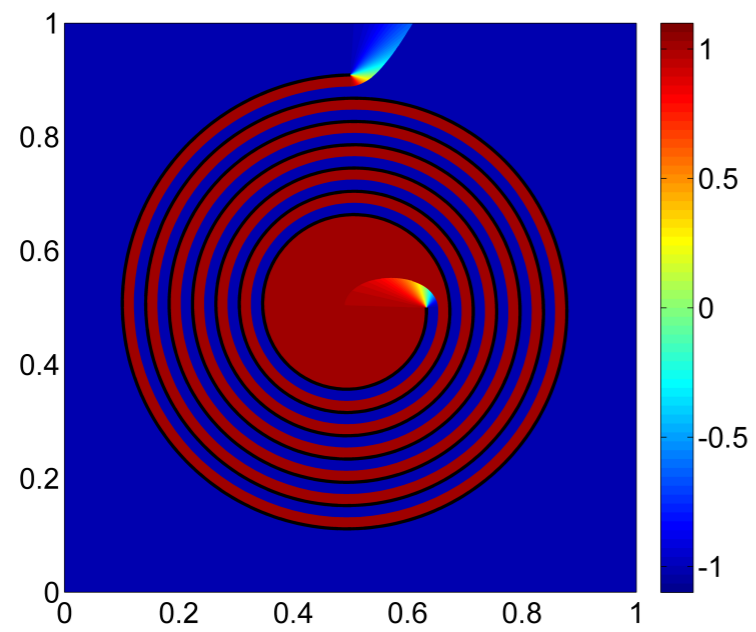




Crack Propagation

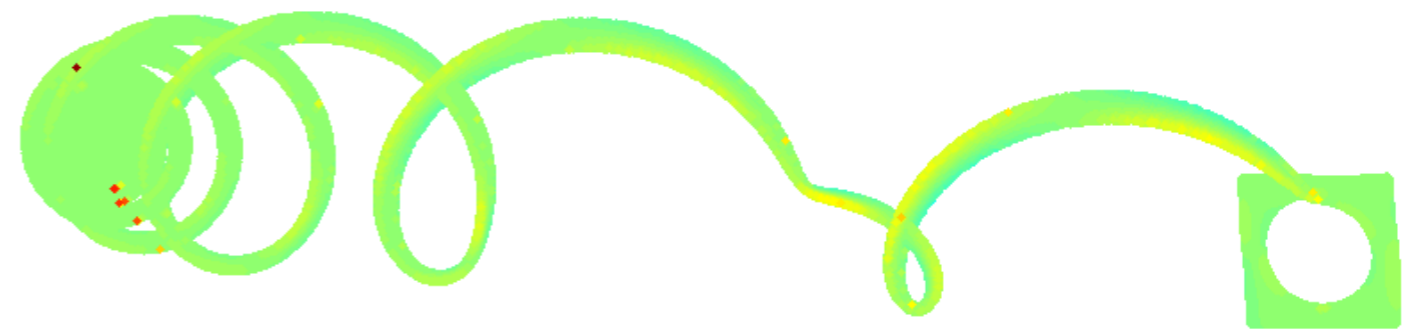
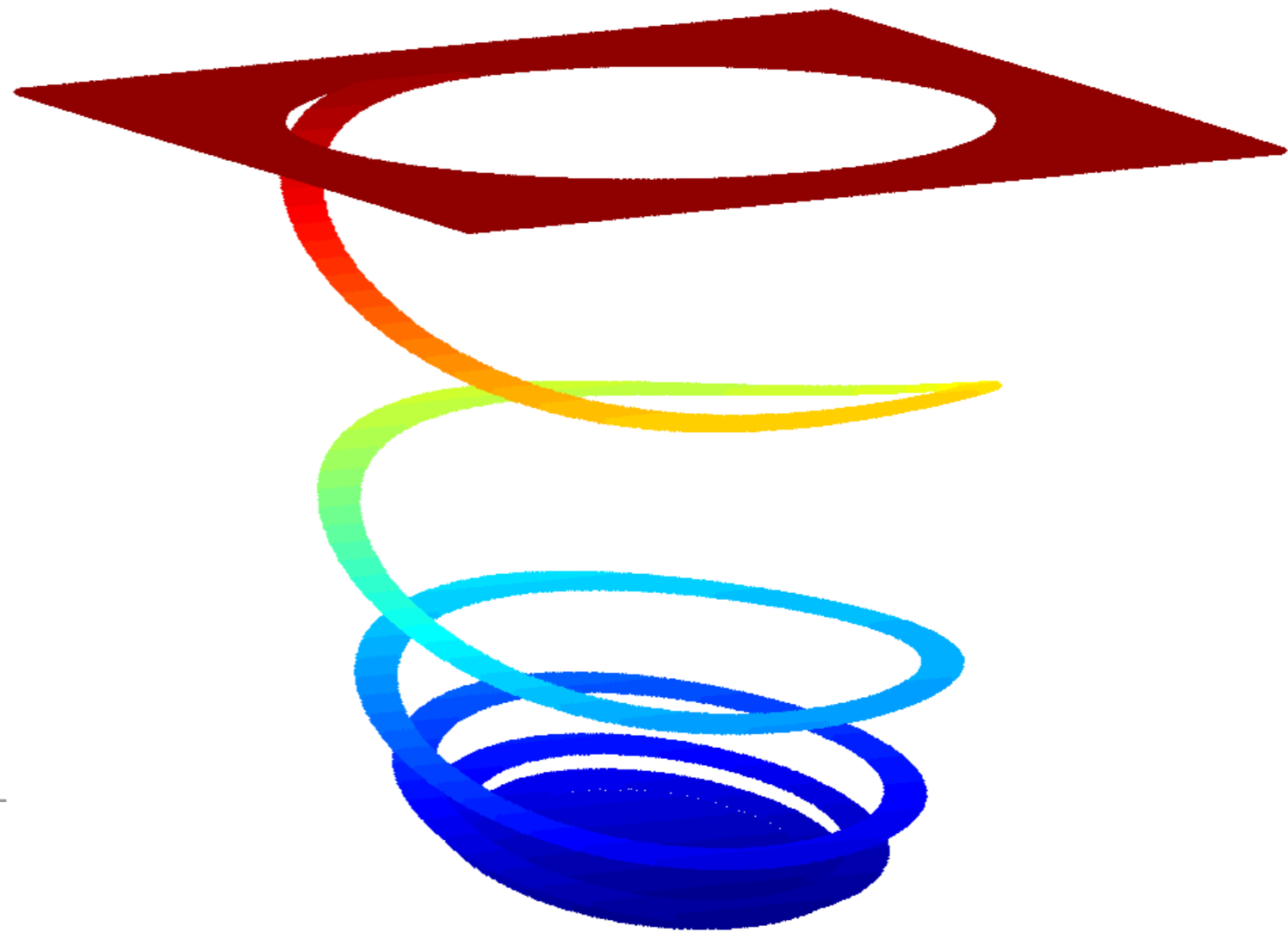
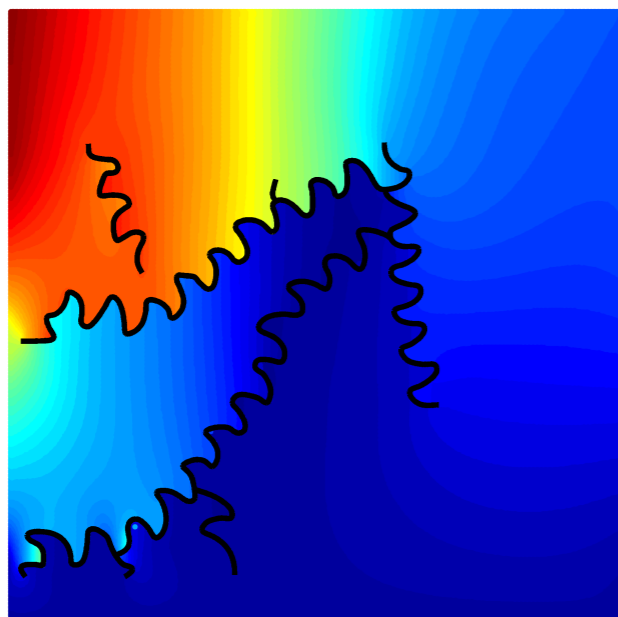
Three-dimensional effects, such as curvature and twisting





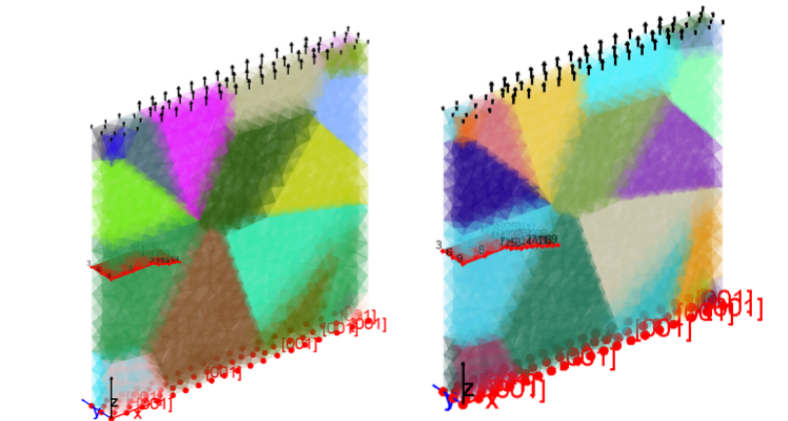
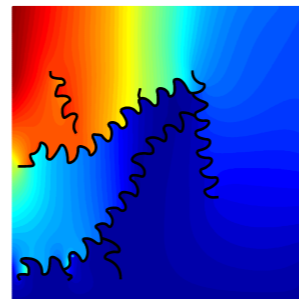
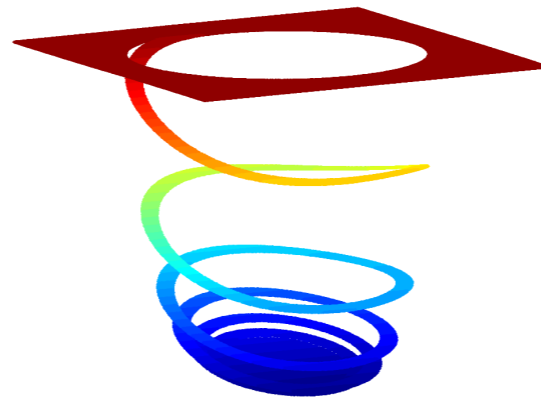
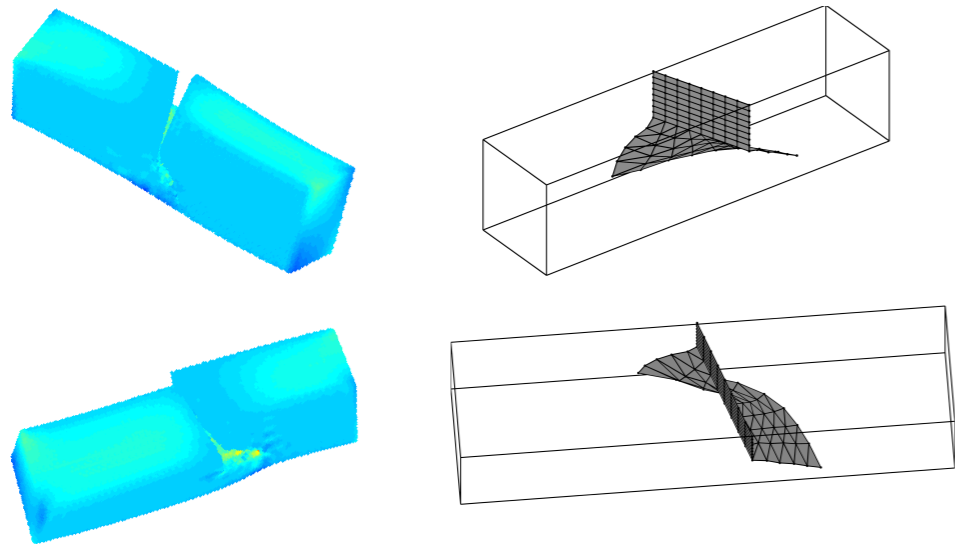
Crack Propagation

Extremely curved crack paths



with Rubén Sevilla, Swansea

Fracture @QMUL



High strain-rate fracture for ceramic armours: a multiscale study
Grant: CDE34762



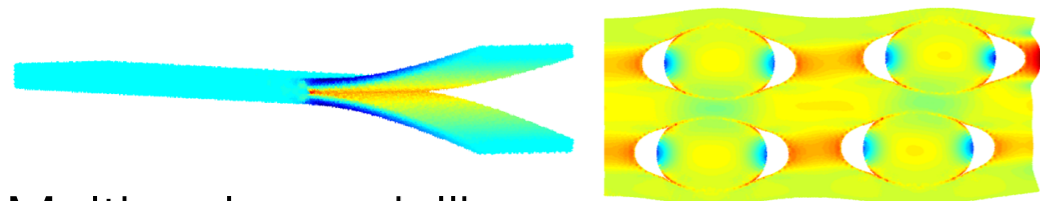
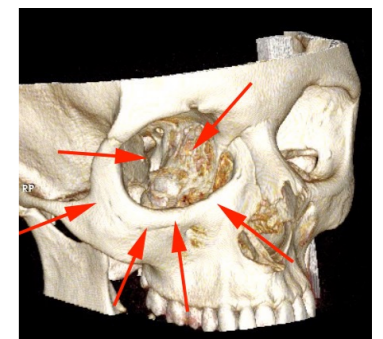
Three-dimensional fracture surfaces in metals

Extremely Curved Cracks:

- study of biological materials
- toughening engineering materials



Craniofacial Trauma



Multiscale modelling of composites in underwater structures



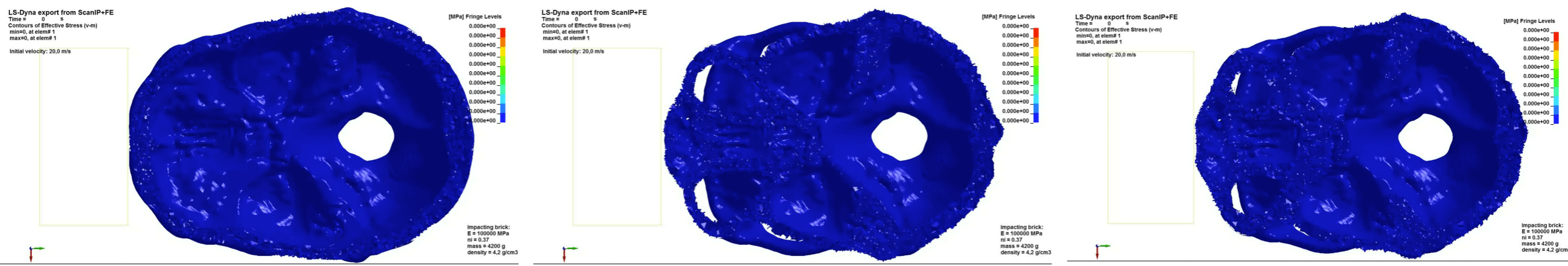
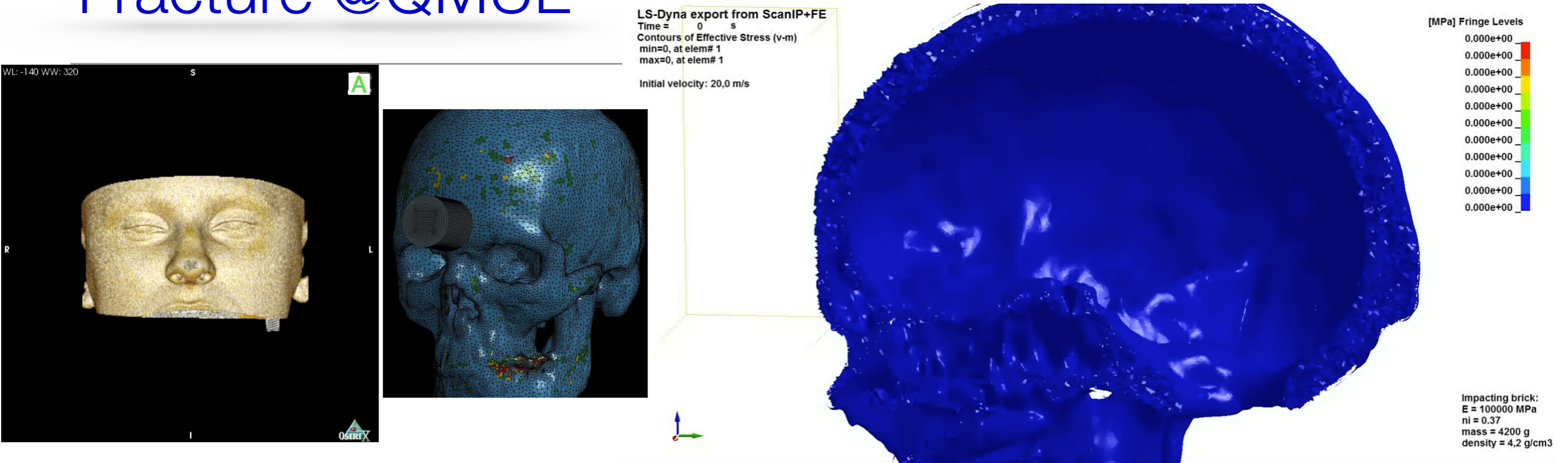
High strain-rate behaviour of rubber suspensions



