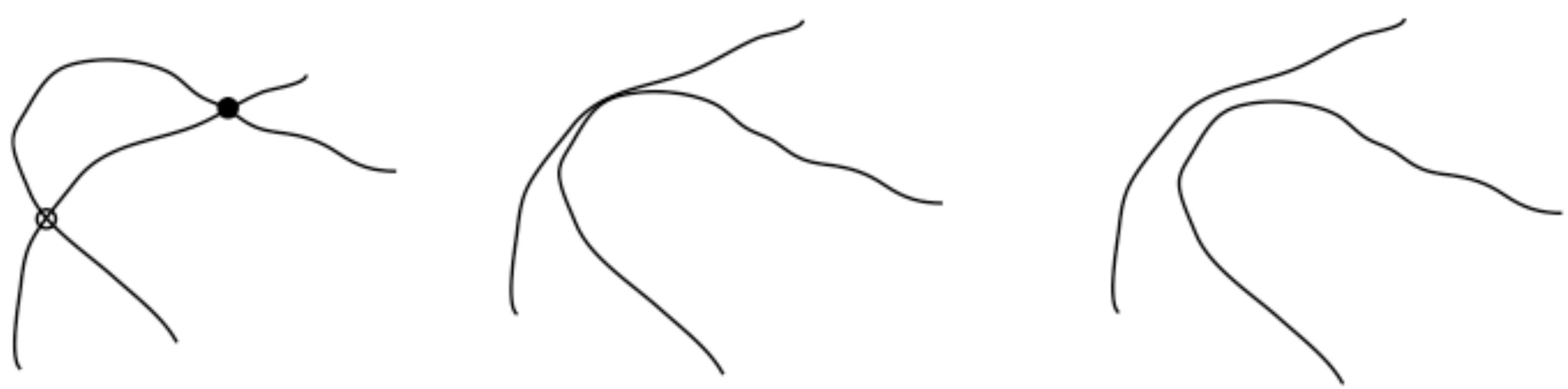
Local and Global bifurcations

Local bifurcation

Saddle-node bifurcation

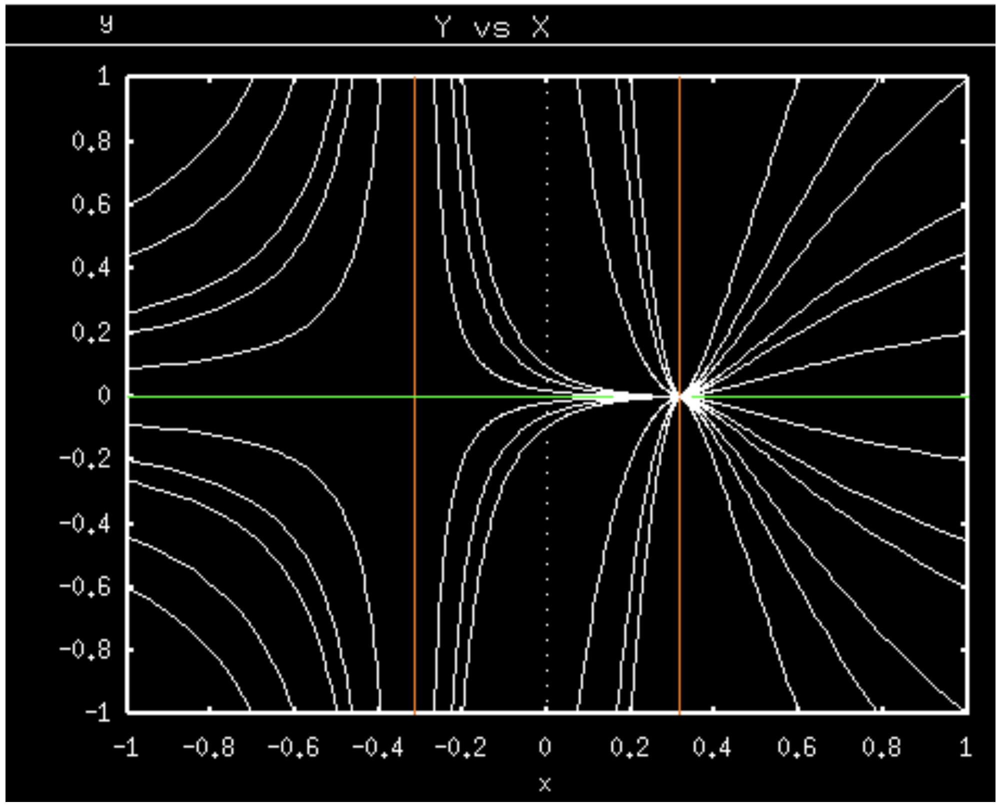
$$\dot{x} = f(x, y)$$
$$\dot{y} = g(x, y)$$



