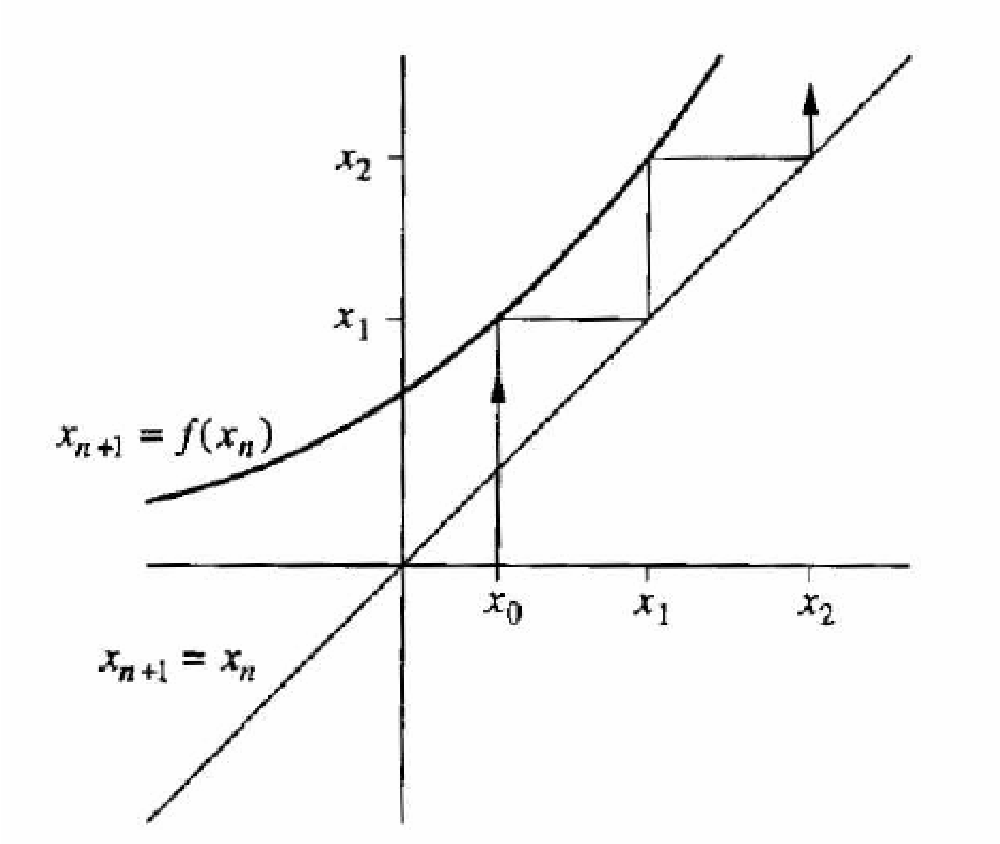
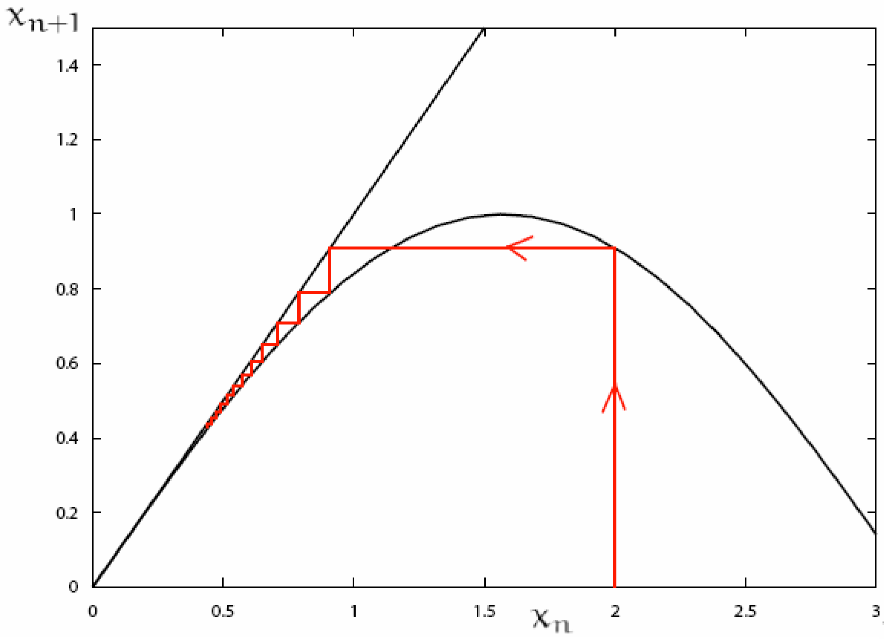
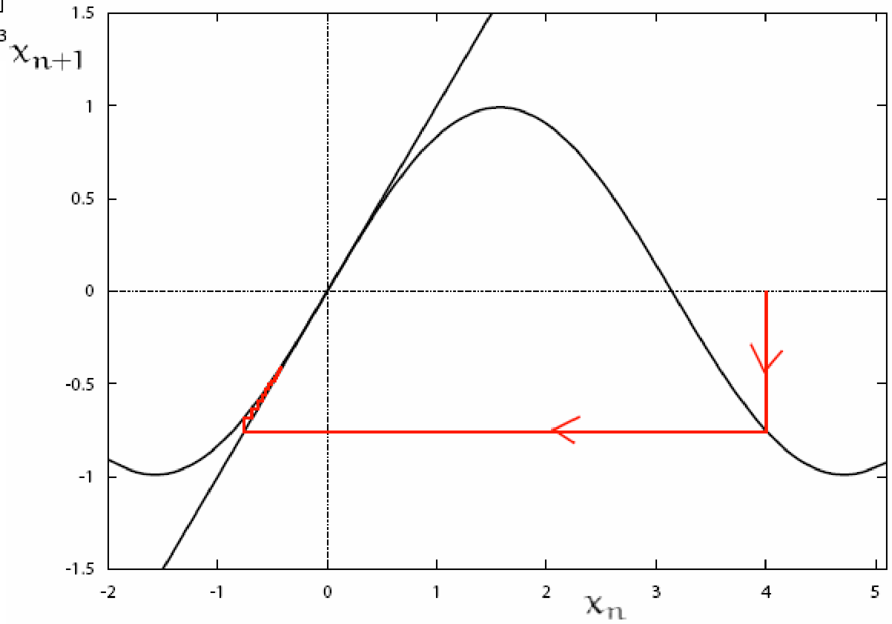
Cobwebs