Fracture @QMUL



Fracture @QMUL

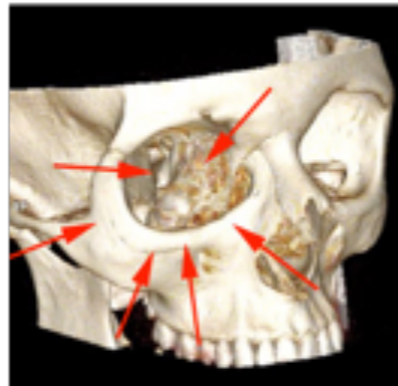


With: Prof Simon Holmes (Royal London Hospital)
Mattia Gaglione

Fracture @QMUL



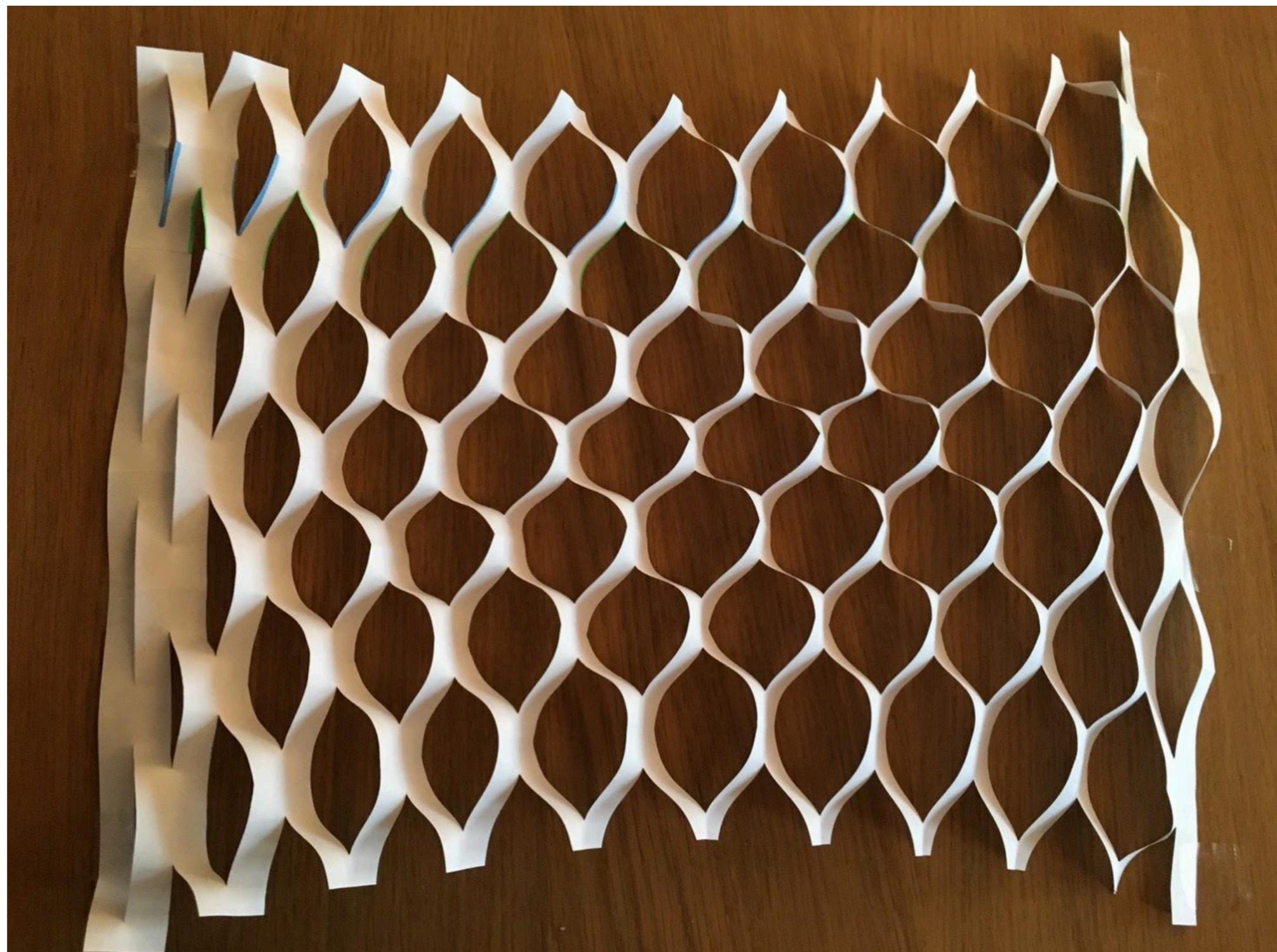
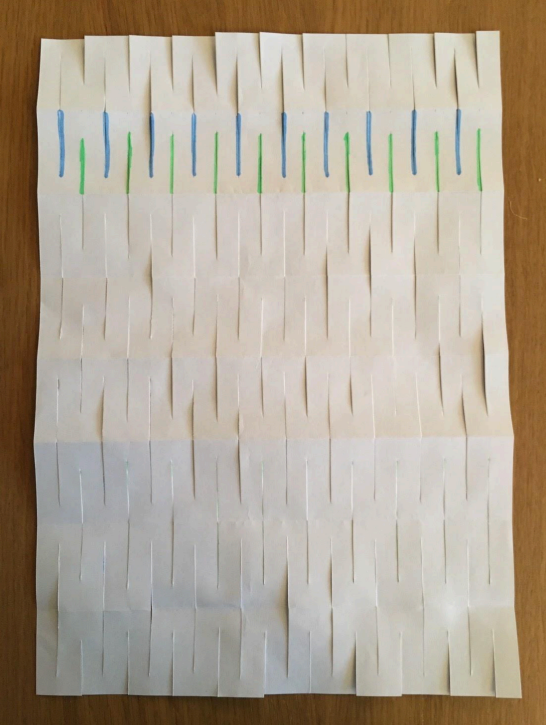
Barts Health 
NHS Trust



ScienceDaily[®]



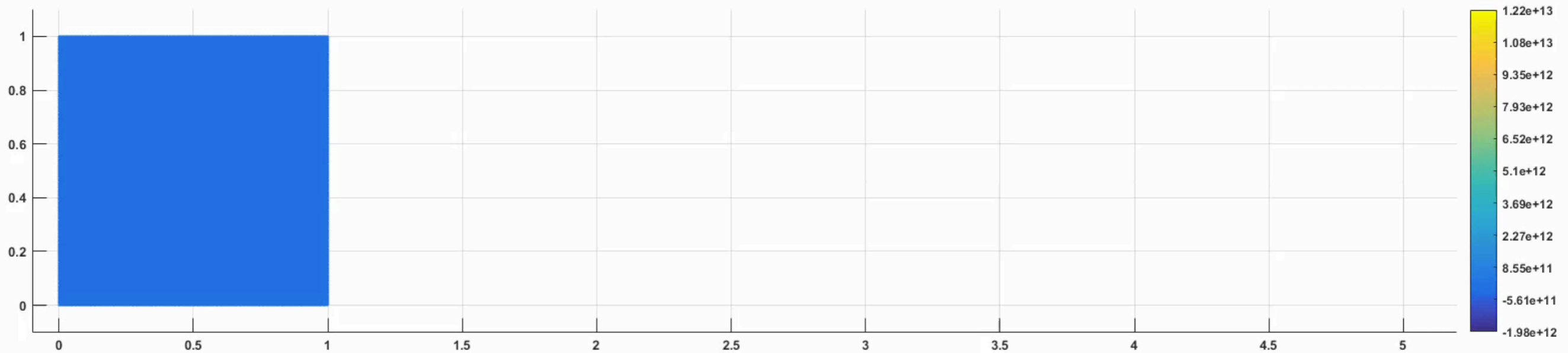
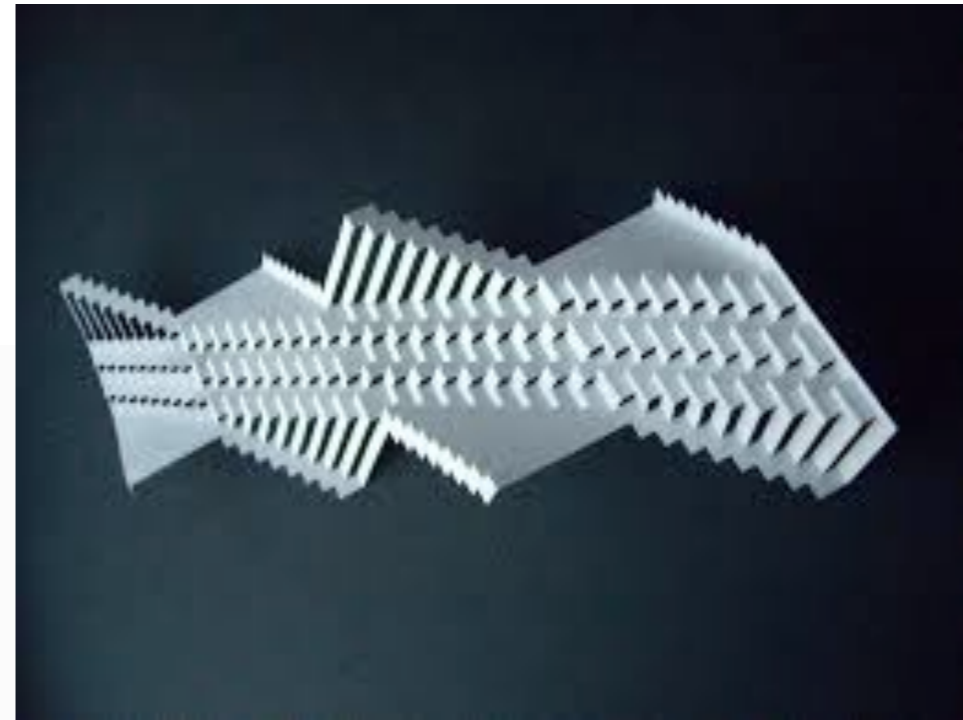
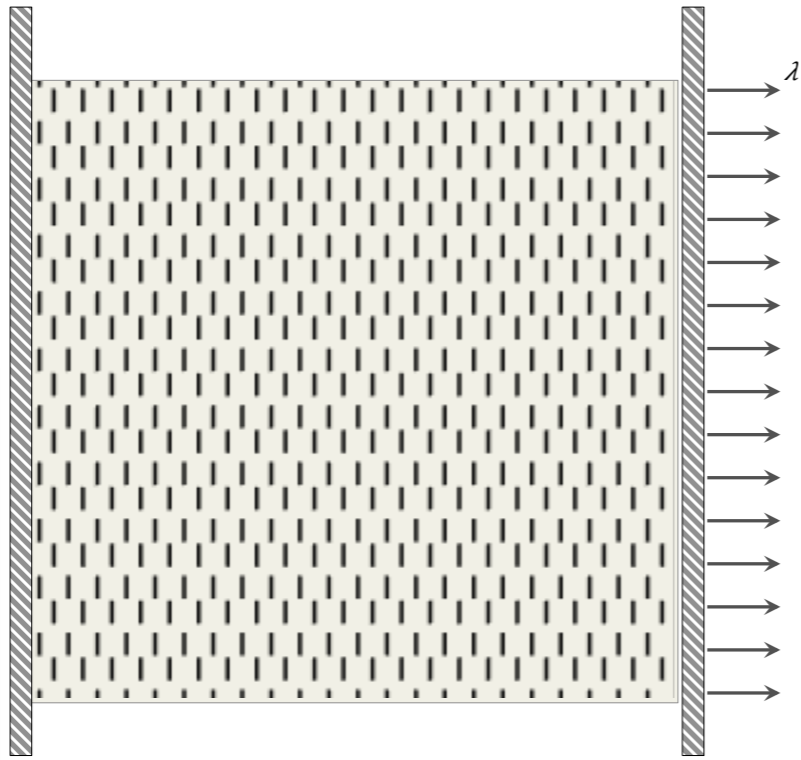
Rudolph's antlers inspire next generation of unbreakable materials

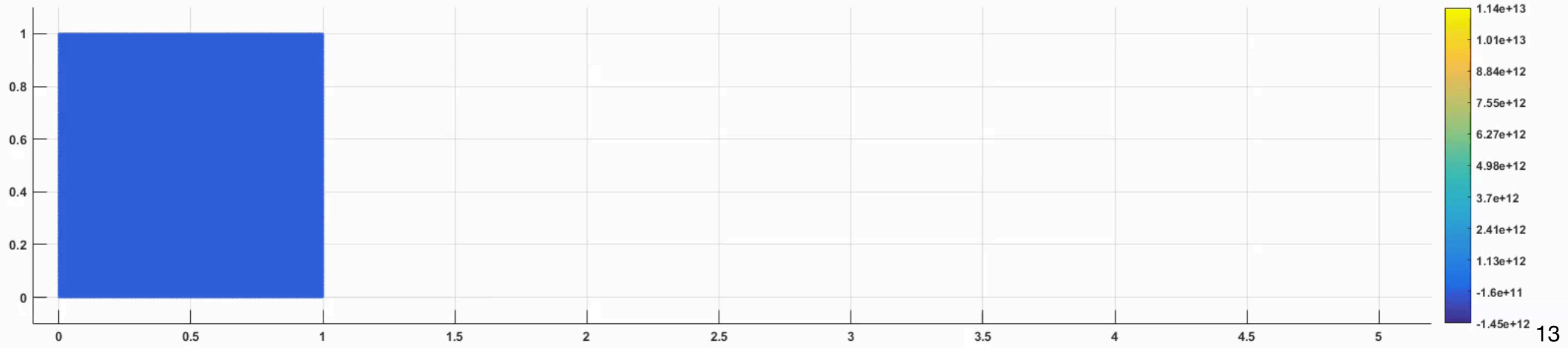
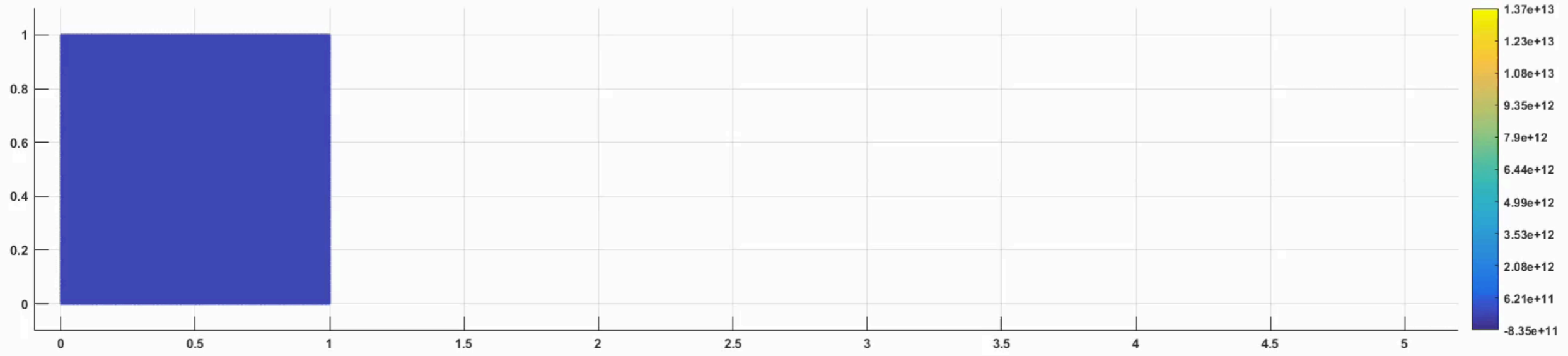


