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# Modular Bayesian uncertainty assessment for Structural Health Monitoring

## Warwick Centre for Predictive Modelling

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June 26, 2017

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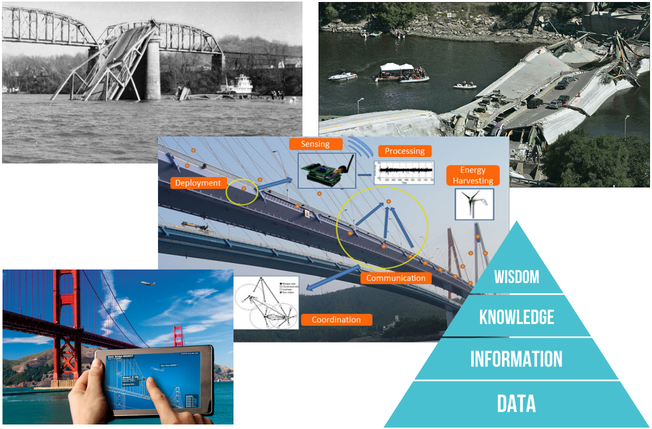
# Structural Health Monitoring
## The BIG picture



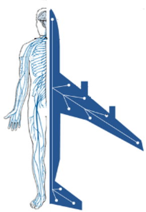



Civil and mechanical engineering; Signal processing; Machine learning; Electronics; Information theory; Computer science . . .

# Problem statement

## Tasks/Approach/Challenges

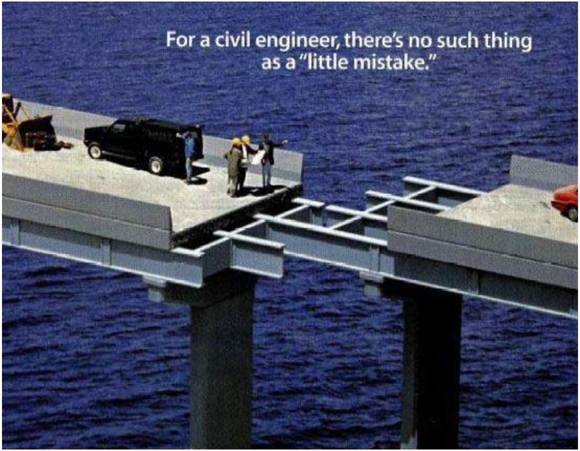
- Tasks
  - Measurement system design;
  - Damage detection;
  - Structural identification;
  - Data interpretation;



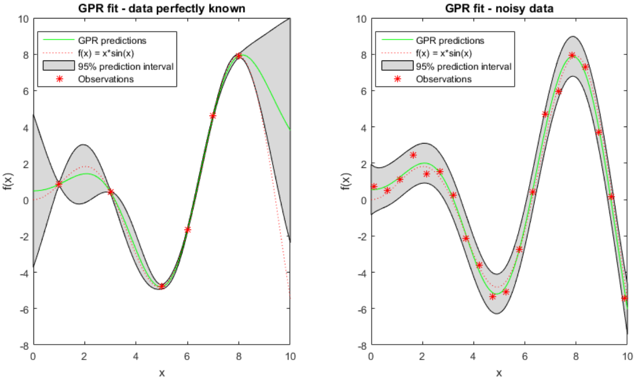
- Approach
  - Data-driven;
  - Model-based;
- Challenges
  - Complexity: structure; monitoring; model $\rightarrow$ uncertainties;
  - Decision-makers need to know how good the model predictions are
  - Model predictions should be accompanied by quantification of uncertainty;

# Problem statement
## Uncertainties





- Workframe for UQ; Reduced computational effort;
- mrGp: Dataset$(X, Y) \rightarrow$ non-parametric probabilistic model




Simulations — Measurements

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Problem statement
Uncertainties
Modular Bayesian approach
Gaussian process surrogate modelling
Bayesian probabilities
Case-studies
Aluminium bridge
Tamar bridge
Lessons learned
WCPM

### Sources of uncertainty

Experimental: Noise; Residual variations
Prediction: Parametric; Model discrepancy; Interpolations

### Bayes' Theorem

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}} \qquad p(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{p(\boldsymbol{D}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(\boldsymbol{D}|\boldsymbol{\theta})p(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}}$$

$$Y^e(\mathbf{X}) = Y^m(\overset{\text{specific}}{\overline{\mathbf{X}}}, \overset{\text{generic}}{\overline{\boldsymbol{\theta}^*}})$$

# Problem statement
## Uncertainty quantification



### Sources of uncertainty

Experimental: Noise; Residual variations
Prediction: Parametric; Model discrepancy; Interpolations

### Bayes' Theorem

$$\text{posterior} = \frac{\text{likelihood} \times \text{prior}}{\text{marginal likelihood}} \qquad p(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{p(\boldsymbol{D}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{\int p(\boldsymbol{D}|\boldsymbol{\theta})p(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}}$$

$$\overbrace{Y^e}^{\text{Measurements}}(\mathbf{X}) = \overbrace{Y^m}^{\text{Simulations}}(\mathbf{X}, \underbrace{\overbrace{\boldsymbol{\theta}^*}^{\text{Prior information}}}_{\text{Structural Parameters}}) + \underbrace{\delta}_{\text{Model Discrepancy}}(\mathbf{X}) + \underbrace{\varepsilon}_{\text{Noise}}$$