$$x_{n+1} = \sin(x_n)$$

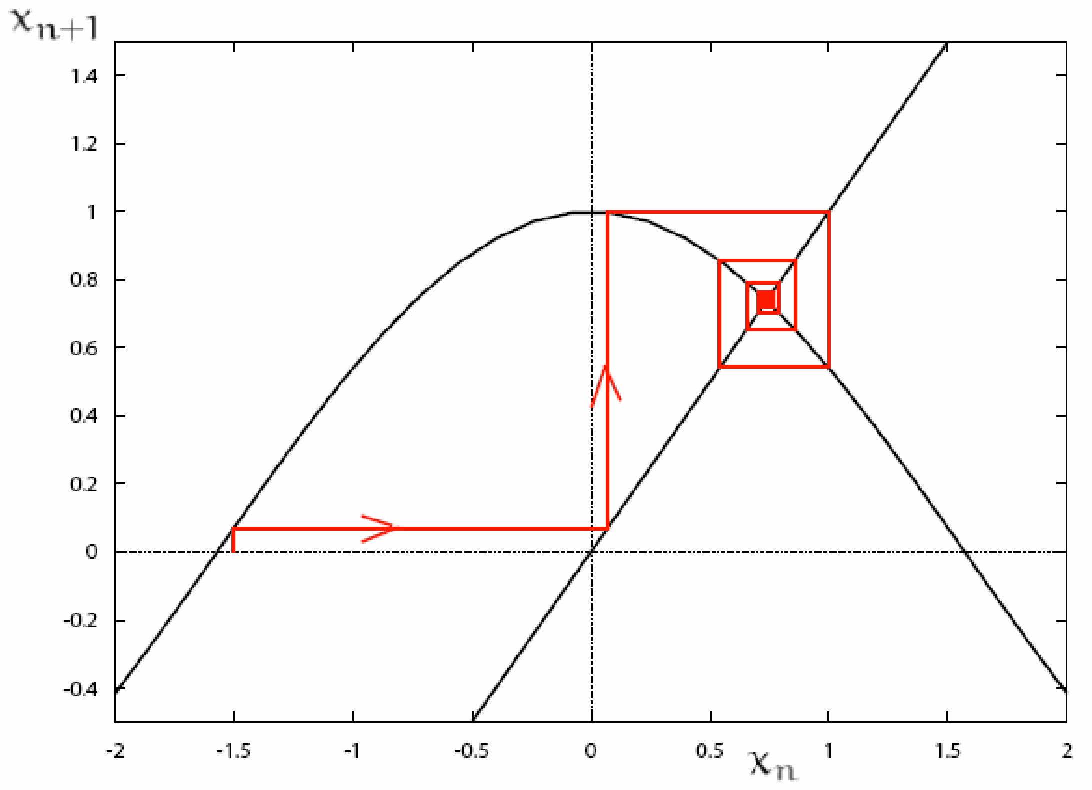


x_{n+1}
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3
x_n



x_{n+1}
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5
x_n

$$x_{n+1} = \cos(x_n)$$