Kirigami

(photo courtesy of artist Mizuho Ozaki)

Kirigami Structures





Arc-Length Solver

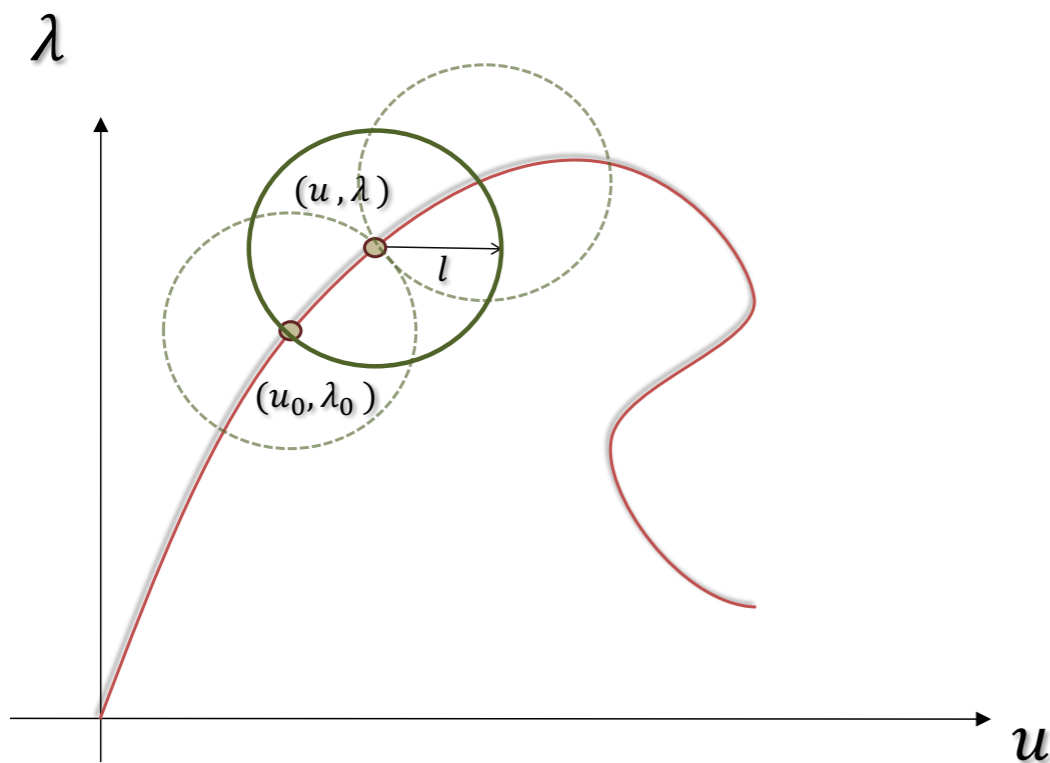
When?

Fracture

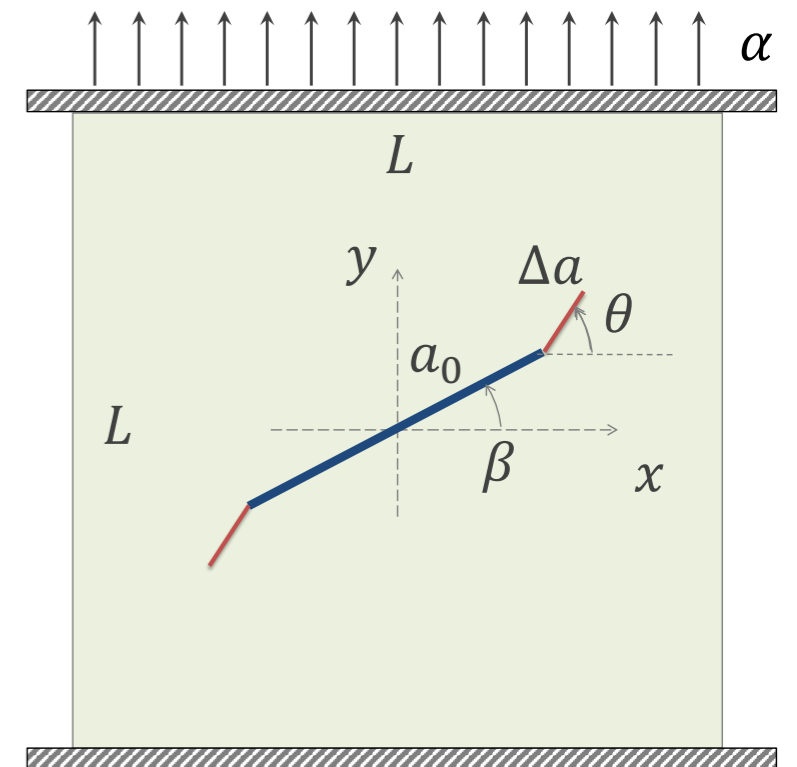
Where?

$$\mathbf{F}^{(i)}(\mathbf{d}) - \lambda \mathbf{F}^{(e)} = 0$$

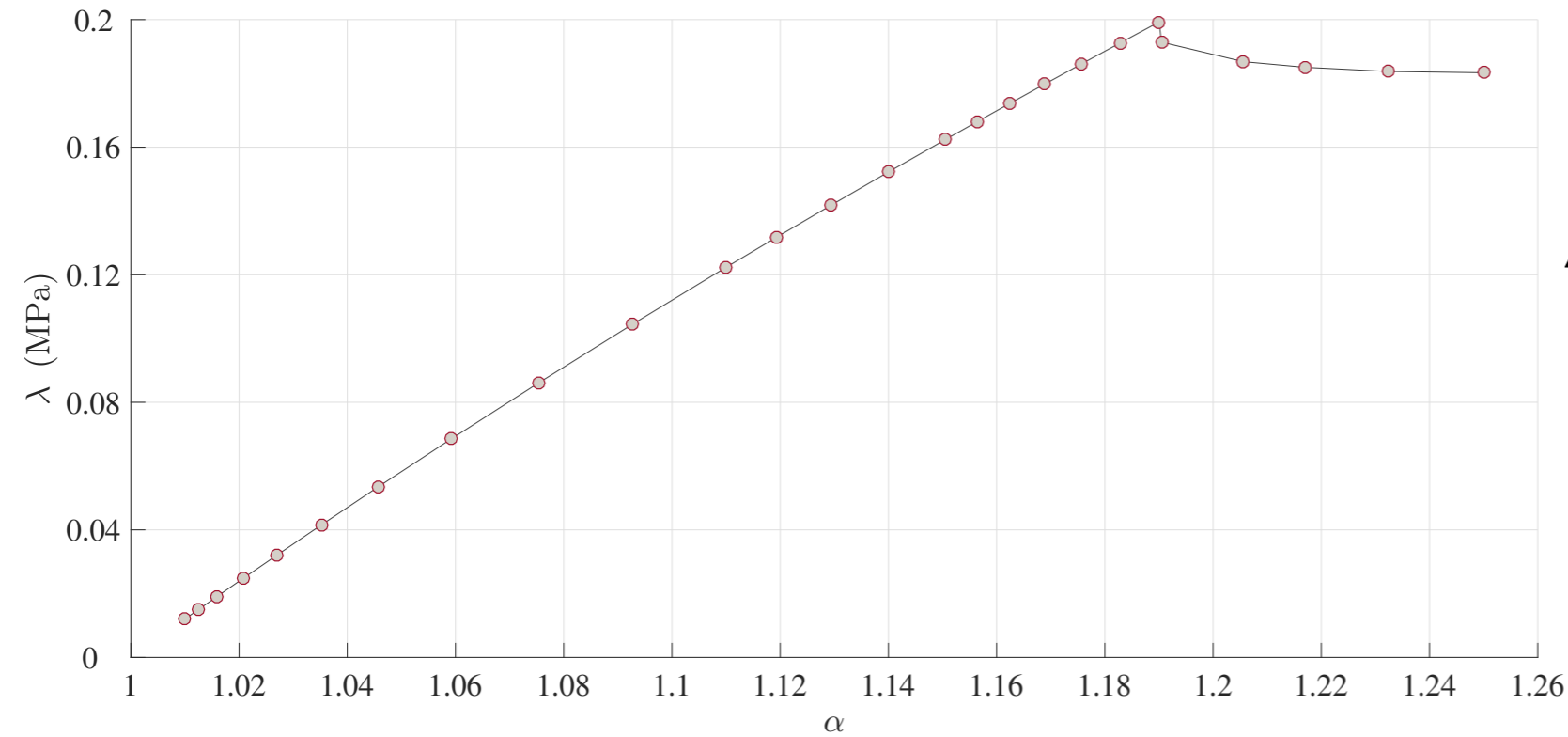
How?



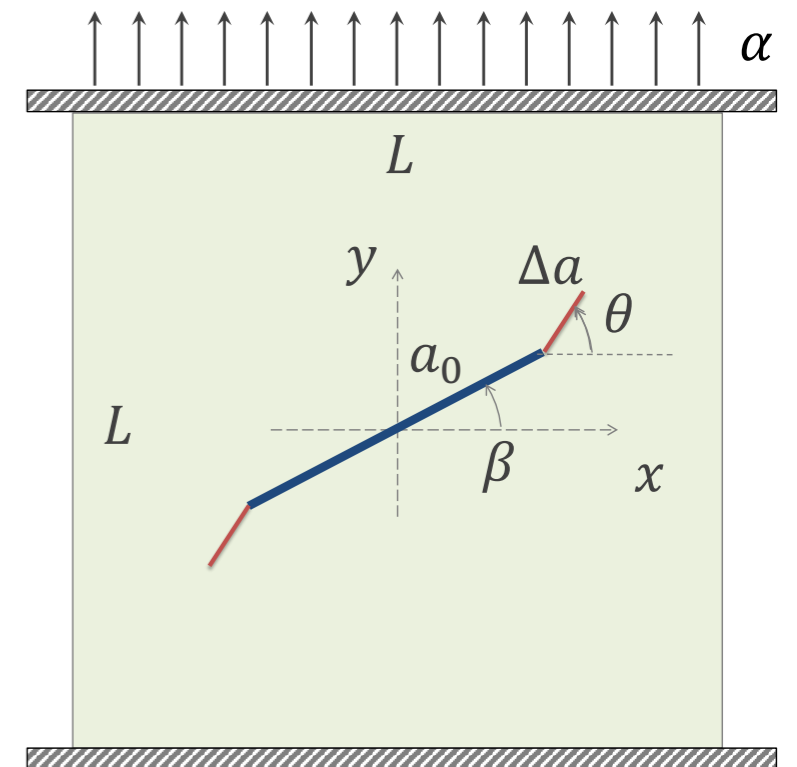
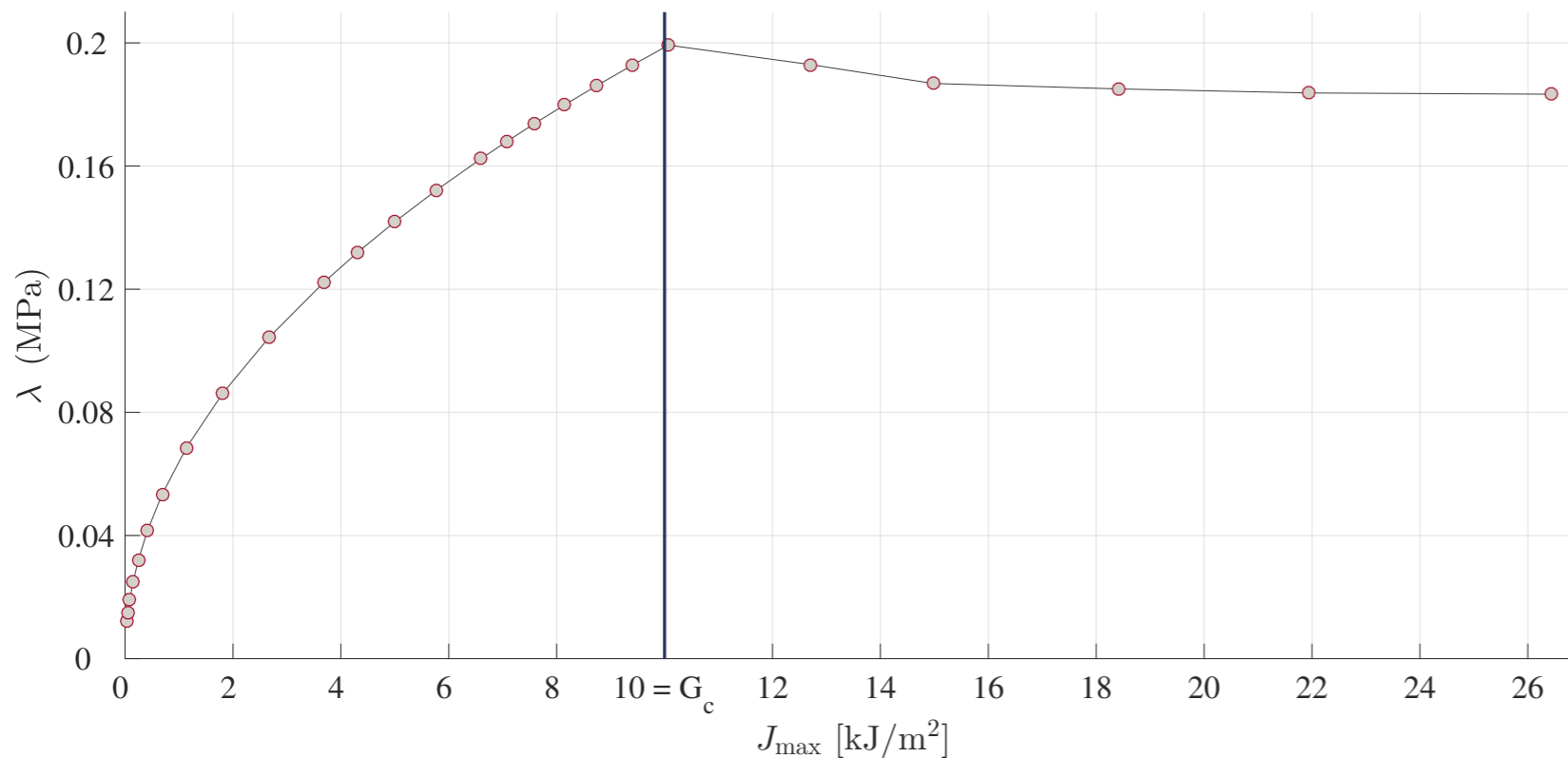
$$\Delta \mathbf{d}^T \Delta \mathbf{d} + \psi^2 \Delta \lambda^2 \mathbf{F}^{(e)T} \mathbf{F}^{(e)} - l^2 = 0$$



Arc-Length Solver



$$\Delta \mathbf{d}^T \Delta \mathbf{d} + \psi^2 \Delta \lambda^2 \mathbf{F}^{(e)T} \mathbf{F}^{(e)} - l^2 = 0$$



Arc-Length Solver

When?