$\mu = \mu_c$

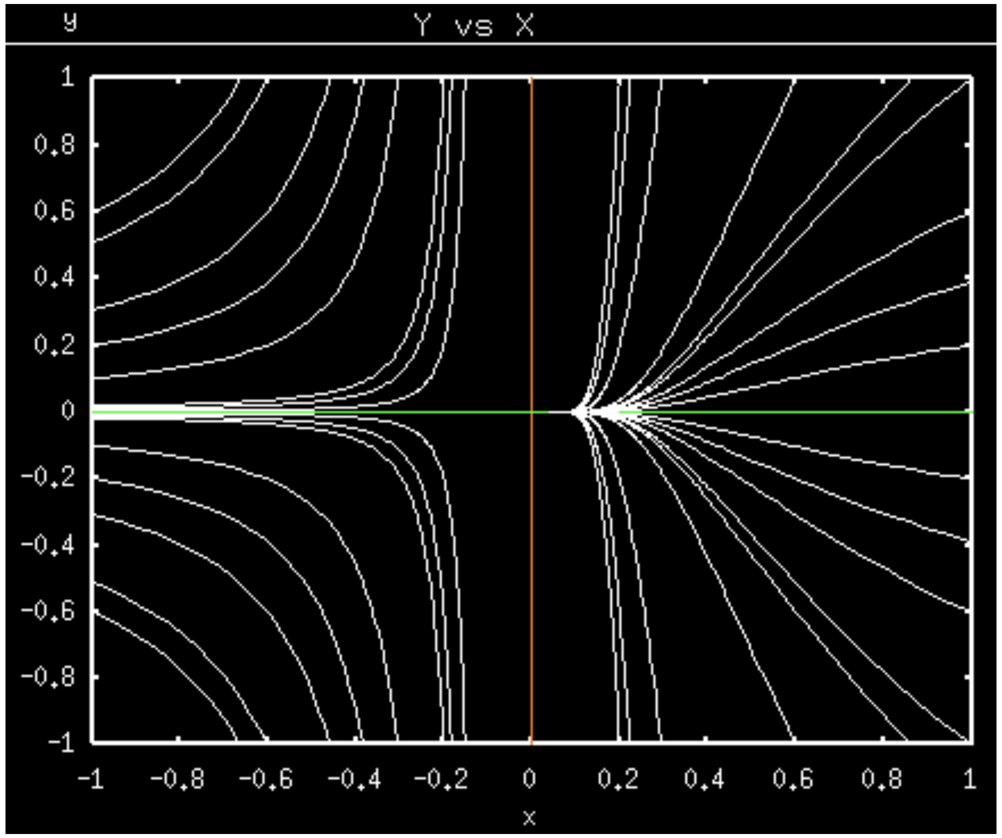
Saddle-node bifurcation

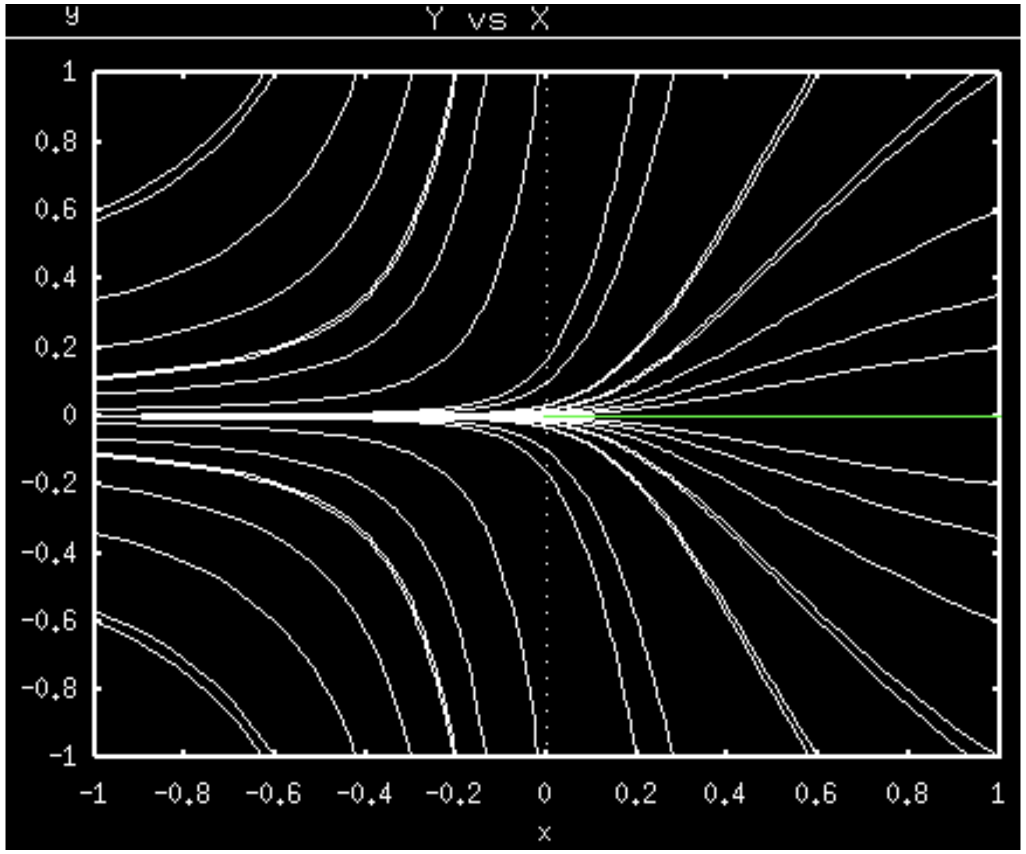
$$\dot{x} = \mu - x^2$$
$$\dot{y} = -y$$