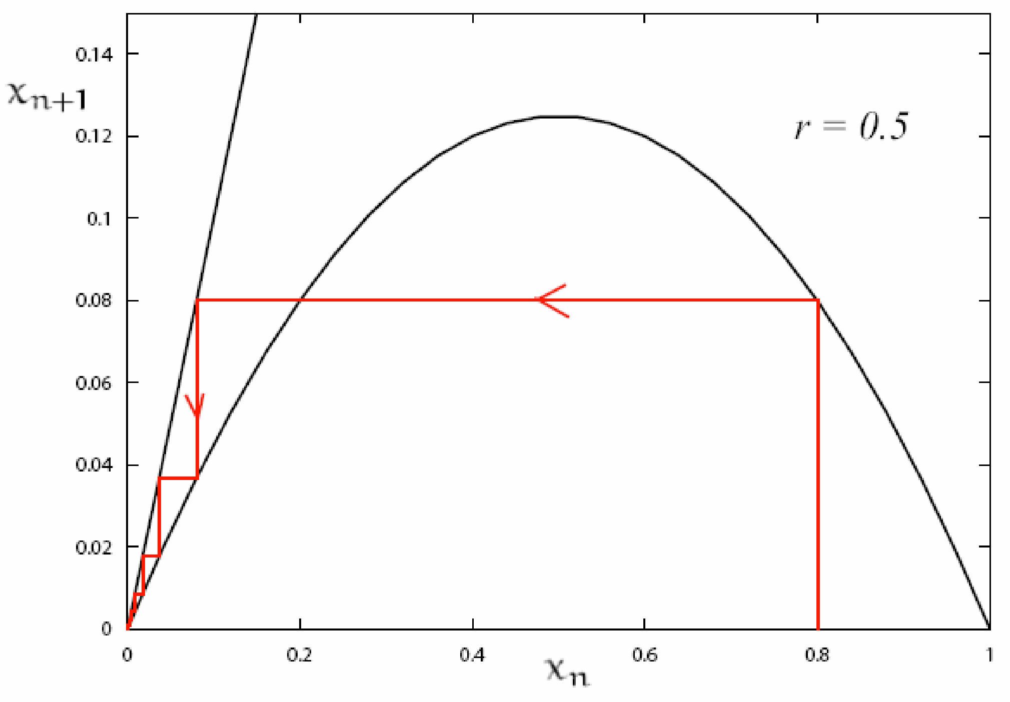
Logistic map

$$x_{n+1} = rx_n(1 - x_n)$$

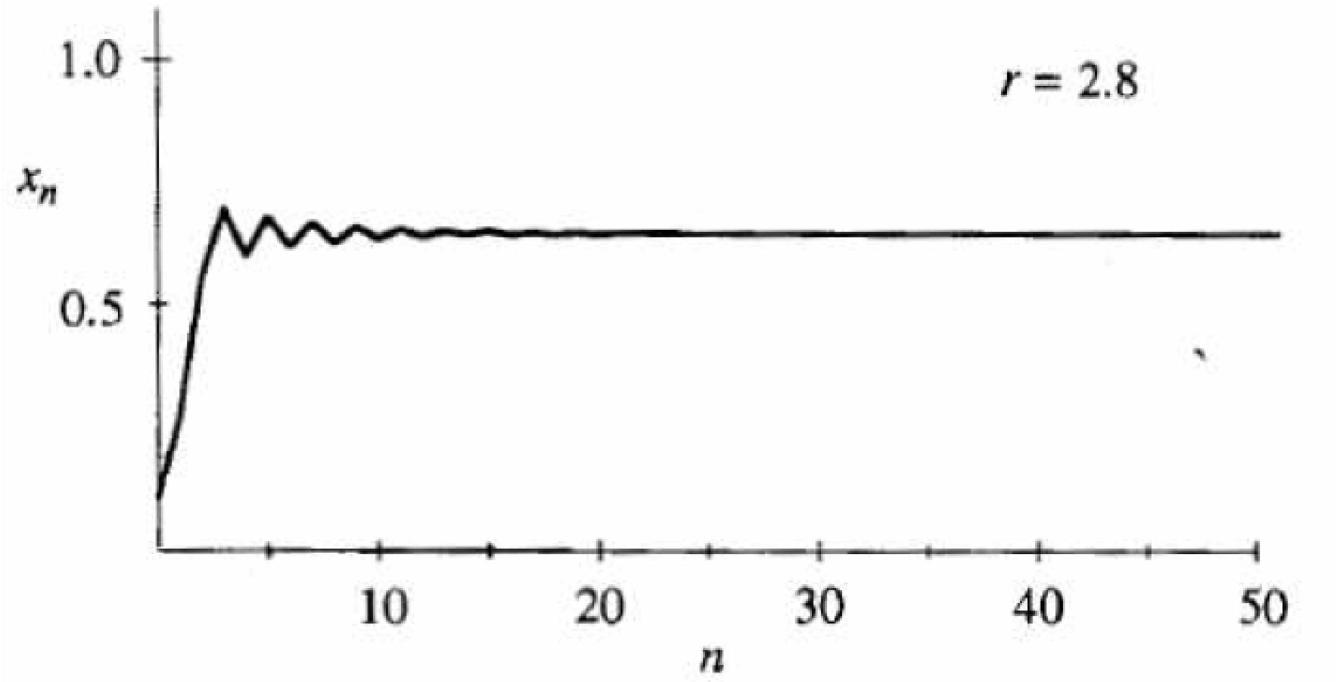
$$0 \leq r \leq 4$$

$$0 \leq x \leq 1$$

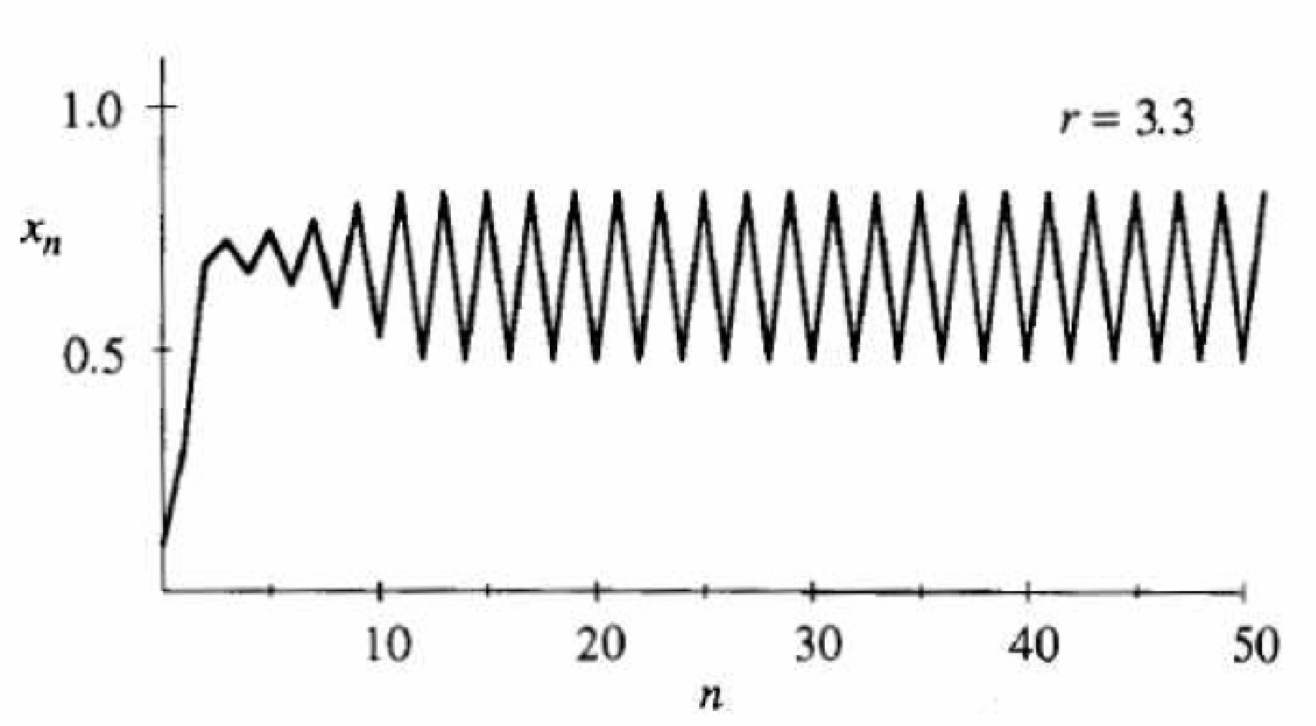
$$r < 1,\ x_n \to 0 \text{ as } n \to \infty$$