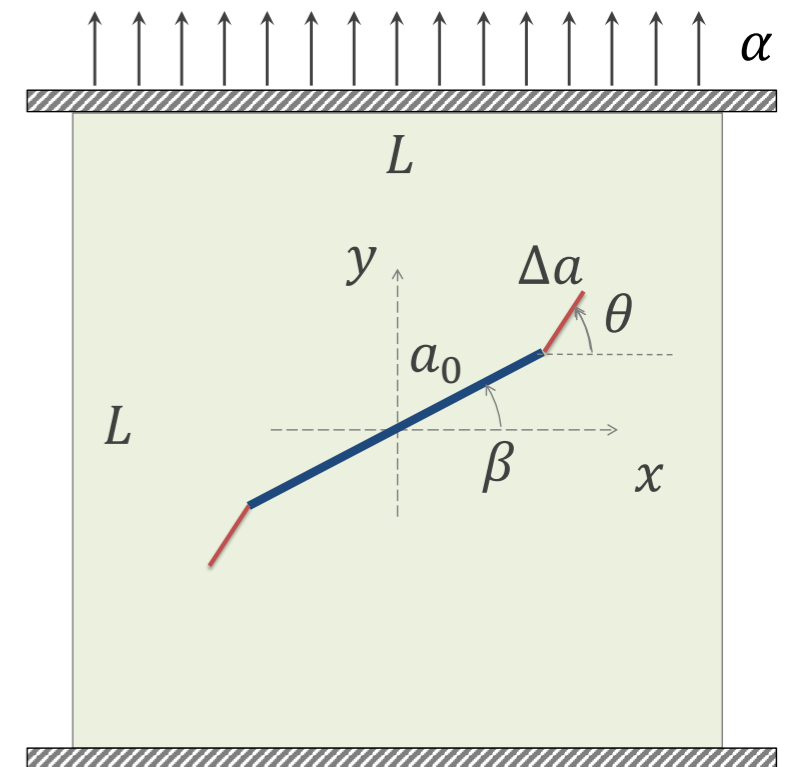
Where?

Fracture

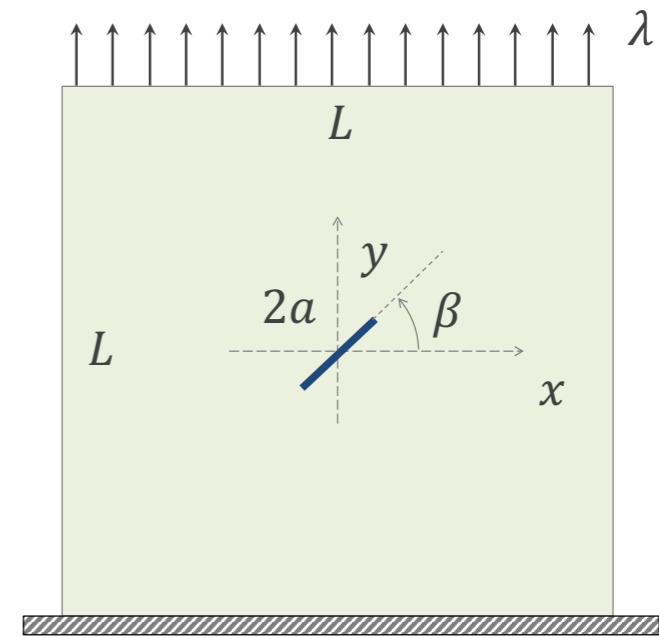
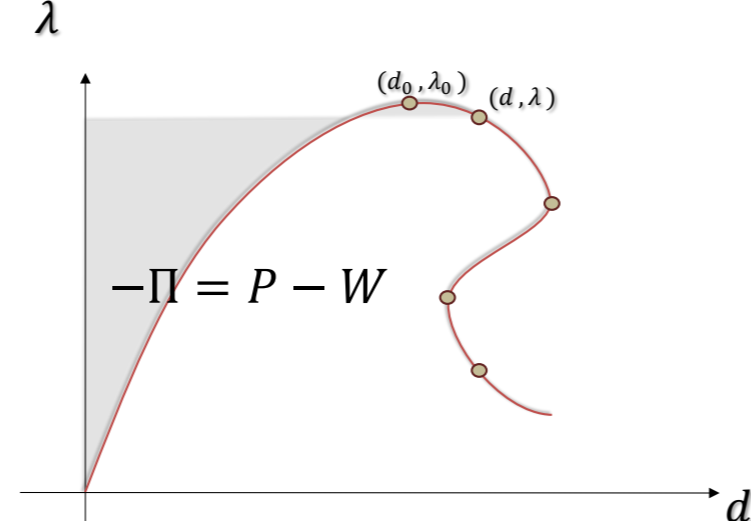
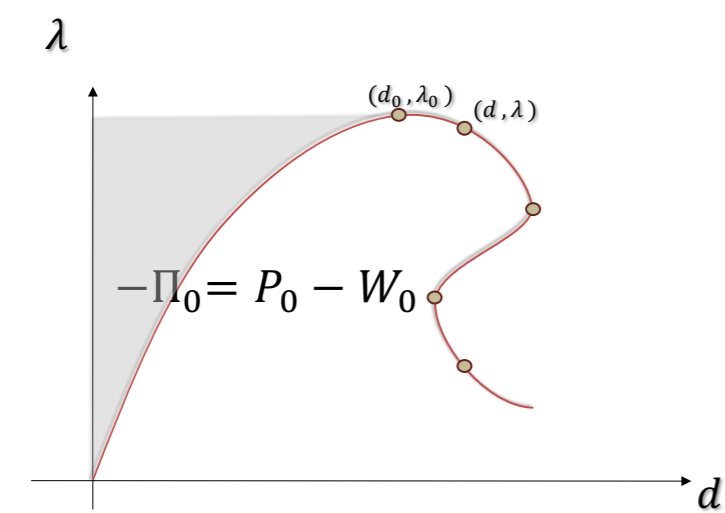
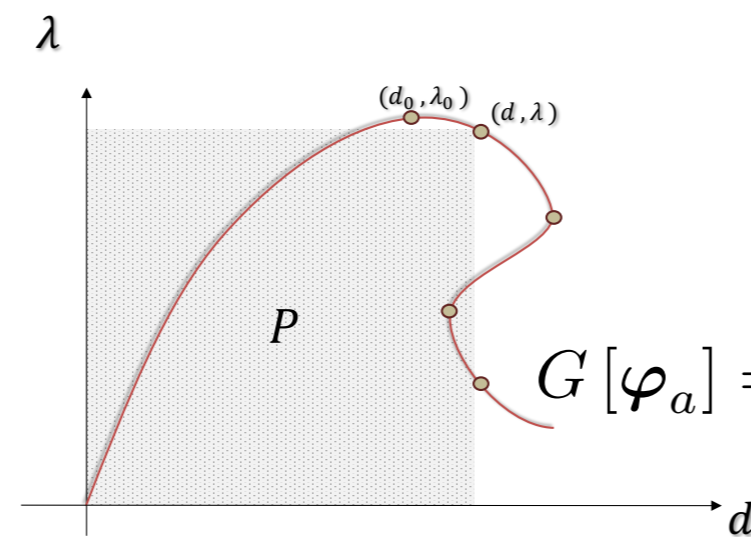
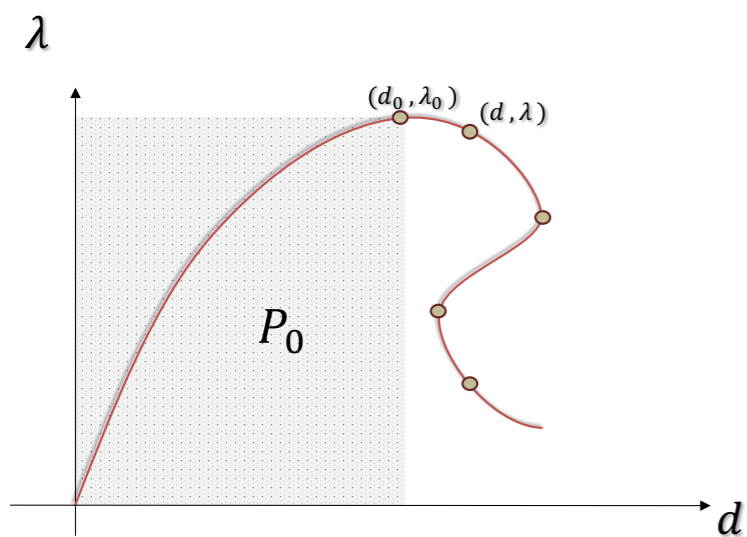
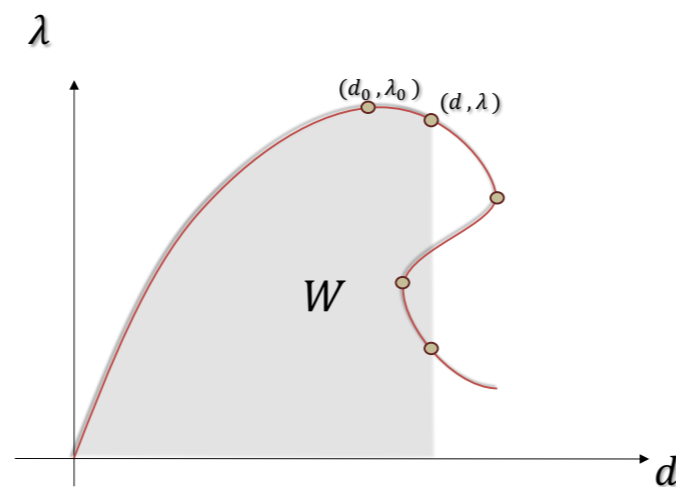
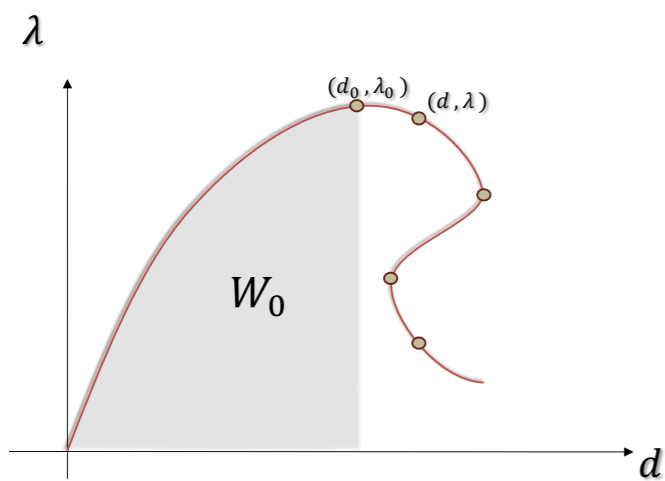
$$\mathbf{F}^{(i)}(\mathbf{d}) - \lambda \mathbf{F}^{(e)} = 0$$

How?

Griffith Energy
Balance!



Griffith Energy Balance



$$G[\varphi_a] = -\frac{\partial \Pi[\varphi_a]}{\partial A} = \frac{\partial P[\varphi_a]}{\partial A} - \frac{\partial W[\varphi_a]}{\partial A} =$$

$$= \frac{1}{2B} \lim_{\Delta a \rightarrow 0} \frac{\Pi[\varphi_{a+\Delta a}] - \Pi[\varphi_a]}{\Delta a}$$

Arc-Length Solver

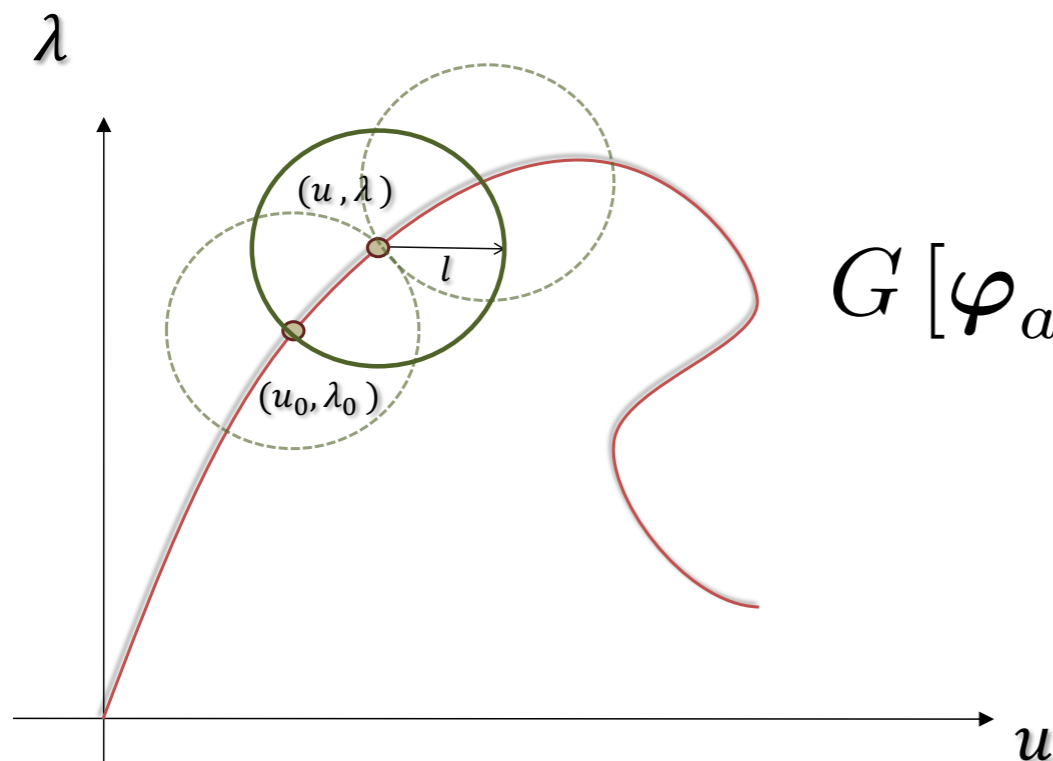
When?

Fracture

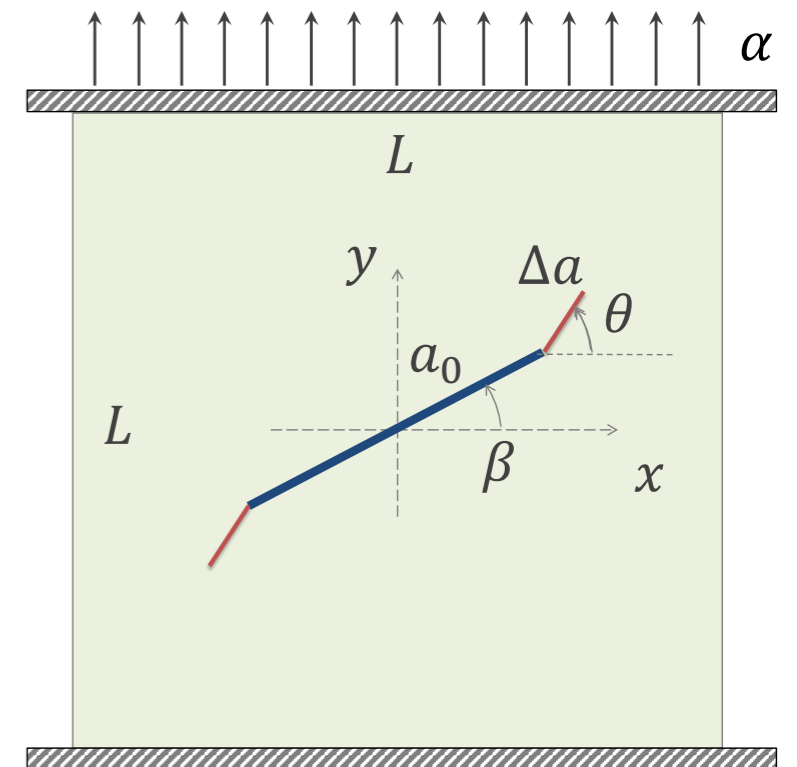
Where?

$$\mathbf{F}^{(i)}(\mathbf{d}) - \lambda \mathbf{F}^{(e)} = 0$$

How?

