# Complexity Science Doctoral Training Centre

### CO903 Complexity and Chaos in Dynamical Systems

## Exam questions for preparation part (preparation time for two questions: 15 min)

1. First-order systems $\dot{x} = f(x),\ x \in \mathbb{R}$: fixed points, stability, graphical analysis of vector field.
2. Linear stability analysis for $\dot{x} = f(x),\ x \in \mathbb{R}$.
3. Existence and uniqueness theorem in $\mathbb{R}$. Potential for $\dot{x} = f(x),\ x \in \mathbb{R}$.
4. Euler and Improved Euler methods in numerical simulations.
5. Bifurcations in 1D (saddle-node, transcritical, pitchfork).
6. Multistability and hysteresis.
7. Flow on the circle (uniform oscillator, non-uniform oscillator).
8. Harmonic oscillator.
9. Solving linear system in $\mathbb{R}^2$ (in $\mathbb{R}^n$ ).
10. Classification of fixed points for linear systems in $\mathbb{R}^2$.
11. Existence and uniqueness theorem in $\mathbb{R}^n$. Definitions of stability (Lyapunov, asymptotic).
12. Linear stability analysis for $\dot{x} = f(x, y),\ \dot{y} = g(x, y)$. Linear stability theorem.
13. Lyapunov theorem.
14. Hopf bifurcation (supercritical, subcritical).
15. Poincare-Bendixson theorem.
16. Relaxation oscillators.
17. Coupled oscillators.
18. Poincare maps and Floquet multipliers.
19. Defining chaos and attractor (strange attractor).
20. Main properties of the Lorenz equations: $\dot{x} = \sigma(y - x),\ \dot{y} = rx - y - xz,\ \dot{z} = xy - bz$.
21. Lyapunov exponent and a time when prediction breaks down.
22. Lorenz map.
23. One-dimensional maps, cobwebs.
24. Fixed points and linear stability of 1D maps.
25. A flip bifurcation in the logistic map.
26. Lyapunov exponent for 1D map.

27. Routes to chaos.

28. Fractal dimensions.

29. Global bifurcation (saddle-node bifurcation of cycles, homoclinic bifurcation, heteroclinic bifurcation).

30. State-space reconstruction