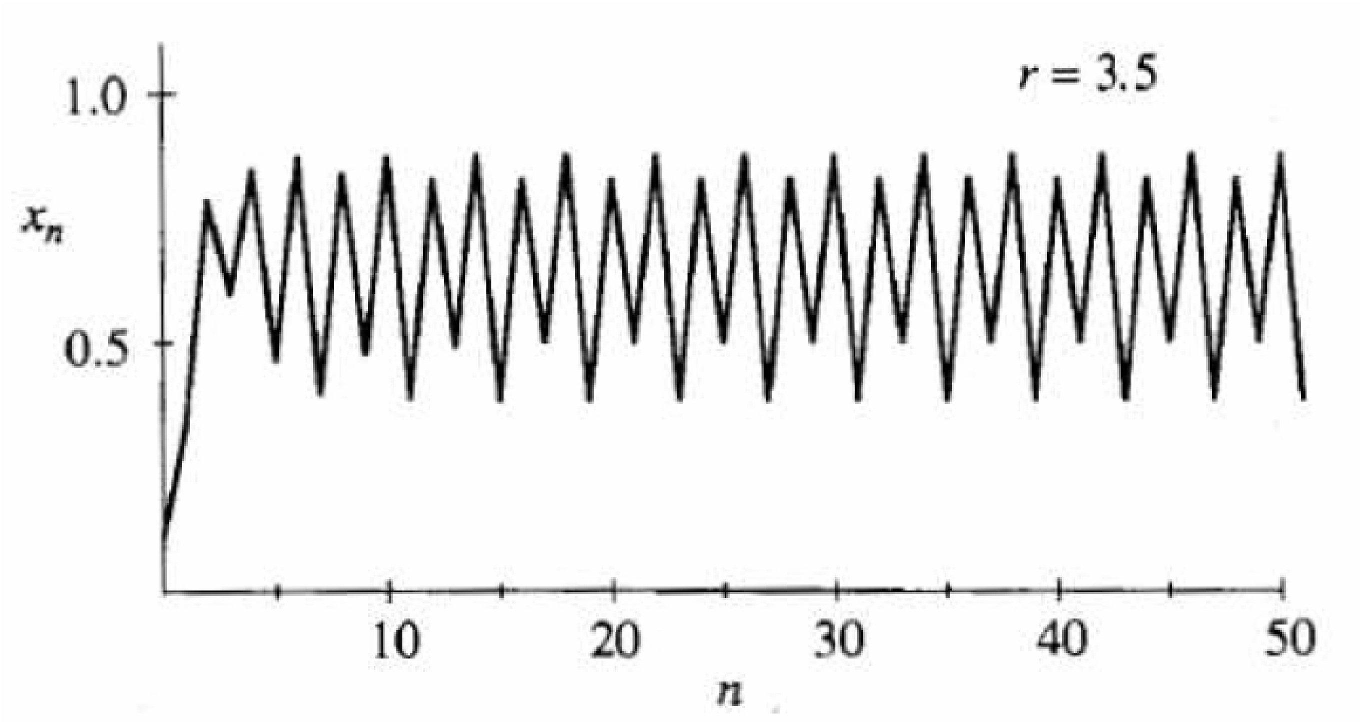
$1 < r < 3$



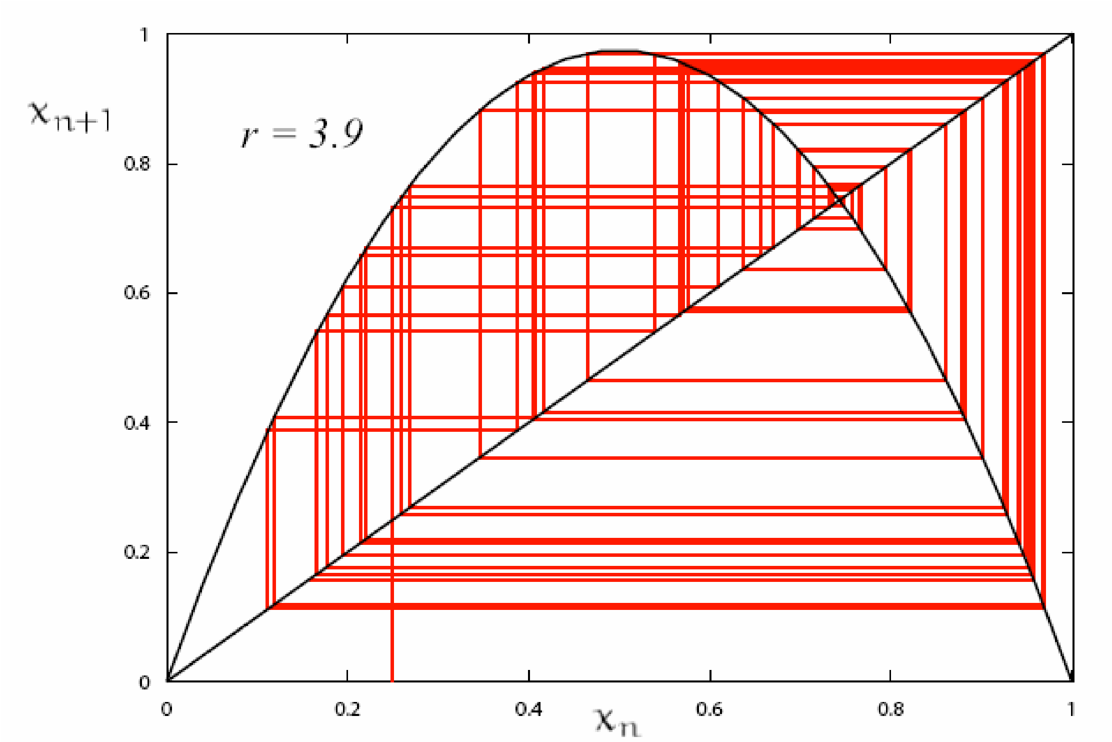
Period-2 cycle



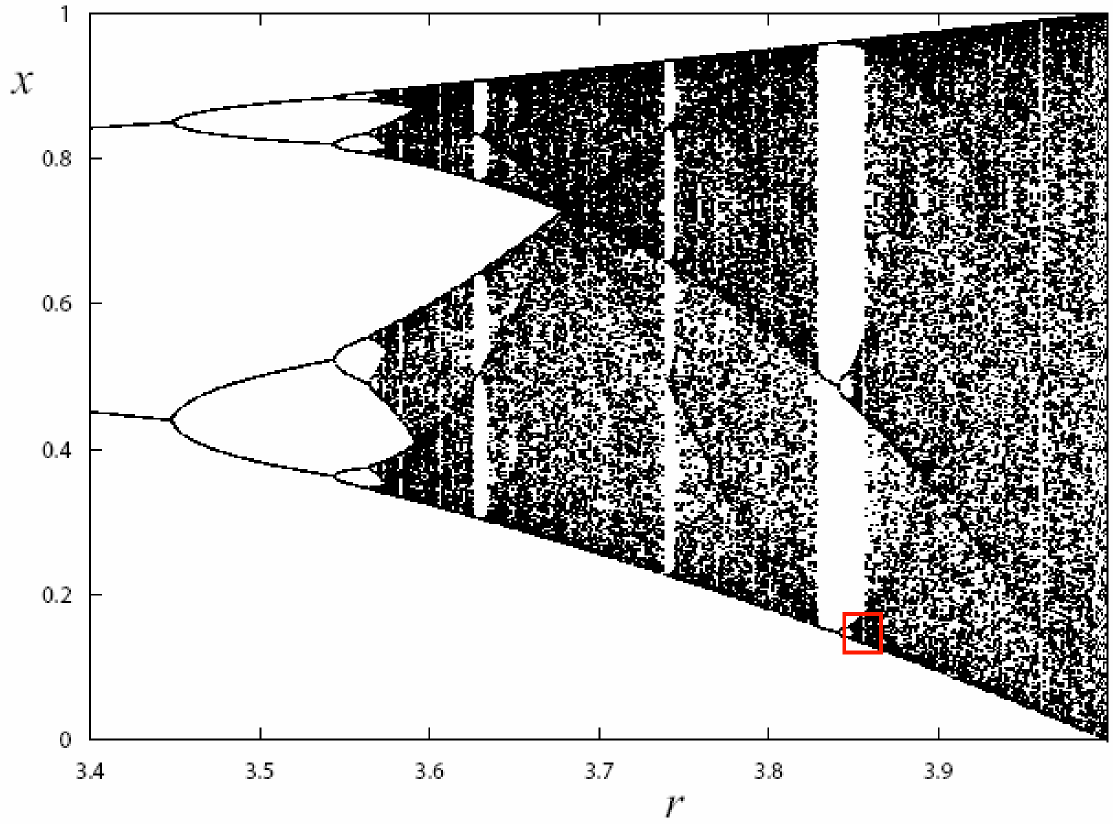
Period-4 cycle