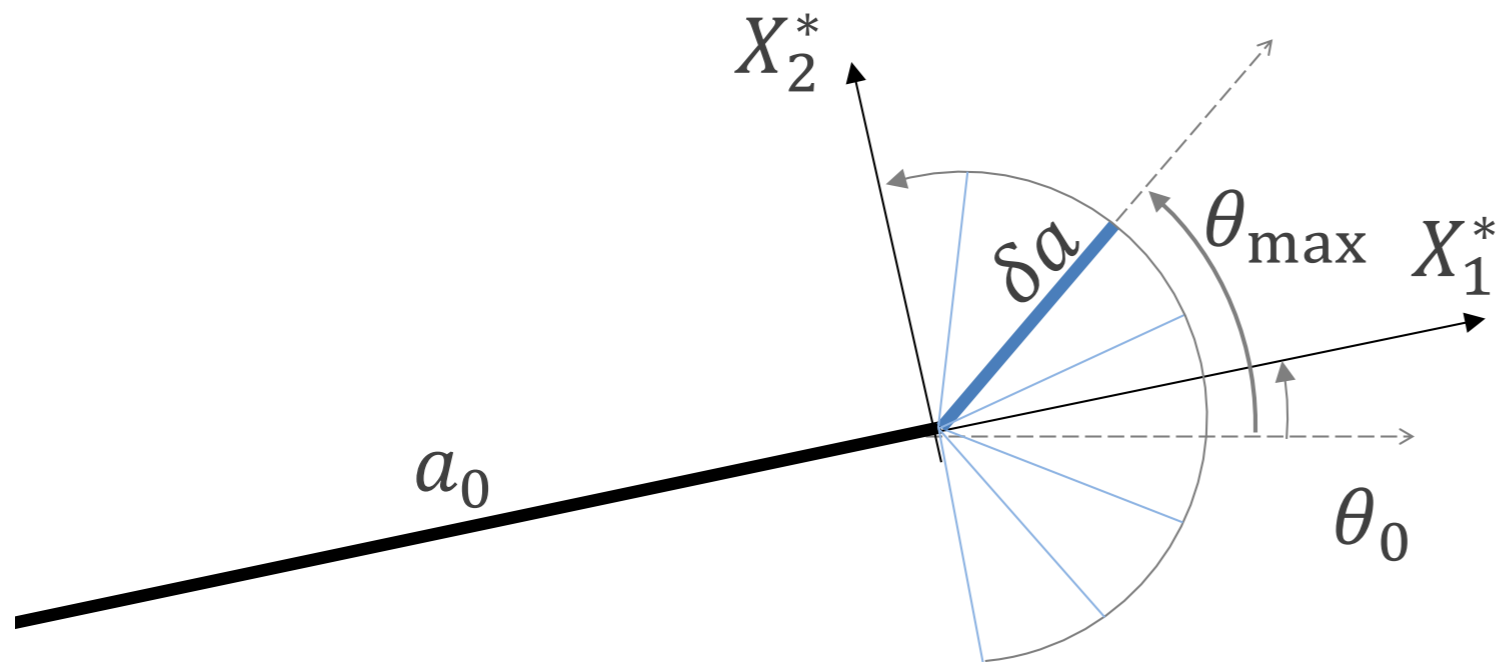


$$G[\varphi_a] - G_c(a) = 0$$



Crack Propagation Direction

$$G[\varphi_a] = J[\varphi_a] = [1 \ 0] \int_{\mathcal{D}_\rho} (\nabla_0^* \mathbf{u}^{*T} \mathbf{P}^* - w \mathbf{I}) \nabla_0^* q dS$$

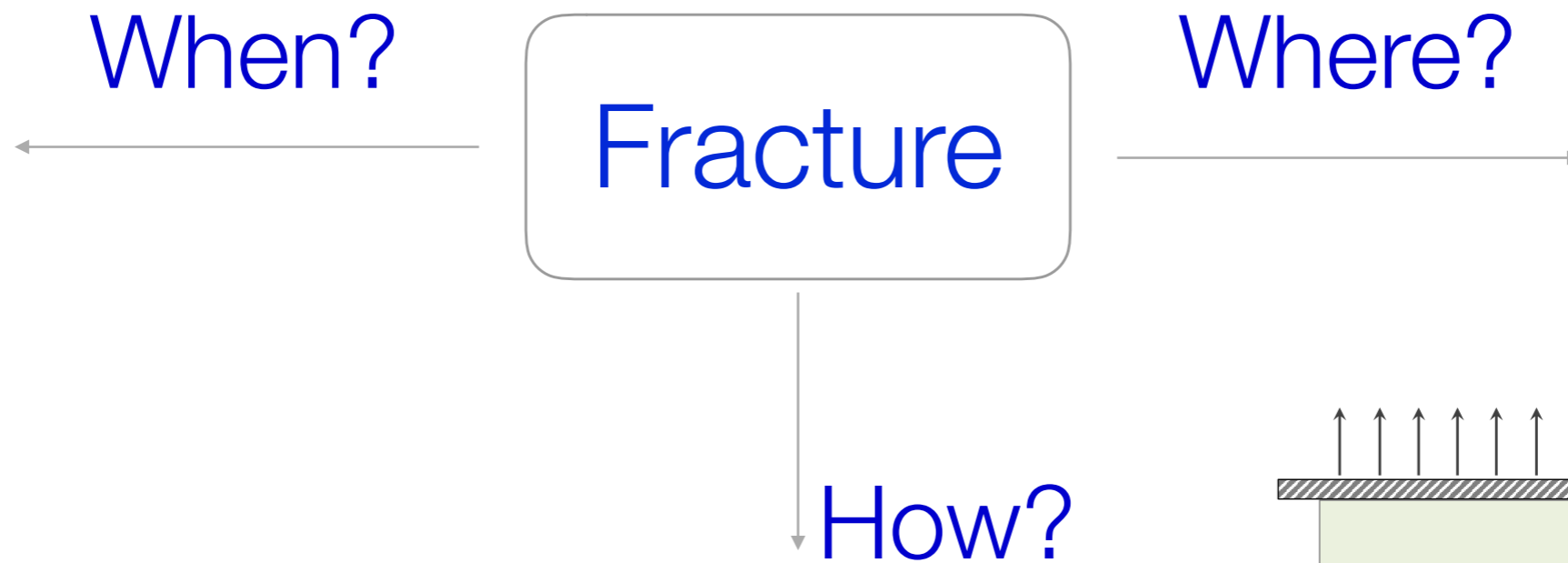


$$J(\mathbf{d}, \theta) = J_1(\mathbf{d}) \cos(\theta) + J_2(\mathbf{d}) \sin(\theta) = \mathbf{j}^h(\mathbf{d}) \cdot \mathbf{e}$$

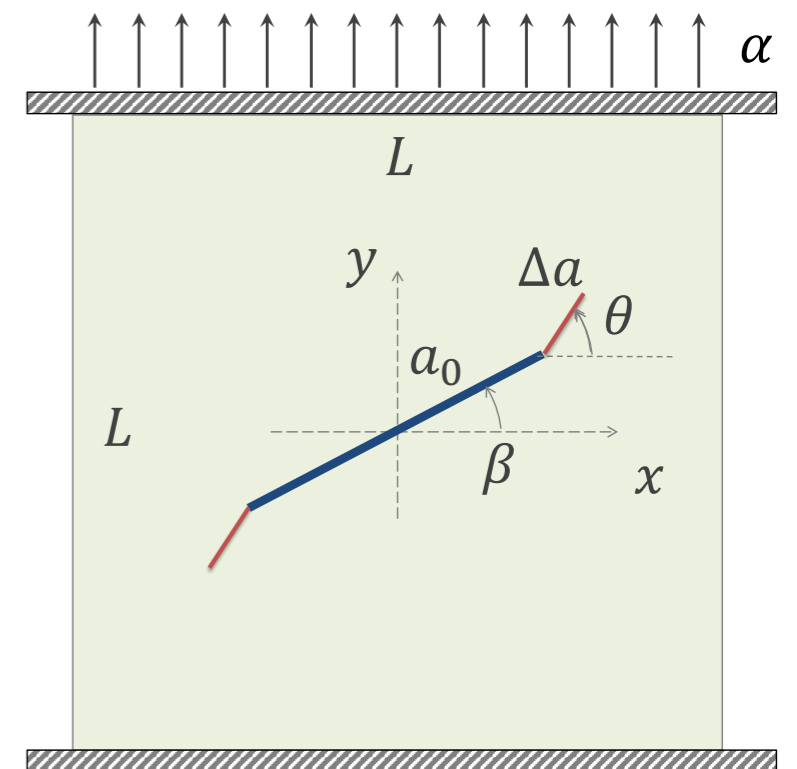
$$\mathbf{e} = \mathbf{e}(\theta) = [\cos(\theta), \sin(\theta)]$$

$$\mathbf{j}^h = \mathbf{j}^h(\mathbf{d}) = [J_1(\mathbf{d}), J_2(\mathbf{d})]$$

Arc-Length Solver

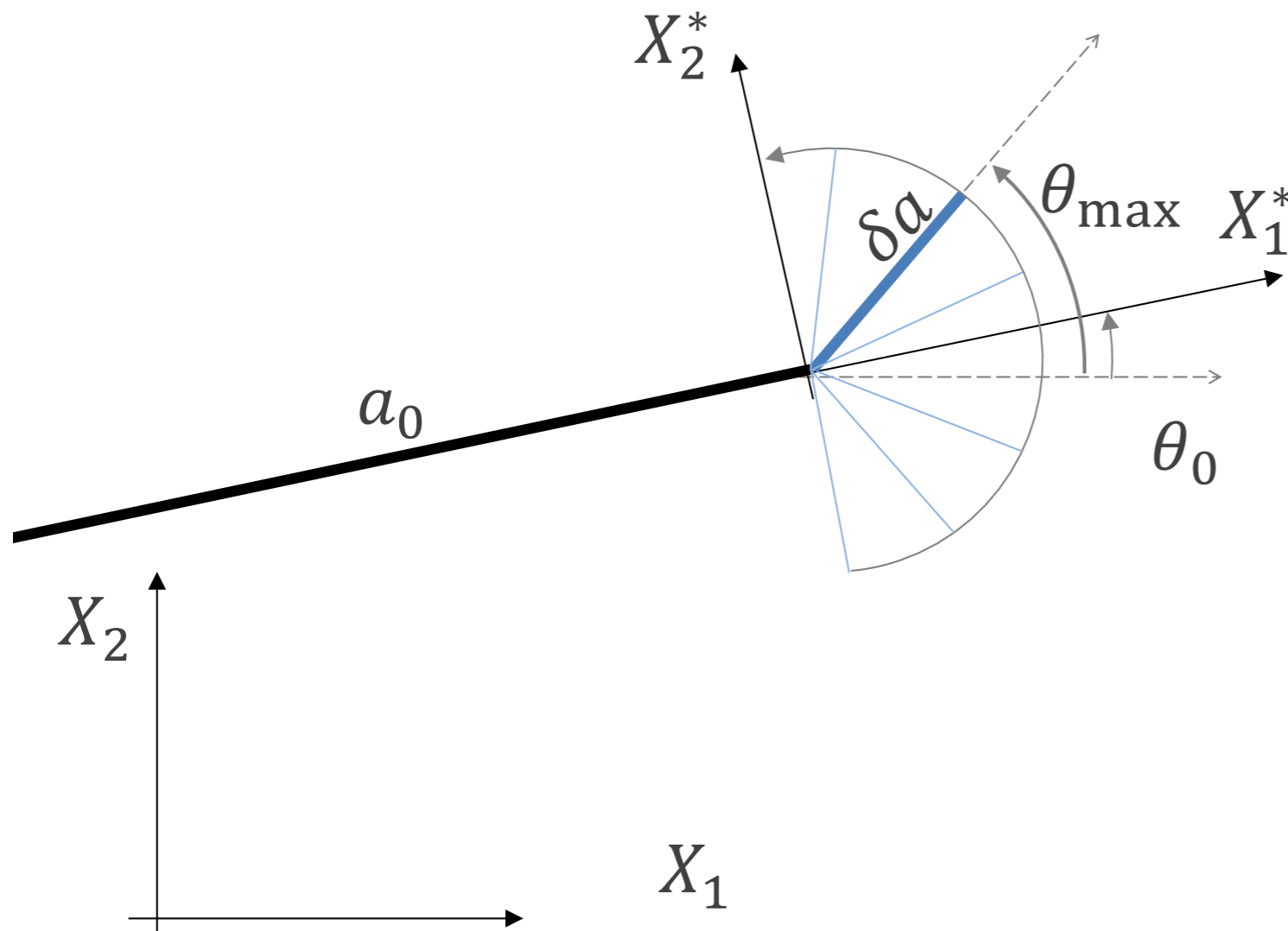


$$\begin{cases} \mathbf{F}^{(i)}(\mathbf{d}) - \lambda \mathbf{F}^{(e)} = \boldsymbol{\Psi}(\mathbf{d}, \lambda) = \mathbf{0} \\ J(\mathbf{d}, \theta) - G_c = g(\mathbf{d}, \theta) = 0 \\ \theta - \theta_{\max} = f(\mathbf{d}, \theta) = 0 \end{cases}$$



Arc-Length Solver

$$J(\mathbf{d}, \theta) = J_1(\mathbf{d}) \cos(\theta) + J_2(\mathbf{d}) \sin(\theta) = \mathbf{j}^h(\mathbf{d}) \cdot \mathbf{e}$$

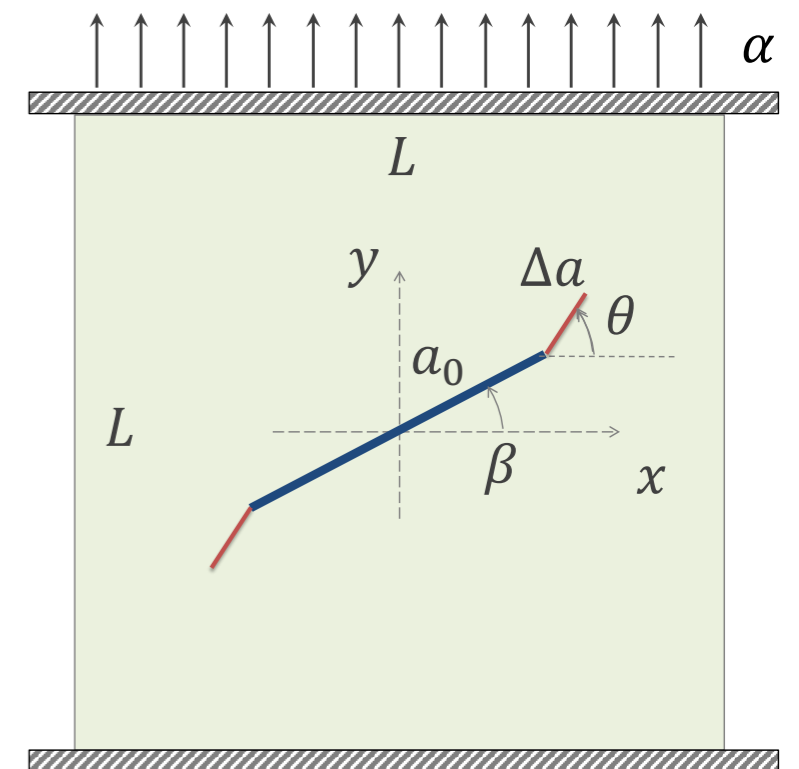
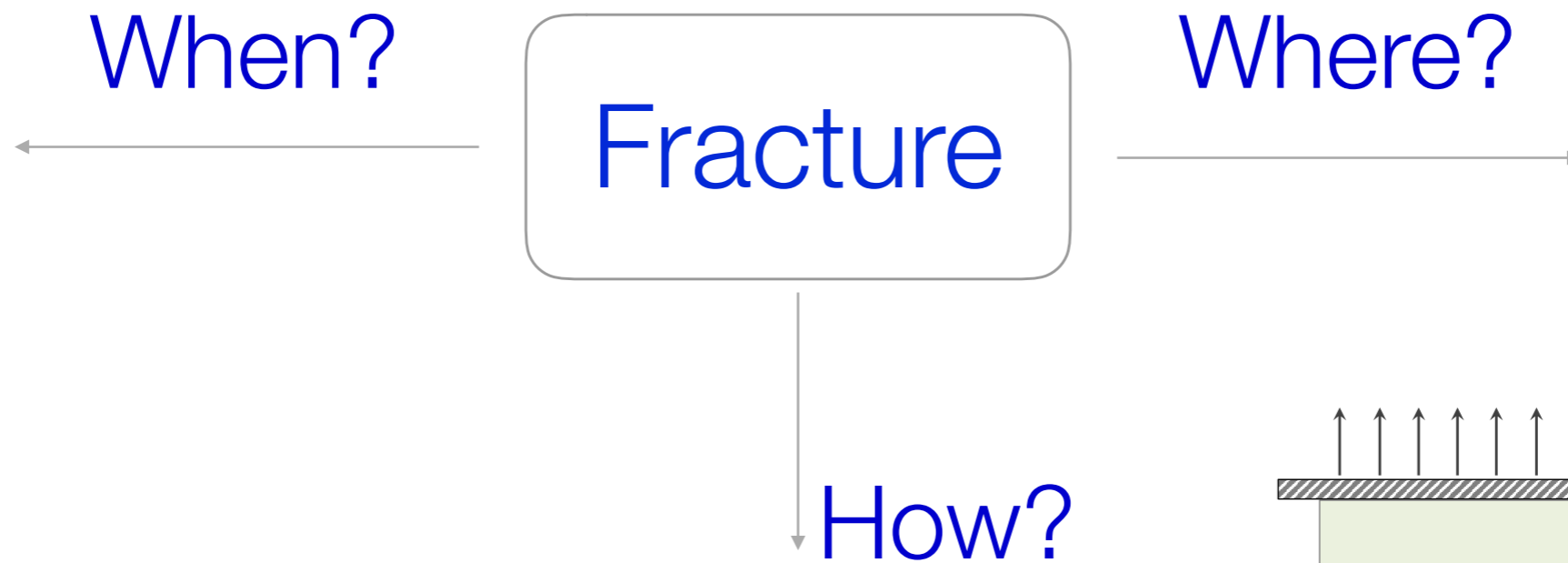


$$\theta_{\max} = \arg \max_{\theta \in [0, 2\pi]} J(\mathbf{d}, \theta)$$

$$\frac{\partial J}{\partial \theta} = 0 \quad \frac{\partial^2 J}{\partial \theta^2} < 0$$