$\mu > 0$

$\mu = 0$

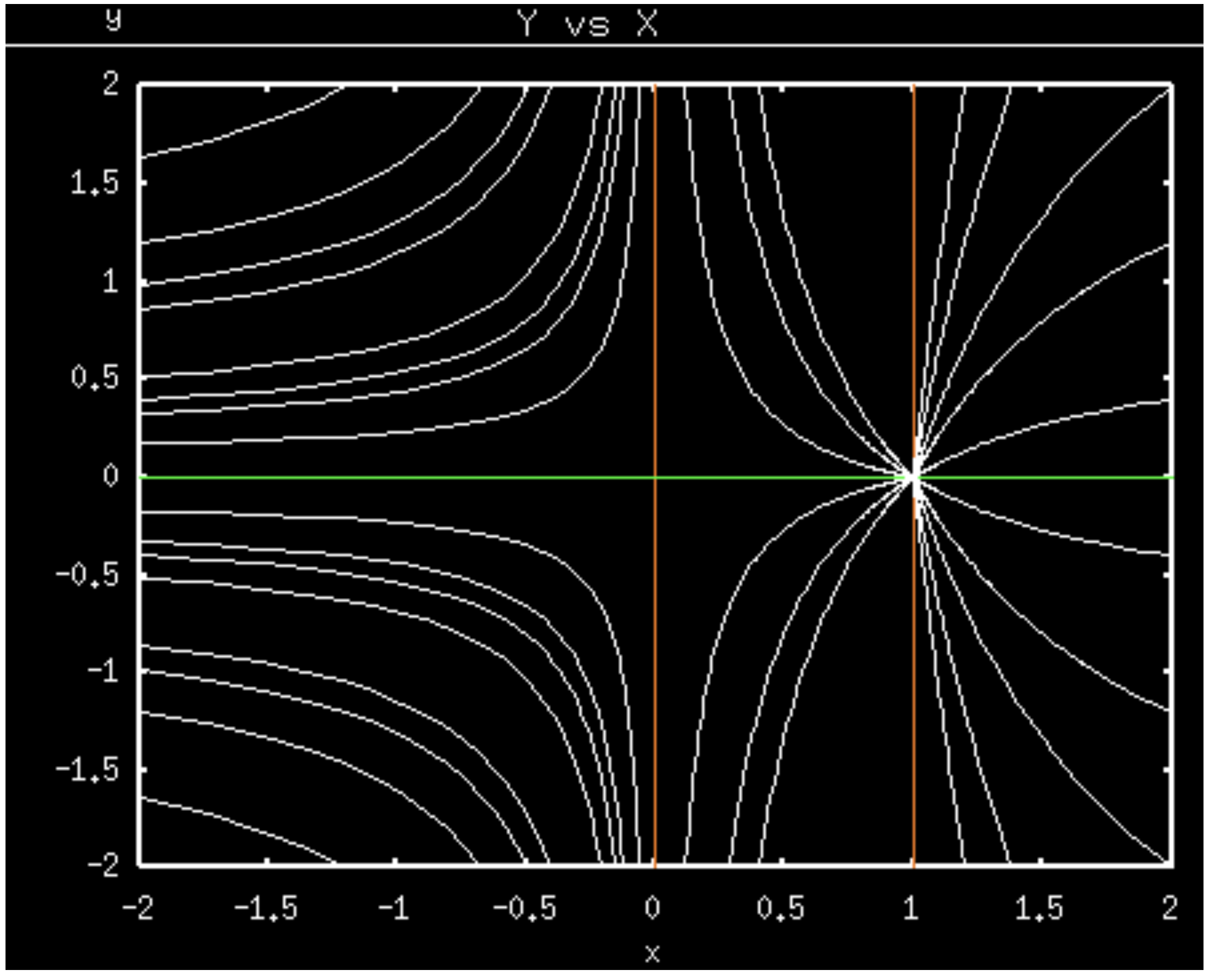