Multiple parameters: Markov Chain Monte Carlo methods

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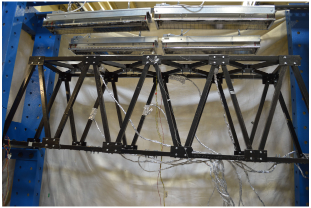
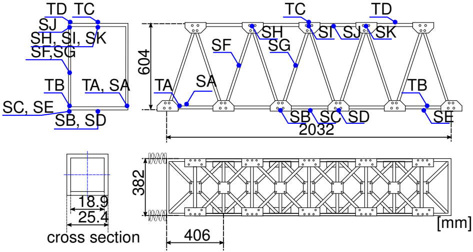
# Aluminium bridge
## Setup

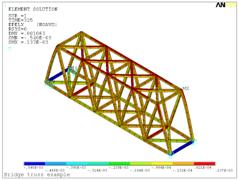


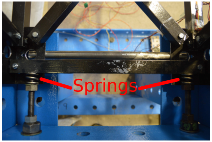
### Experiment

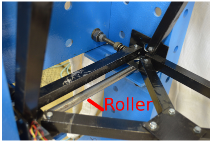





### Model

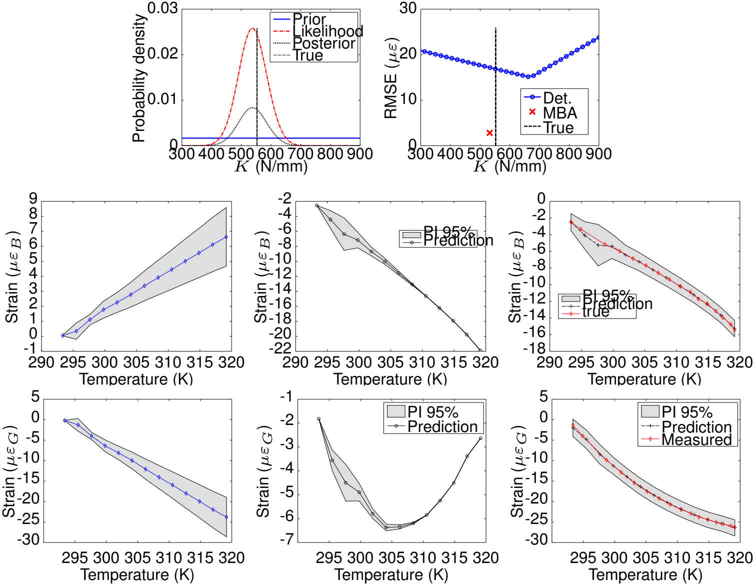


# Aluminium bridge
## Results





# Tamar bridge
## Setup

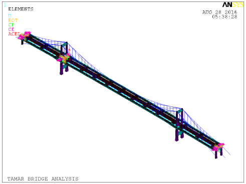


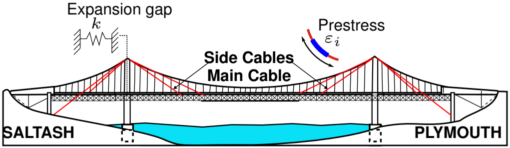
### Experiment

Measurements during a one year span

- $X$ – Temperature; traffic
- $Y$ – Natural frequencies; Mid-span displacement;

### Model



# Tamar bridge
## Results

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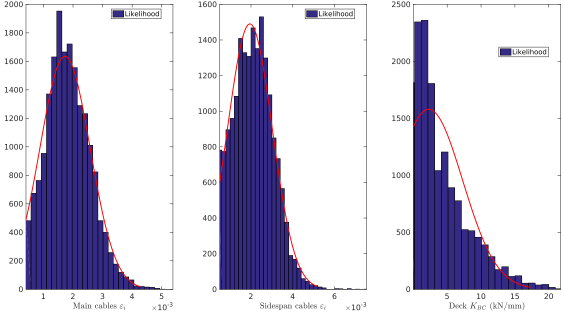


| | Forces (kN) | | |
|---|---|---|---|
| Year | Main cable | Side cable | Method |
| 1961 | 20296 | 20597 | iterative shape finding |
| 2001 | 20296 | 20597 | iterative shape finding |
| 2009 | 23564 | 25985 | modular Bayesian approach |

A 13% increase in the cables forces was identified

# Lessons learned



- Credibility of modelling should always be assessed by uncertainty quantification (UQ);
- Sufficiently informative responses improve UQ of the Modular Bayesian approach;
- Methodology was applied in reduced and full-scale examples of Structural Health Monitoring, allowing identification of critical parameters;
- Enhancement of the methodology for multiple parameter identification;
- Acknowledgementes: EPSRC funding; supervisors & colleagues; Exeter research group;

EPSRC
Pioneering research and skills

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# Thank you for your attention.

# Questions?

# Modular Bayesian approach
## Workflow



1. Fit model with mrGp

$$Y^m(X^m, \Theta^m) \approx \mathrm{mrGp}^m$$

2. Fit discrepancy function with mrGp

$$Y^e(X^e) - \int \mathrm{mrGp}^m(X^e, \theta) p(\theta) \mathrm{d}\theta \approx \mathrm{mrGp}^\delta$$

3. Bayes' theorem

$$p(\boldsymbol{\theta}|\boldsymbol{D}) = \frac{p(\boldsymbol{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta})}{\int p(\boldsymbol{D}|\boldsymbol{\theta}) p(\boldsymbol{\theta}) \mathrm{d}\boldsymbol{\theta}}$$

4. Predictions with updated metamodel

$$Y^e \approx \mathrm{mrGp}^m + \mathrm{mrGp}^\delta$$

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**Lessons learned**