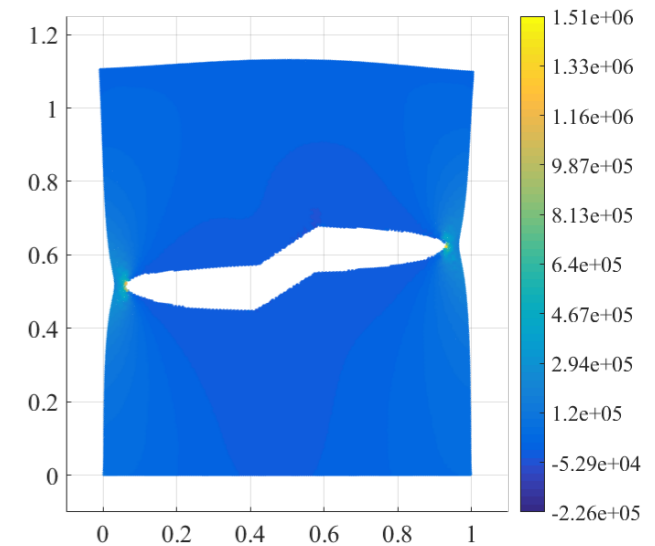
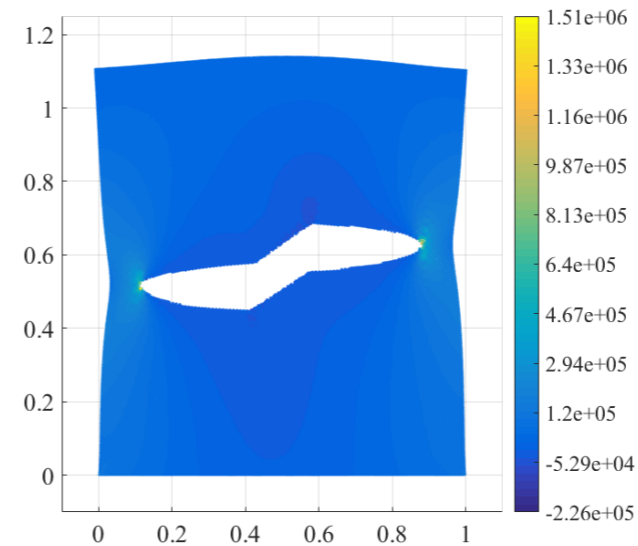
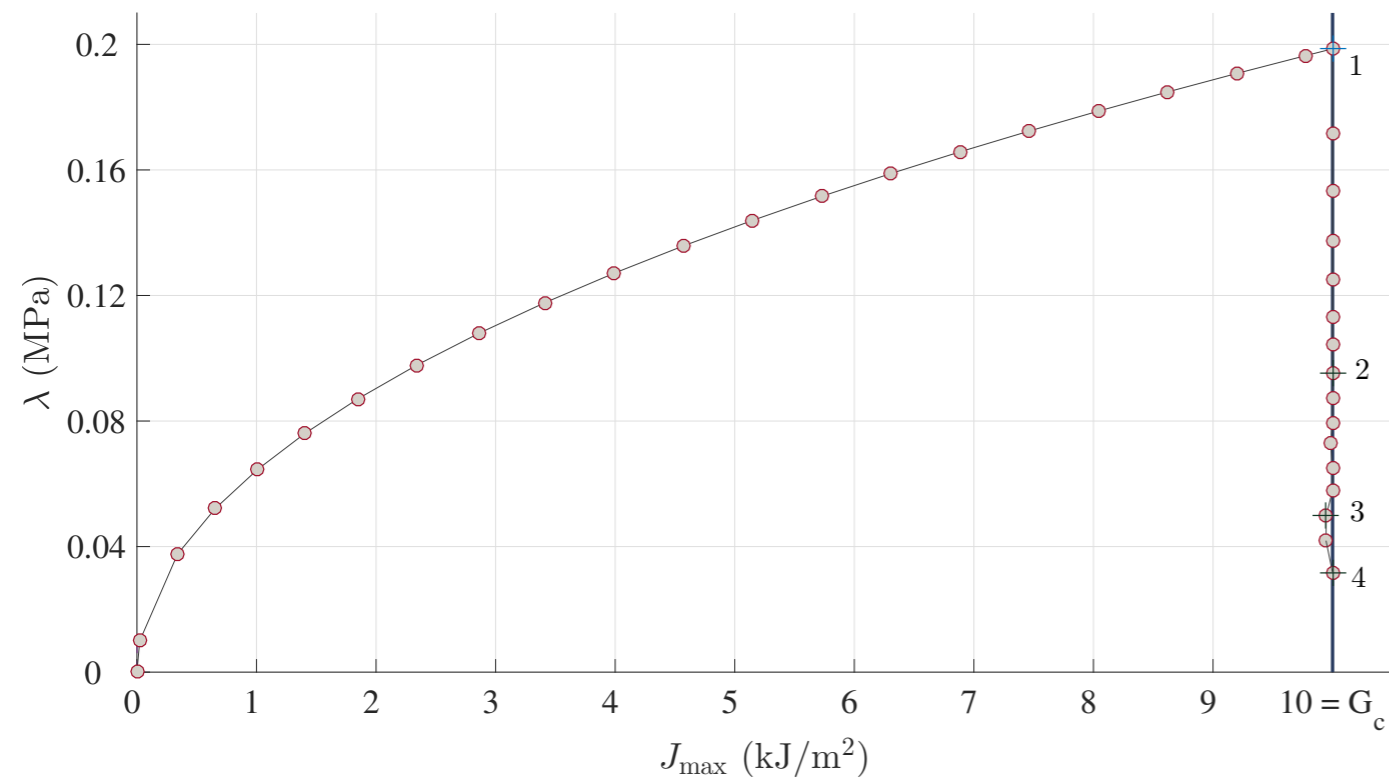
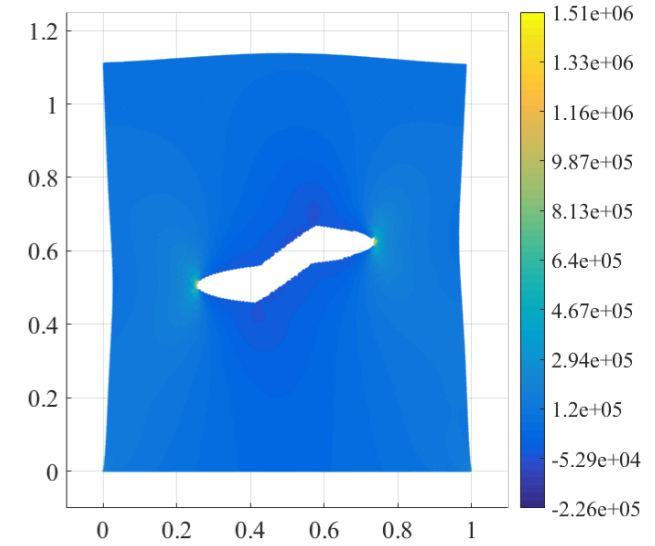
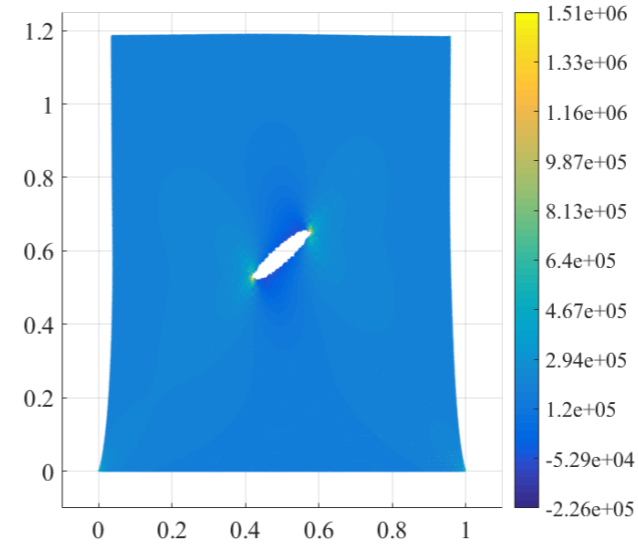
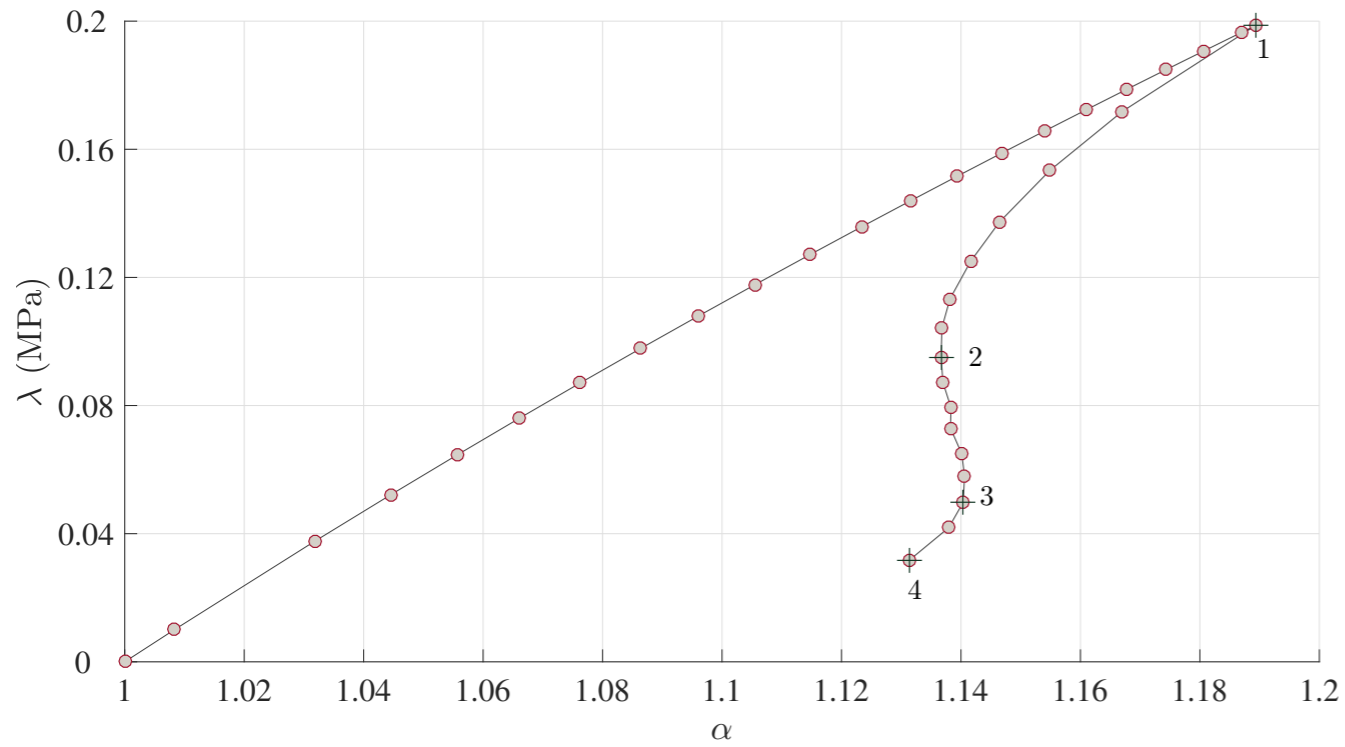
$$\theta_{\max} = \text{arctan}_2(J_2, J_1)$$

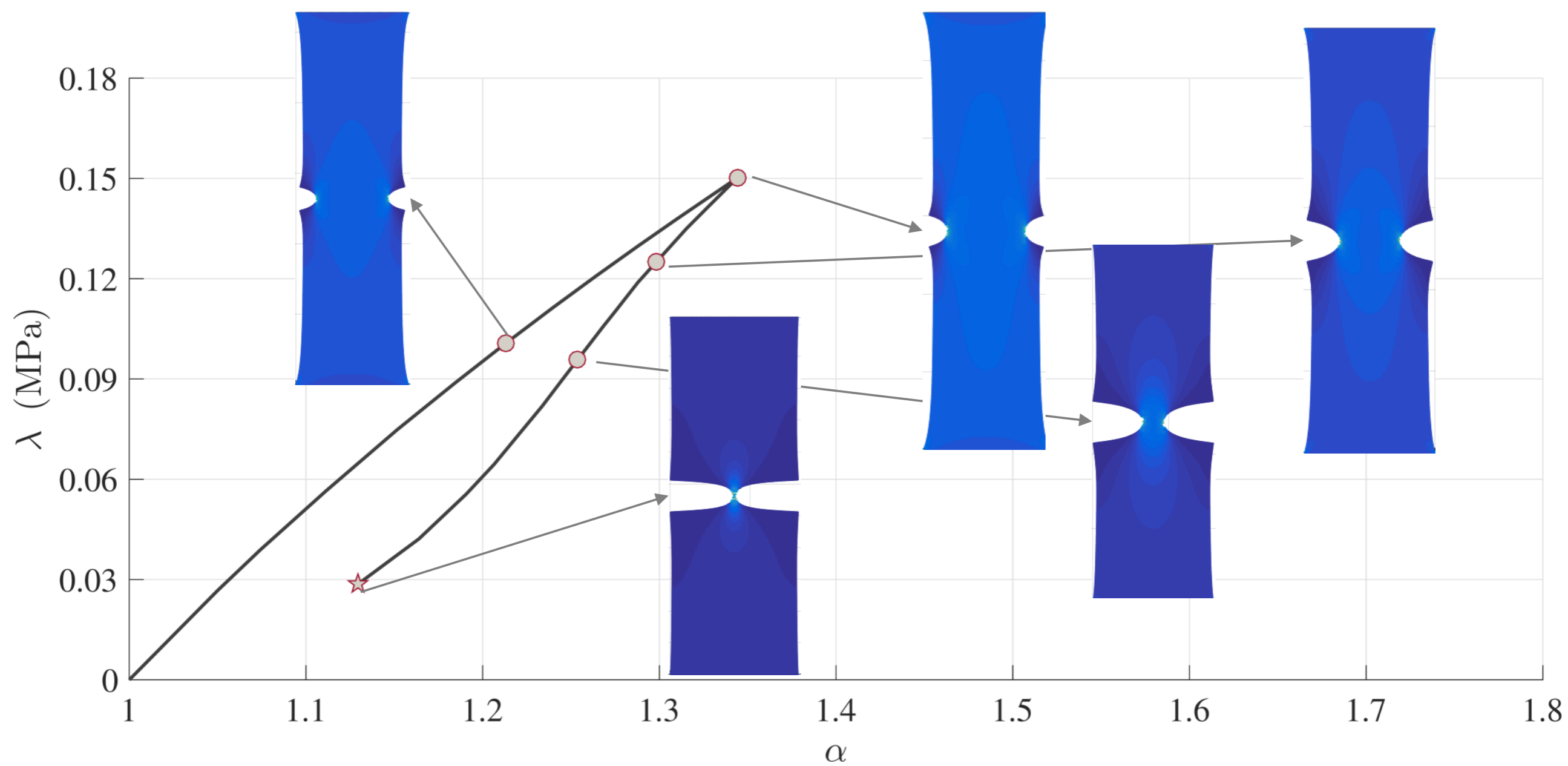
Arc-Length Solver



$$\begin{cases} \mathbf{F}^{(i)}(\mathbf{d}) - \lambda \mathbf{F}^{(e)} = \boldsymbol{\Psi}(\mathbf{d}, \lambda) = \mathbf{0} \\ J(\mathbf{d}, \theta_{\max}) - G_c = g(\mathbf{d}) = 0 \end{cases}$$