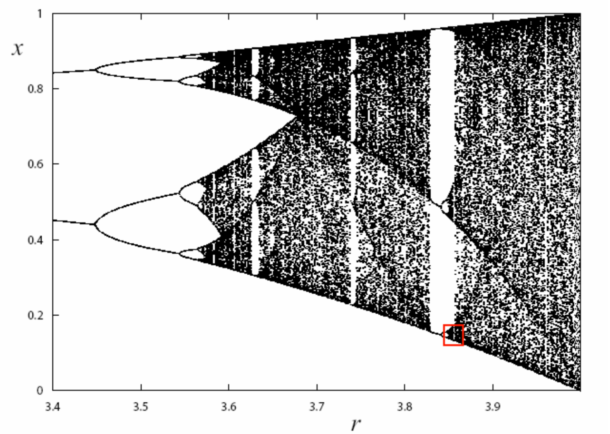


Aperiodic long-term behaviour

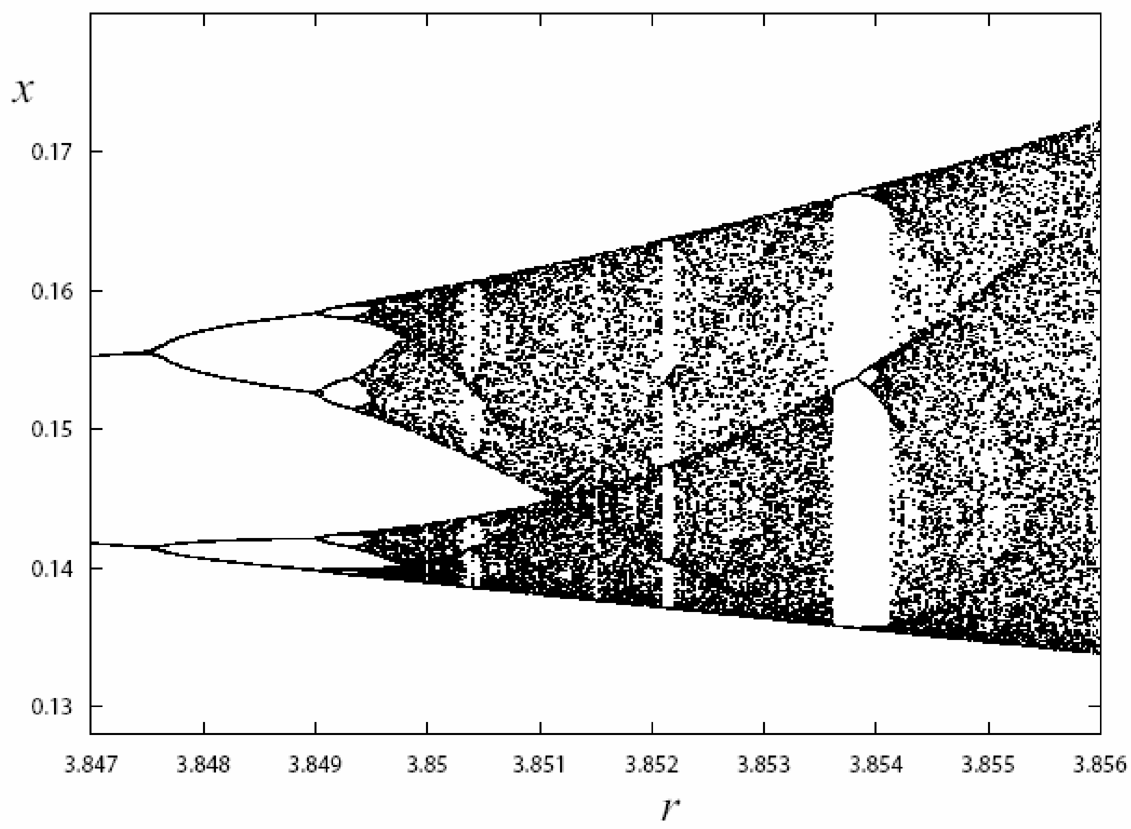


Orbit diagram (the system's attractor)