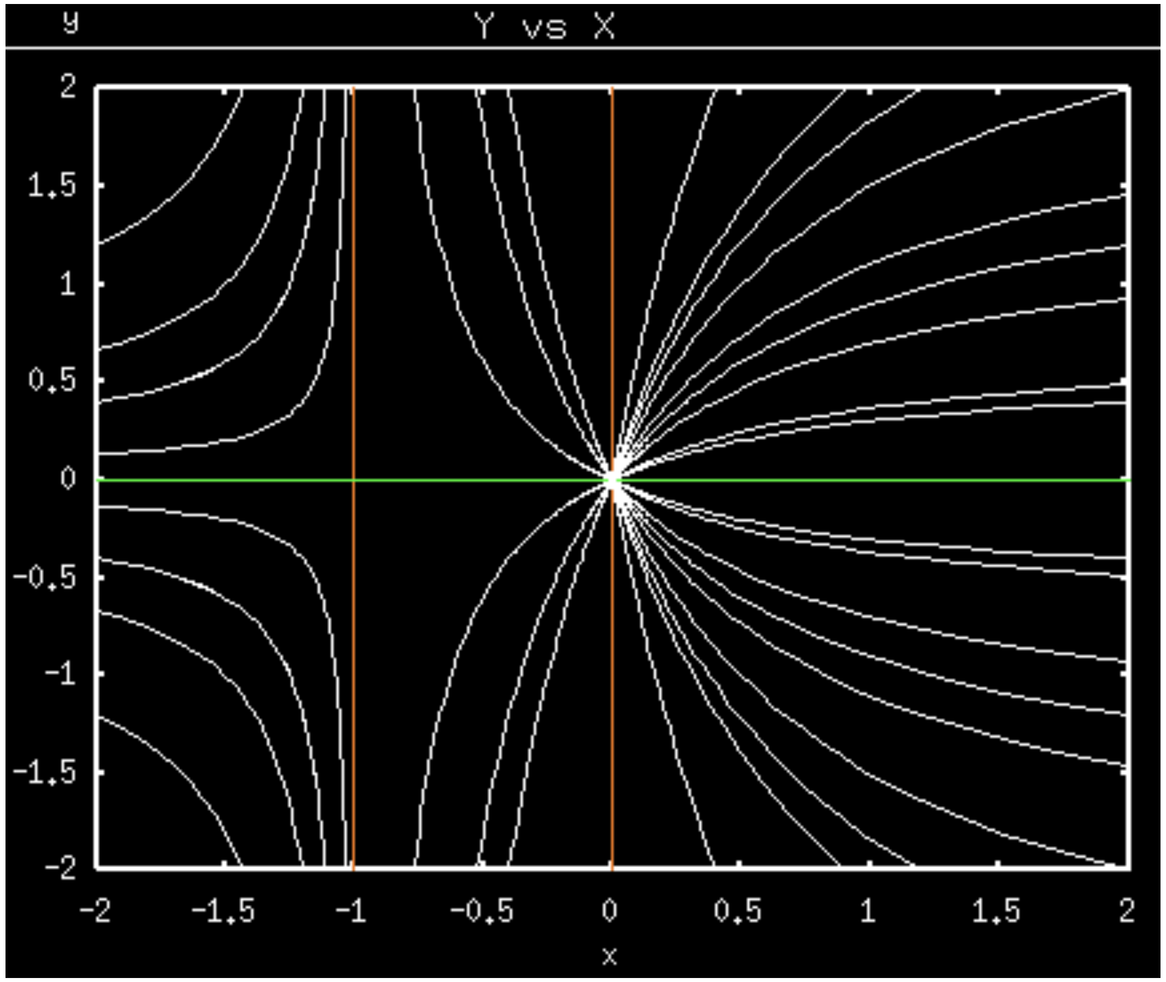
$\mu < 0$

Transcritical bifurcation

$$\dot{x} = \mu x - x^2$$