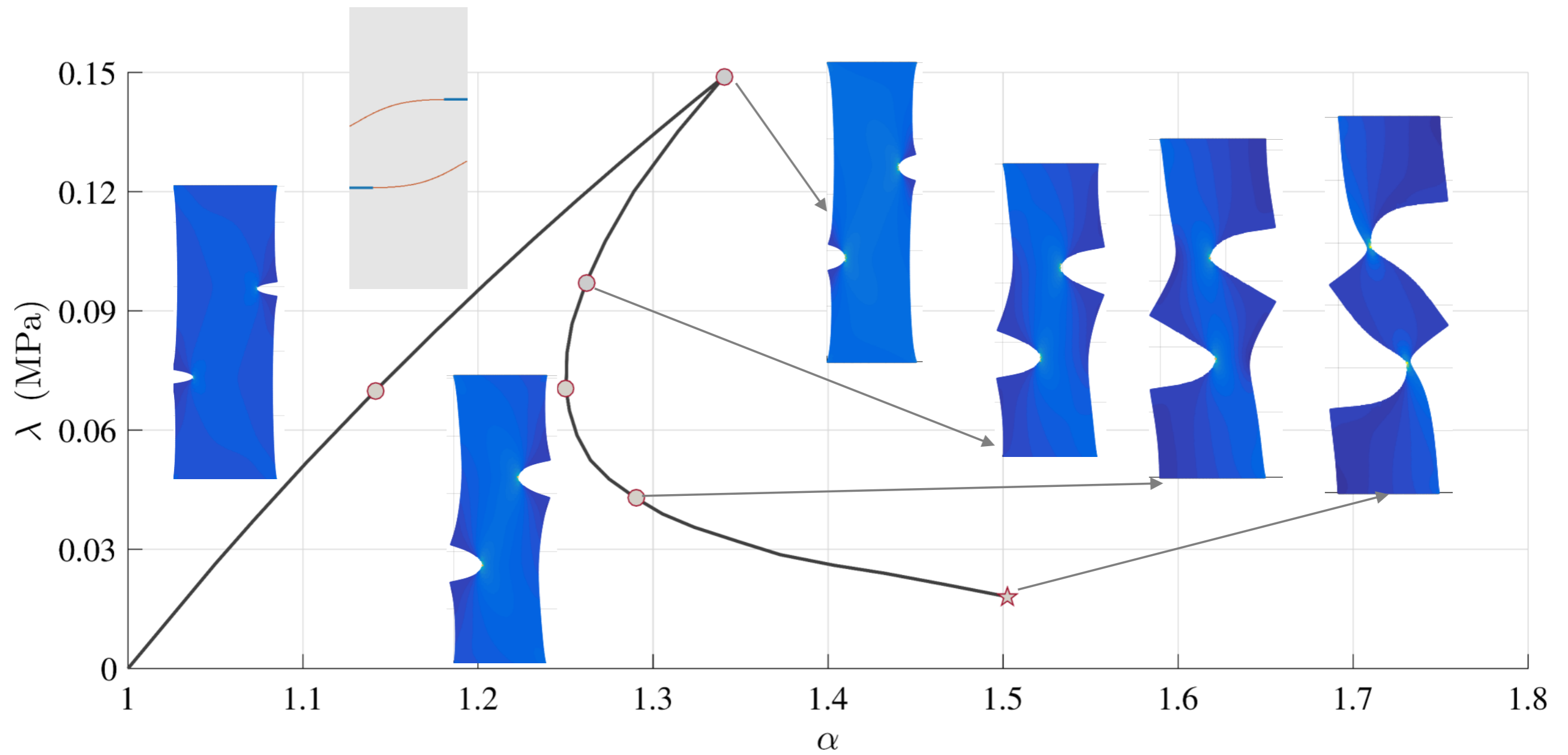
Arc-Length Solver





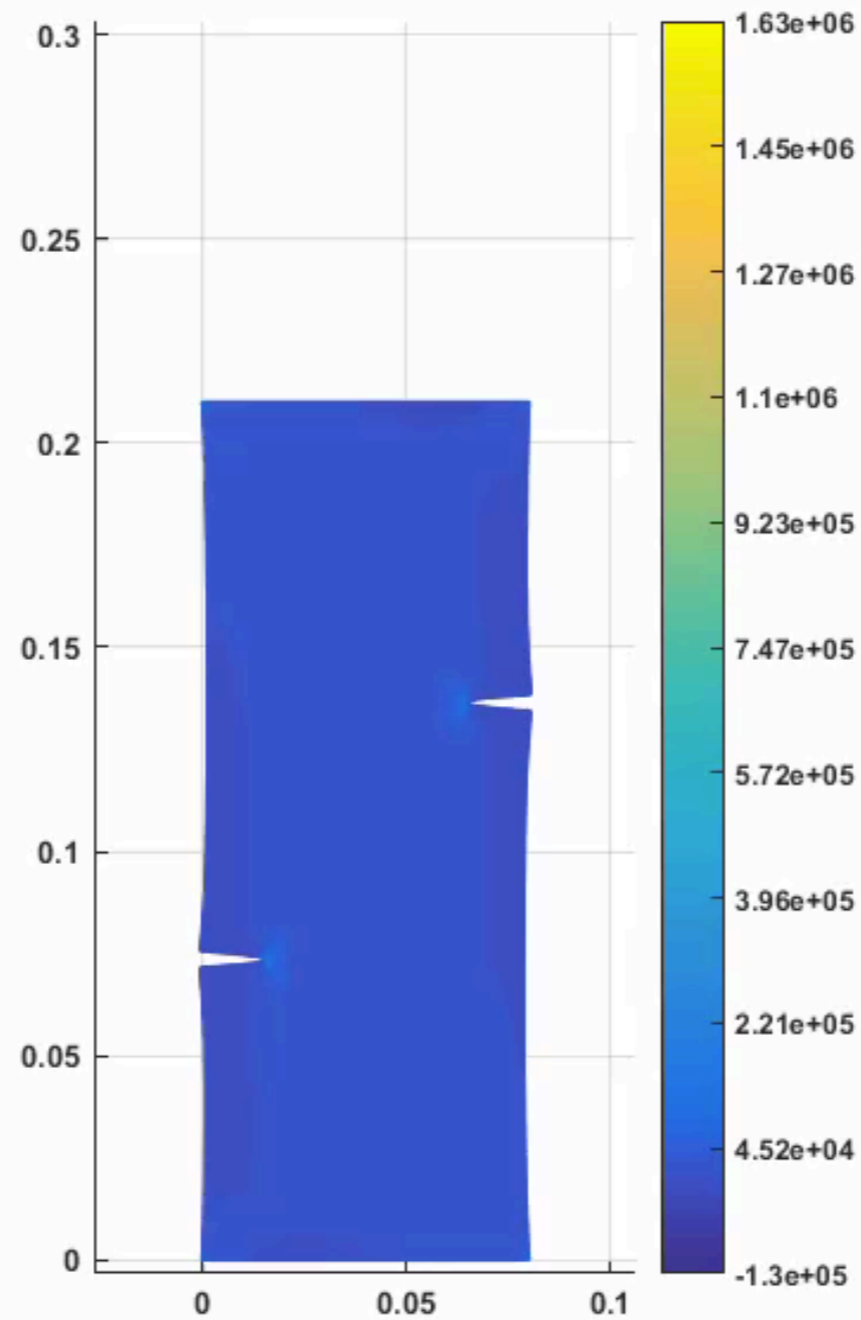
Rubber sheet with
aligned cracks

Not really stretchable



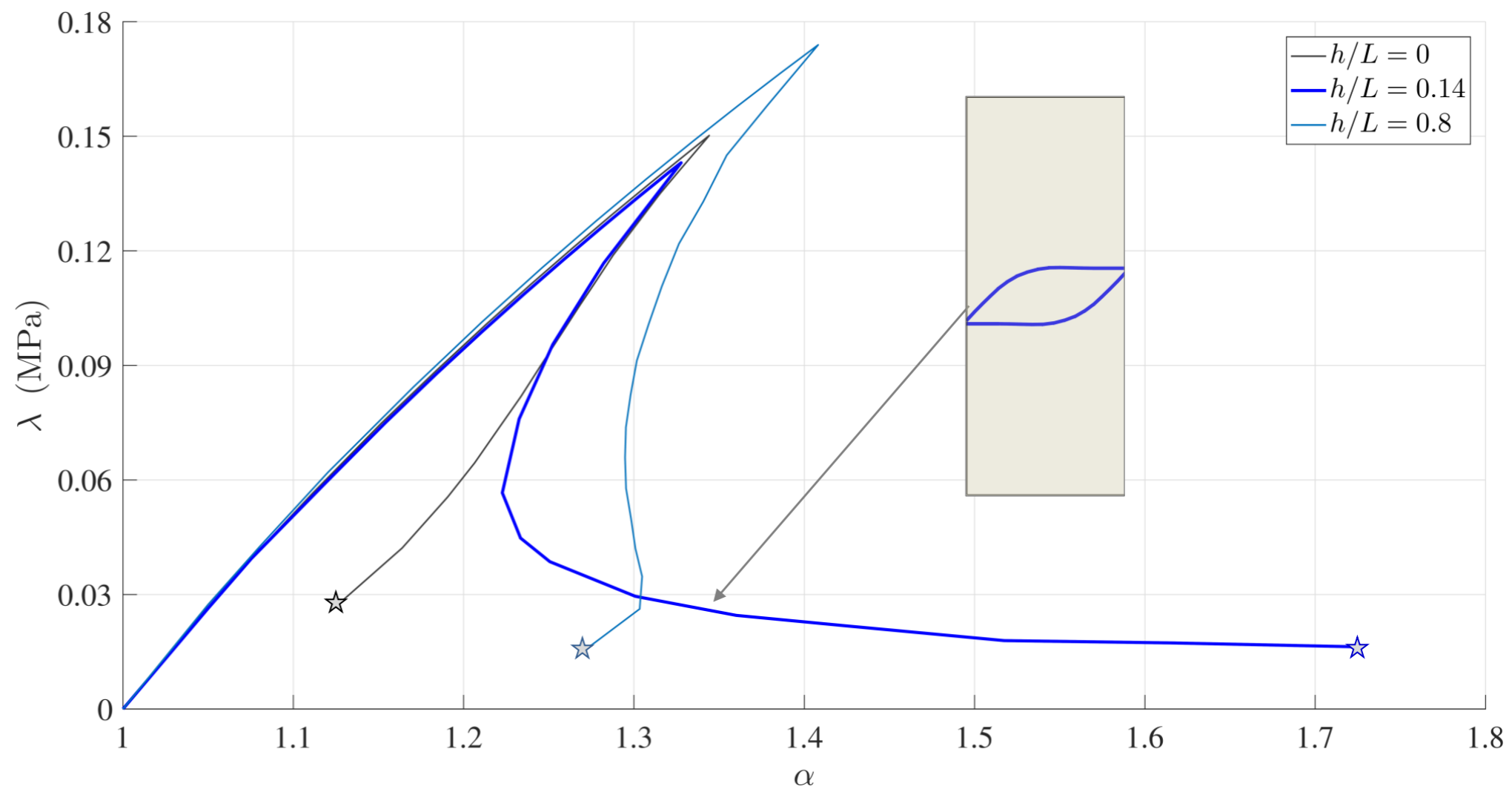
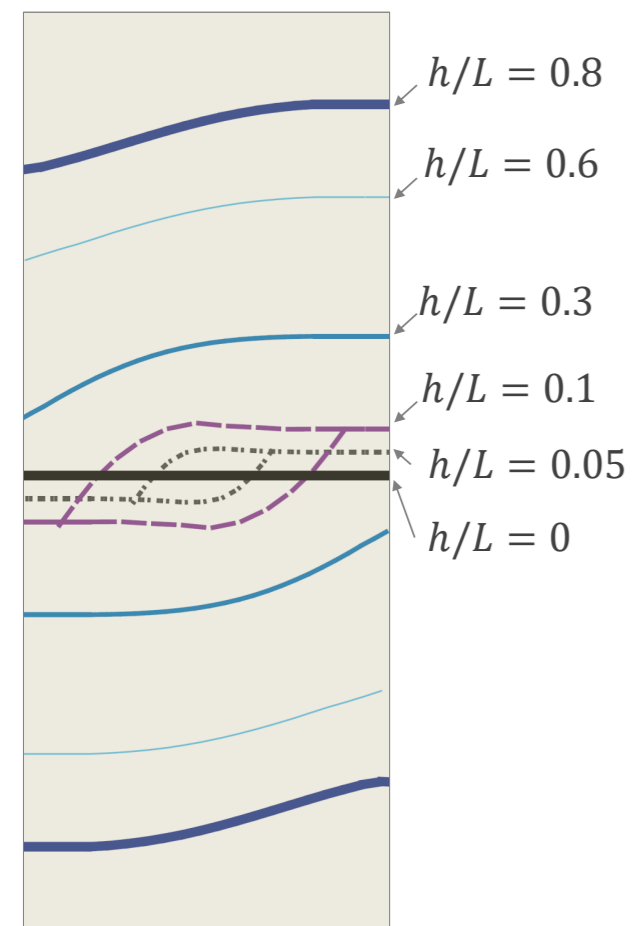
Rubber sheet with
misaligned cracks