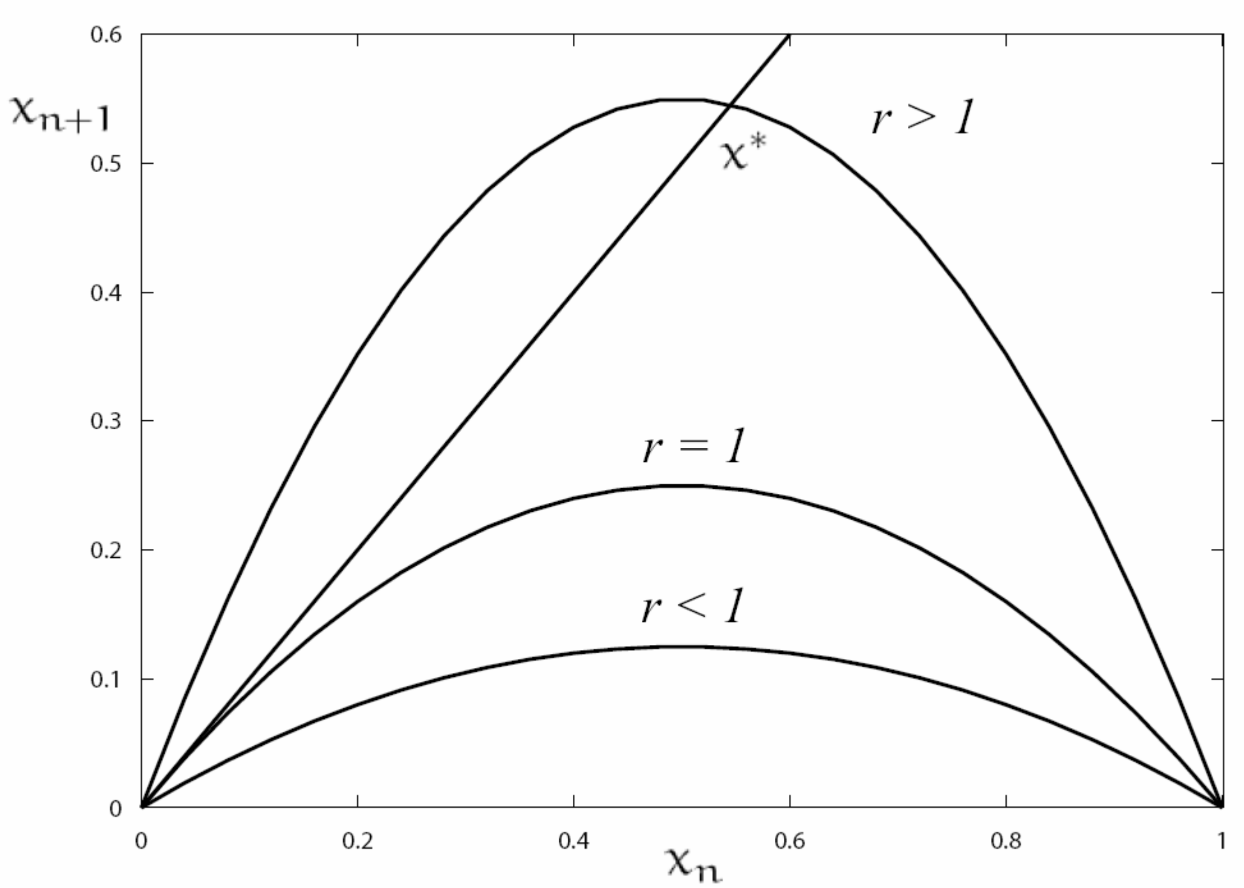


1
x
0.8
0.6
0.4
0.2
0
3.4
3.5
3.6
3.7
3.8
3.9
r



x
0.17
0.16
0.15
0.14
0.13
3.847
3.848
3.849
3.85
3.851
3.852
3.853
3.854
3.855
3.856
r