$$\dot{y} = -y$$



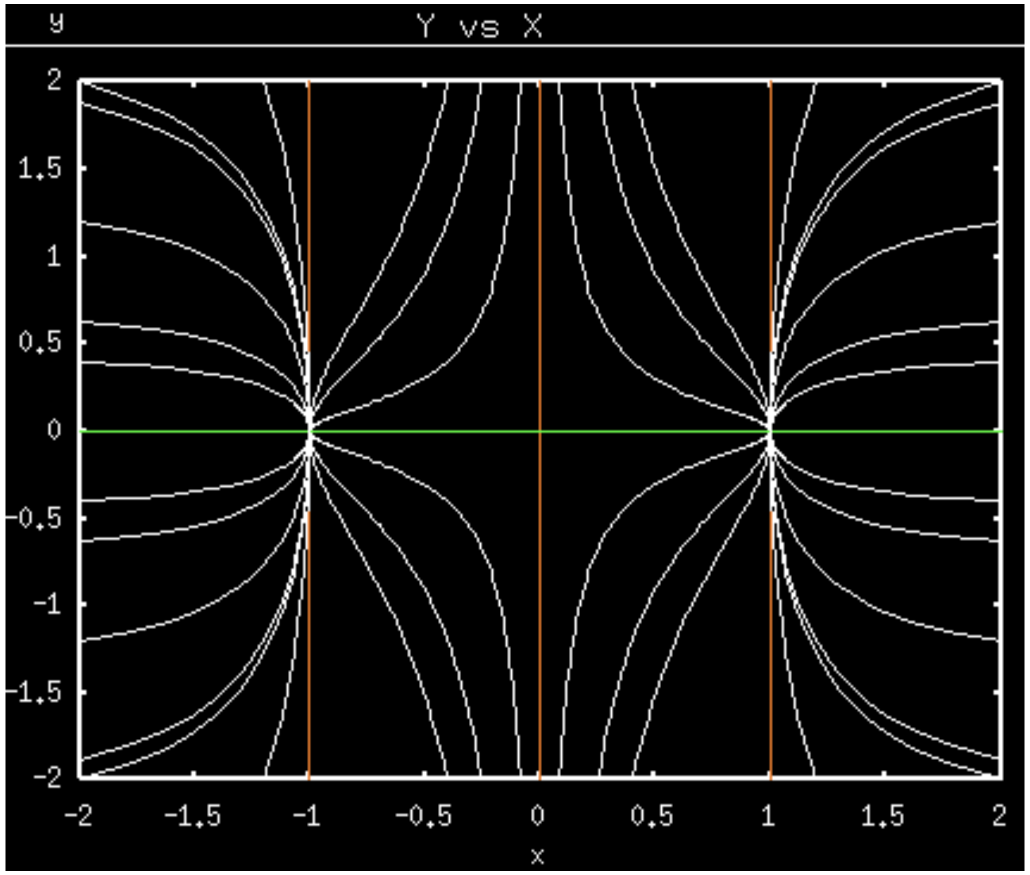
$\mu > 0$