Much more stretchable



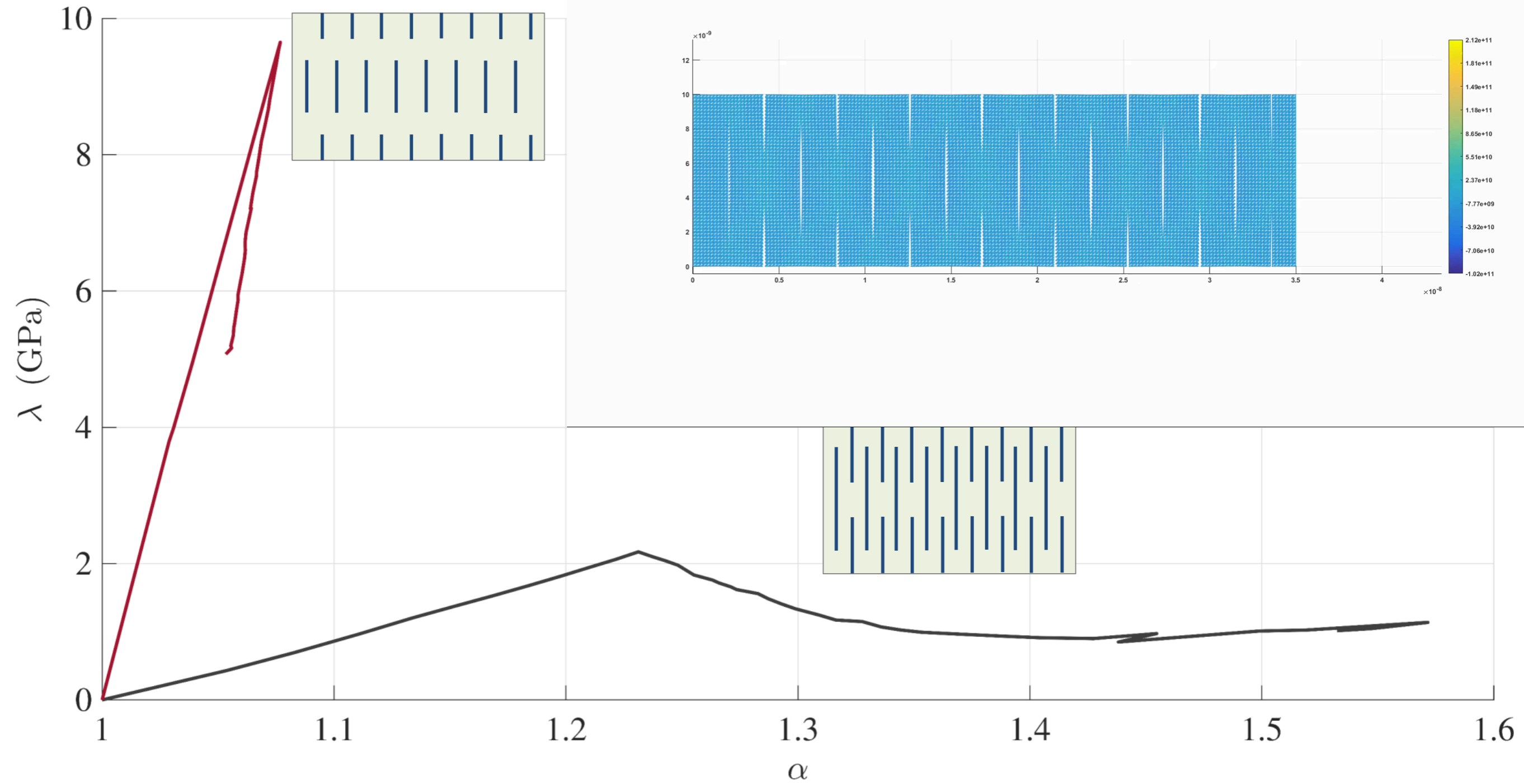
Rubber sheet with
misaligned cracks

Much more stretchable!



Rubber sheet with
misaligned cracks

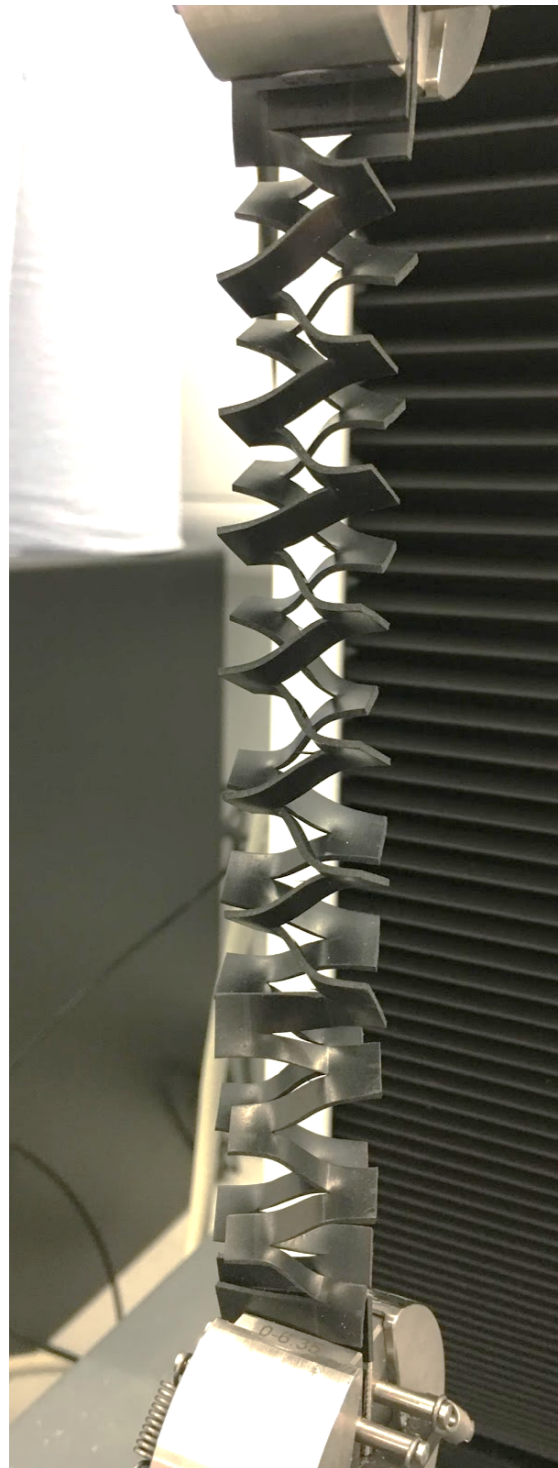
Much more stretchable!



Kirigami Structures

Making structures more stretchable...by cracking it!

Some designs...



LAYERING of patterns

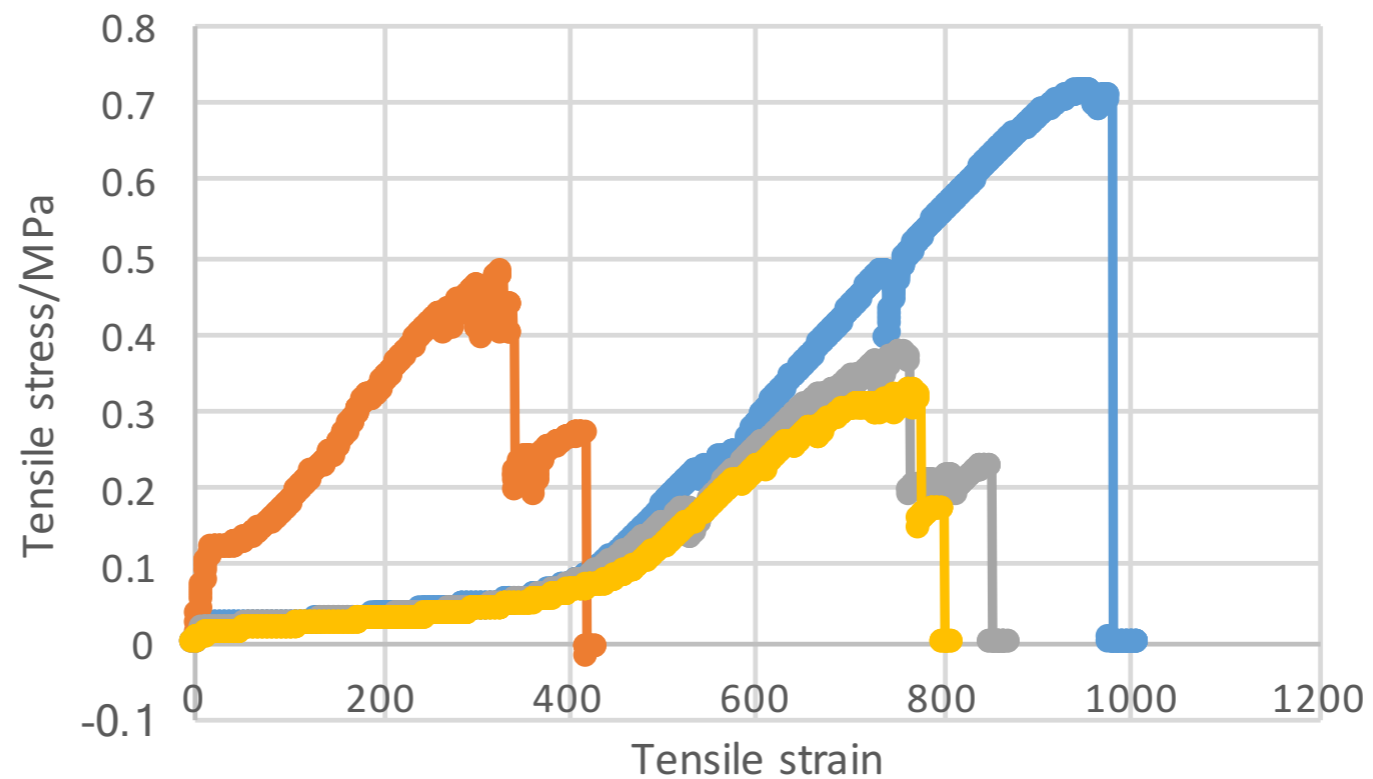
- increases strength
- promotes helicoidal motion

ALTERNATING of patterns determines

- stress distribution
- delay of failure
- opponent motion

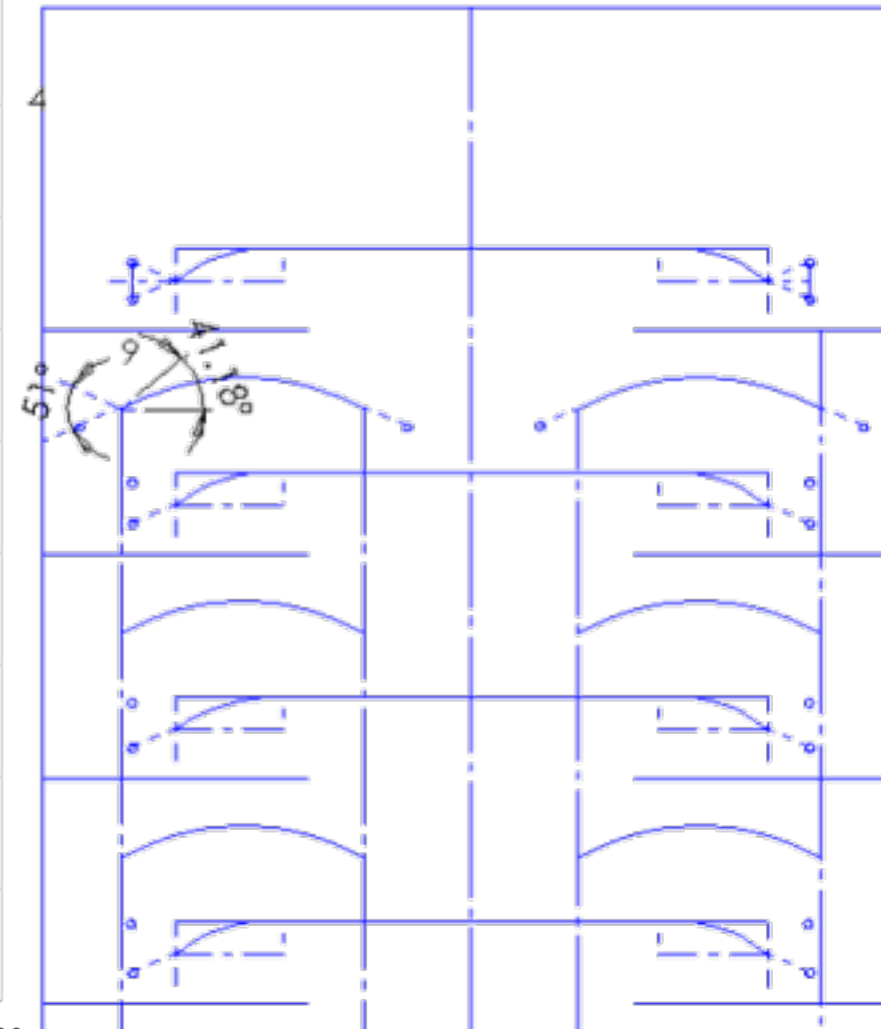
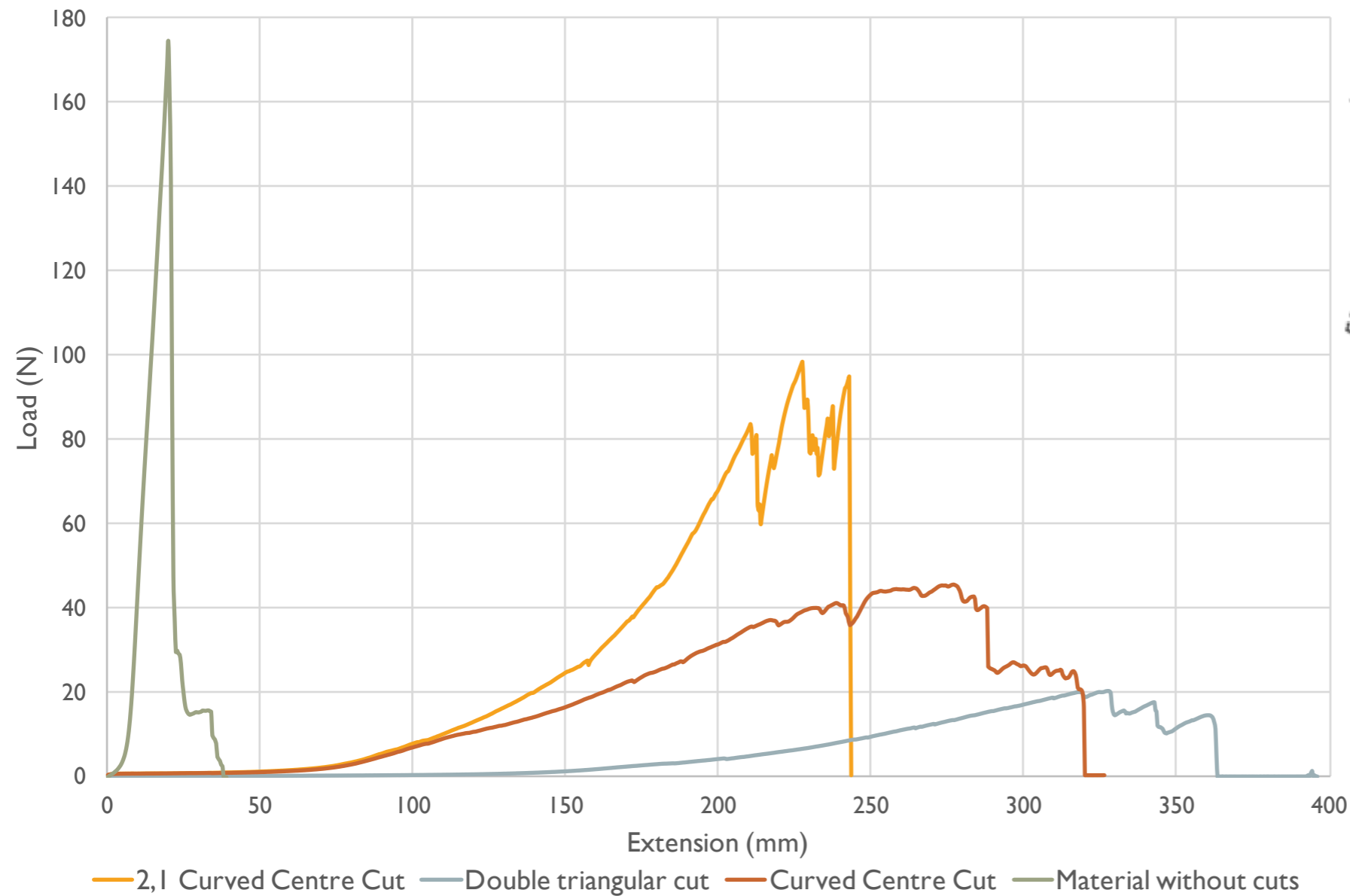
Single Layer

Figure 3: Polypropylene



- Sample 7
- Sample 8
- Sample 9
- Sample 10

Some designs...



Fracture @QMUL

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 @EttoreBarbieri

Federica Ongaro
Francesca Carleo
Paolino De Falco
Mattia Gaglione
Simone Falco

