Stochastic synaptic integration in a spatial neuron with voltage activated currents

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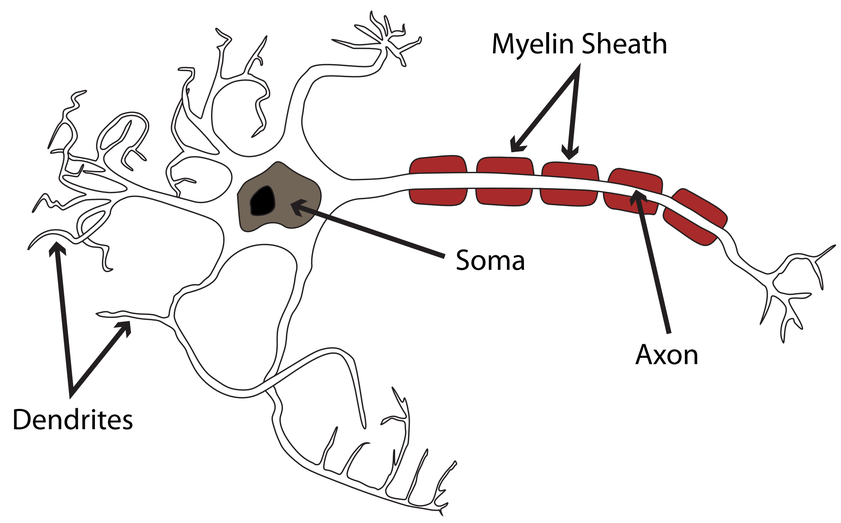
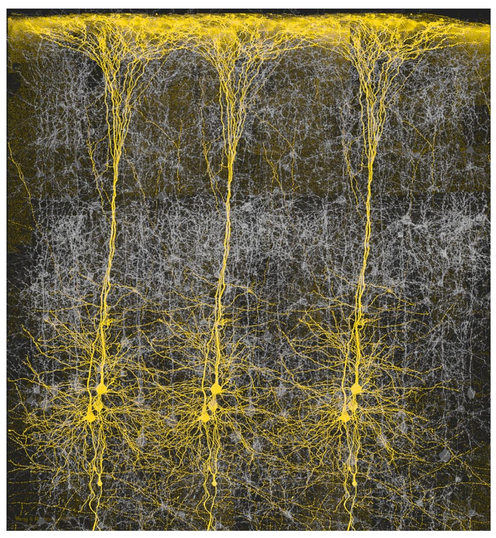
Introduction

The neocortex has neuronal cells forming a complex network where each cell individually has interesting response properties, which is the main reason for its computational power. The principal cells in the neocortex are pyramidal cells, which are highly elongated, normal to the surface of the cortex. This extended cell does not have a homogeneous membrane voltage but instead the voltage satisfies a so-called cable equation.





Each cortical neuron can receive up to 10,000 synapses. Though the rate at which cells fire action potentials can be very low (∼0-5Hz) they are subject to a very large stochastic synaptic input, where each synapse has either an inhibitory or excitatory effect.

Single compartment model

The starting point to study stochastic input into dendrites of a neuron, is to consider the dendrites to be point like, i.e. without spatial dependence. The input is considered to be Poissonian in time, with each incoming pulse at the synapses changing the conductance of the postsynaptic cell by a fixed γ_s [1, 3, 4].

To ensure the resulting voltage relation and its mean and variance have a closed form solution some approximations need to be made. First the Poisson process is written as a mean and the excess from it is Gaussian distributed eqn. (1). Secondly the closing of synaptic channels is assumed to be instantaneous. The conductance is in the form:

$$g_s = g_{s0} + g_{sF}. \tag{1}$$

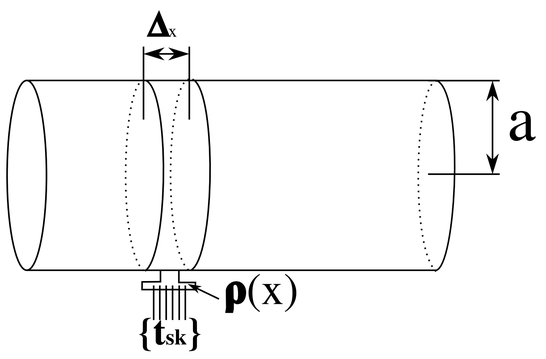
The resulting Voltage relation is in the form of the Ornstein-Uhlenbeck process

$$\tau \frac{dV}{dt} = E - V + \sigma_V \sqrt{2\tau}\xi(t), \tag{2}$$

where τ is the effective membrane time constant, E is an effective equilibrium potential, σ_V is the variance of the voltage and $\xi(t)$ is time dependant white noise.