Some analysis of logistic map

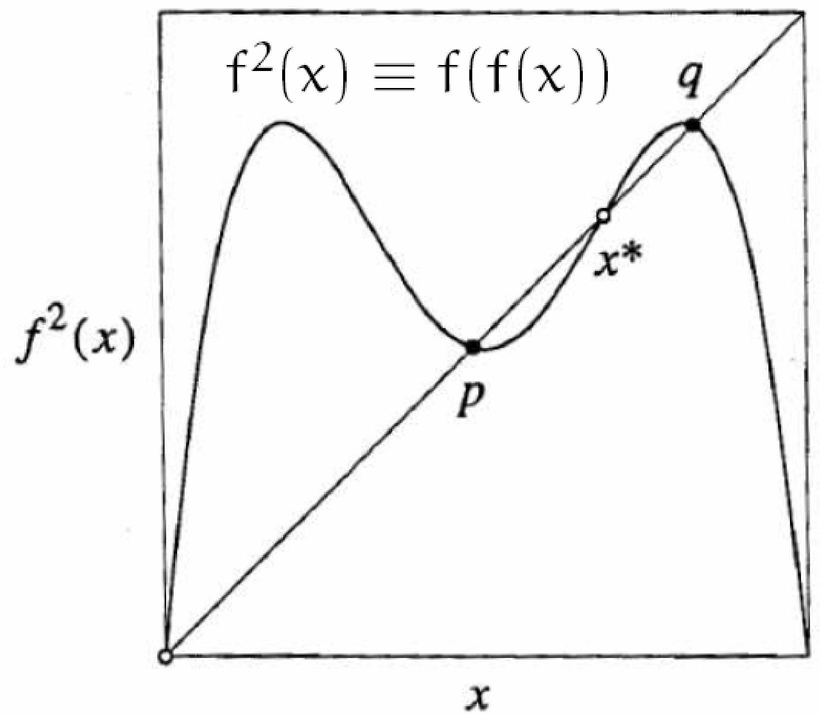


$f'(x^*) = -1$ when $r = 3$

a **flip bifurcation**

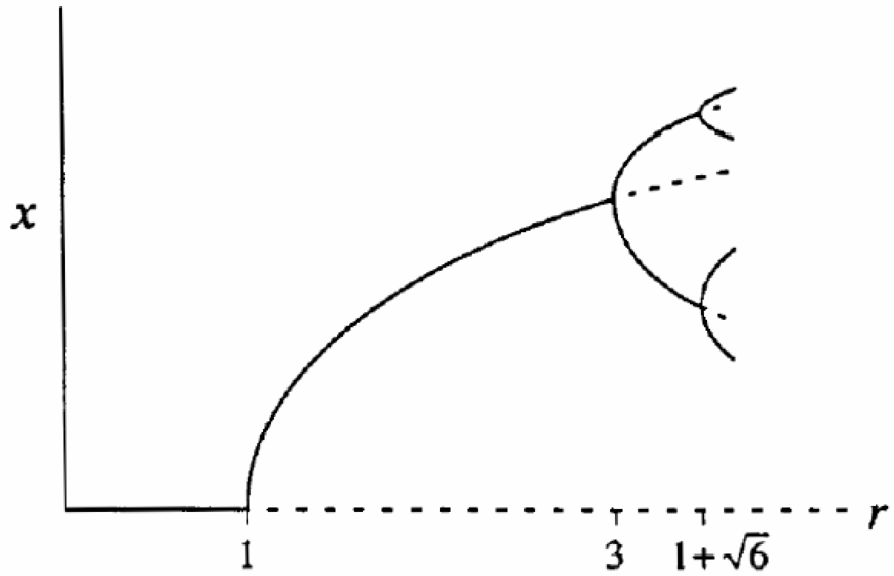
2-cycle for $r > 3$

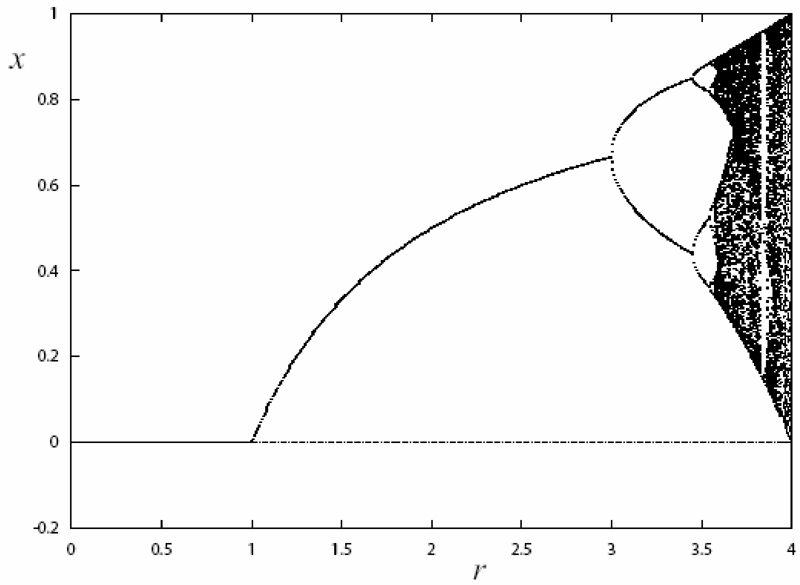
$$f(p) = q \text{ and } f(q) = p$$



$$p,\ q = \frac{r + 1 \pm \sqrt{(r-3)(r+1)}}{2r}$$

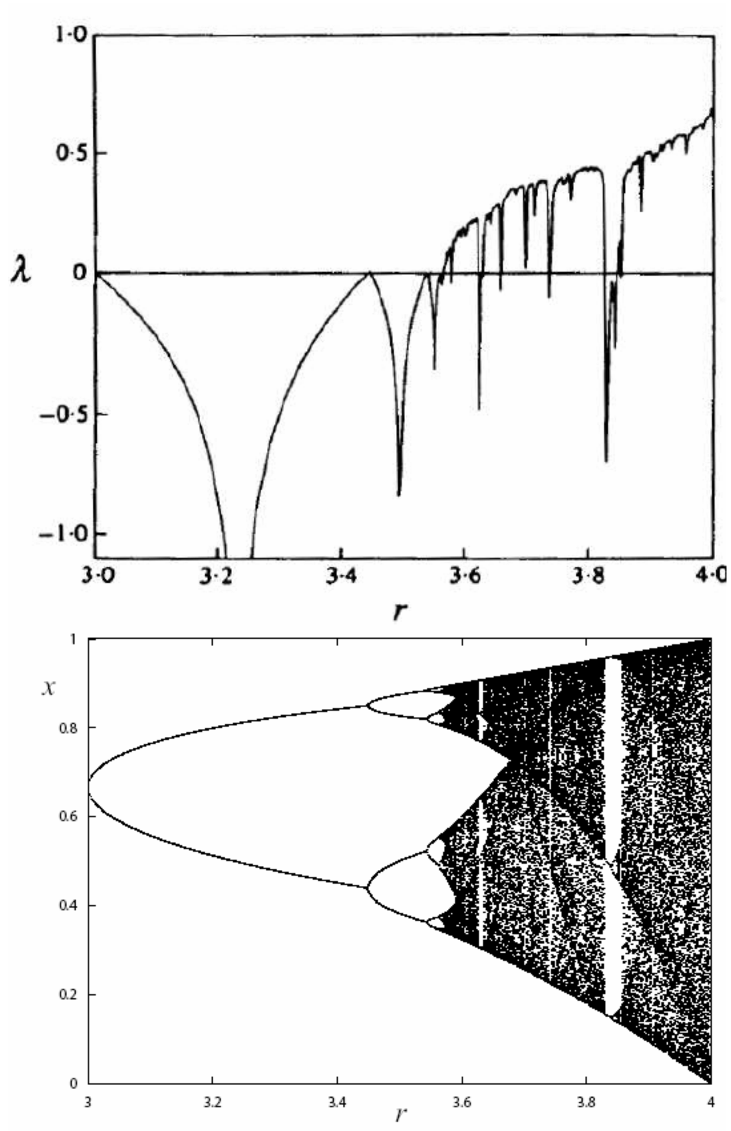