Extended noise model

During this project the Point model was extended to a cable of radius a where the synapses are distributed along the membrane with a density ρ_s ($s = e, i$ stands for excitatory and inhibitory synapses) and along each infinitesimal section of the cable Δ_x is a set of synaptic pulses $\{t_{sk}\}$.



The voltage in the extended cell is described by a diffusion like cable equation

$$\tau \partial_t v = -v + \lambda^2 \partial_x^2 v + I_{syn}(t,x)/G, \tag{3}$$

where I_{syn} is the synaptic drive, τ, λ are the membrane time and space constants respectively and G is a conductance. In this case the synaptic drive is in the form of a sum of excitatory and inhibitory current inputs. For the extended model we make the same approximations as for a single compartment model and get a voltage relation of the form

$$\tau \partial_t v = -v + \lambda^2 \partial_x^2 v + \lambda \tau a_e \sqrt{R_e \lambda}\zeta_e(t,x) + \lambda \tau a_i \sqrt{R_i \lambda}\zeta_i(t,x), \tag{4}$$

$$\text{with } a_s = \frac{\gamma_s E \tau_s}{\lambda \sqrt{G_\lambda g}}, \tag{5}$$

where a_s is the amplitude of voltage jumps due to synaptic pulses, R_s is the number of pulses per unit time per unit area, $\zeta_s(t,x)$ is a white noise term in time and space, τ_s is the decay time constant for the conductance of the excitatory and inhibitory pulses, G_λ is the characteristic conductance term and g is the sum of the leak conductance and the mean of the excitatory and inhibitory conductances.

Solution to new noise model

The cable equation eqn. (3) has a unique solution depending on the boundaries [5], [7]. The Greens function of the cable equation for an infinite cable due to a current impulse $\delta(x-x')\delta(t-t')$ takes the form [5]

$$g(x;t) = \frac{1}{\lambda\tau}\frac{\exp[t/\tau]}{\sqrt{4\pi t/\tau}} \times \exp\left[\frac{-(x/\lambda)^2}{4t/\tau}\right].$$

The closed form solution for $v(t,x)$ can be found by convolving the specific form of the synaptic current $I_{syn}(t,x)$ (from eqn. (4)) in space and time with the Greens function $g(x;t)$. Using the properties of the white noise term we can conclude that $\langle V(x,t)\rangle_{t,x} = 0$. From the solution for the covariance $\langle V(x,t)V(x',t')\rangle$ it is also possible to calculate the variance of the voltage

$$\sigma^2 = \frac{a_e\sqrt{R_e\lambda}}{4}\,\mathrm{erf}\left(\sqrt{2t}\right) + \frac{a_i\sqrt{R_i\lambda}}{4}\,\mathrm{erf}\left(\sqrt{2t}\right).$$

The next step is to calculate the variance and covariance of voltages in the case of a finite cable. The method consists of using an infinite sum of Greens functions [6].

Outlook

There are many interesting directions in which this project can be extended. The first step would be to include an active input into the cell and explore the properties of the resulting voltage relation. In addition to that one could try to include coloured noise to see how the type of synaptic noise affects the voltage.

Once the voltage relations with active input for a single cell are known, it would become possible to explore properties of neuronal cells in networks using mean-field approaches or using a simplified version of the single cell voltage relation.

References

[1] L. Badel, W. Gerstner, and M. Richardson, Neurocomputing, 69 (2006), pp. 1062–1065.

[2] S. Coombes, Y. Timofeeva, et al., Biological Cybernetics, 97 (2007), pp. 137–49.

[3] W. Gerstner and G. Kistler, *Spiking Neuron Models*, Cambridge University Press, 2002.

[4] A. Manwani and C. Koch, Neural Computation, 11 (1999), pp. 1797–1829.

[5] A. Manwani and C. Koch, Neural Computation, 11 (1999), pp. 1831–73.

[6] H. Tuckwell, *Introduction to theoretical neurobiology Volume 1*, Cambridge University Press, 1988.

[7] H. Tuckwell and J. Walsh, Biological Cybernetics, 49 (1983), pp. 99–110.