Can study stability


x
r
1
3
1+√6

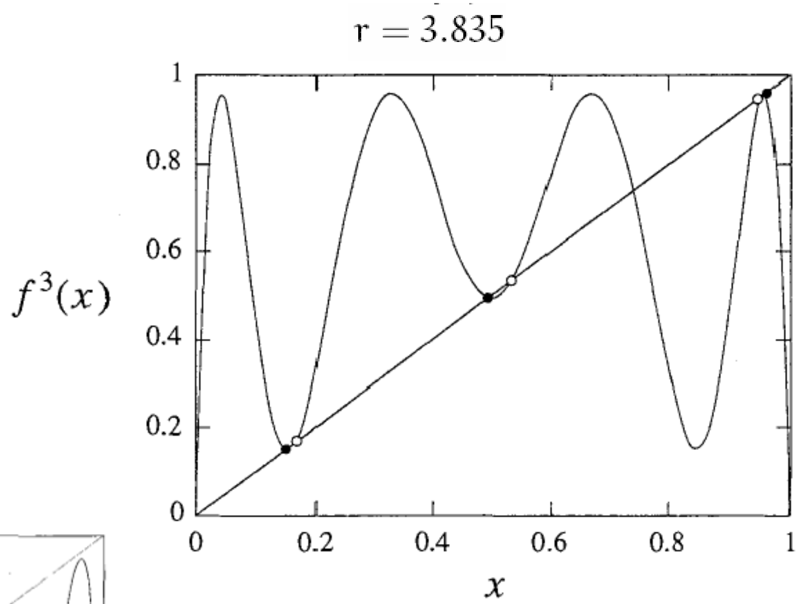

x
r
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
3
3.5
4

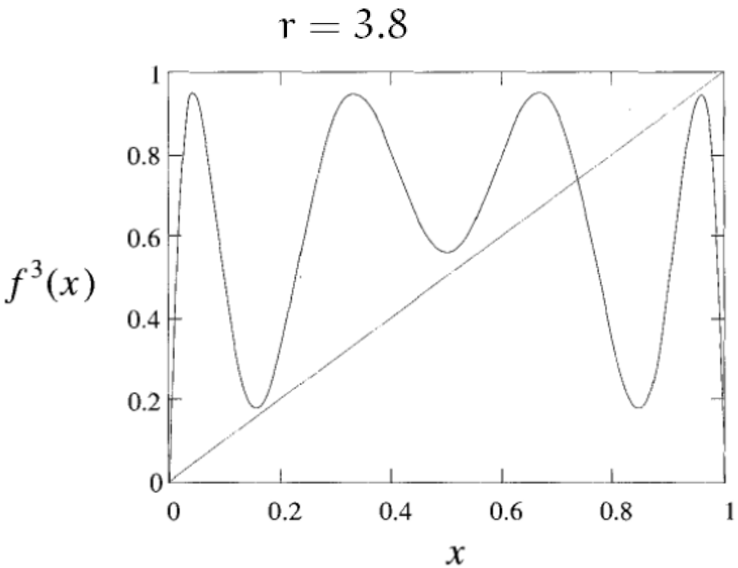
Lyapunov exponent

$$\lambda = \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=0}^{n-1}\ln|f'(x_i)|\right]$$



Period-3 cycle





A **tangent bifurcation** at r_c