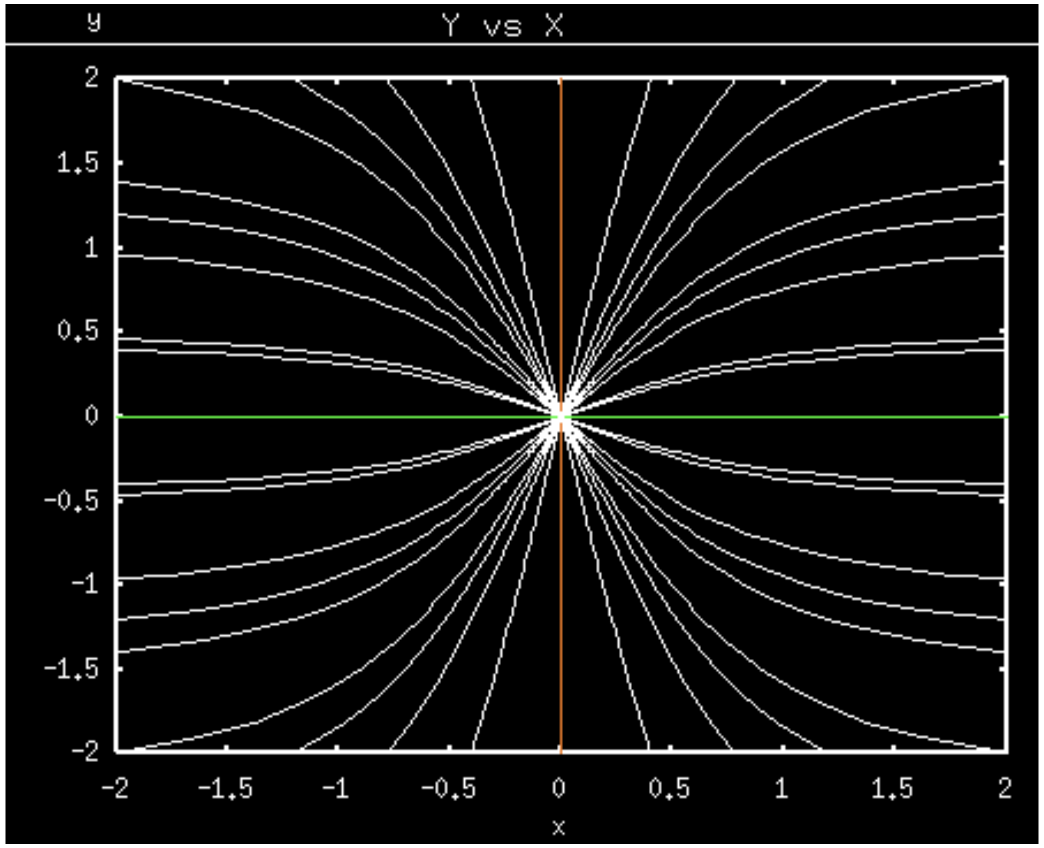
$\mu < 0$

Supercritical pitchfork bifurcation

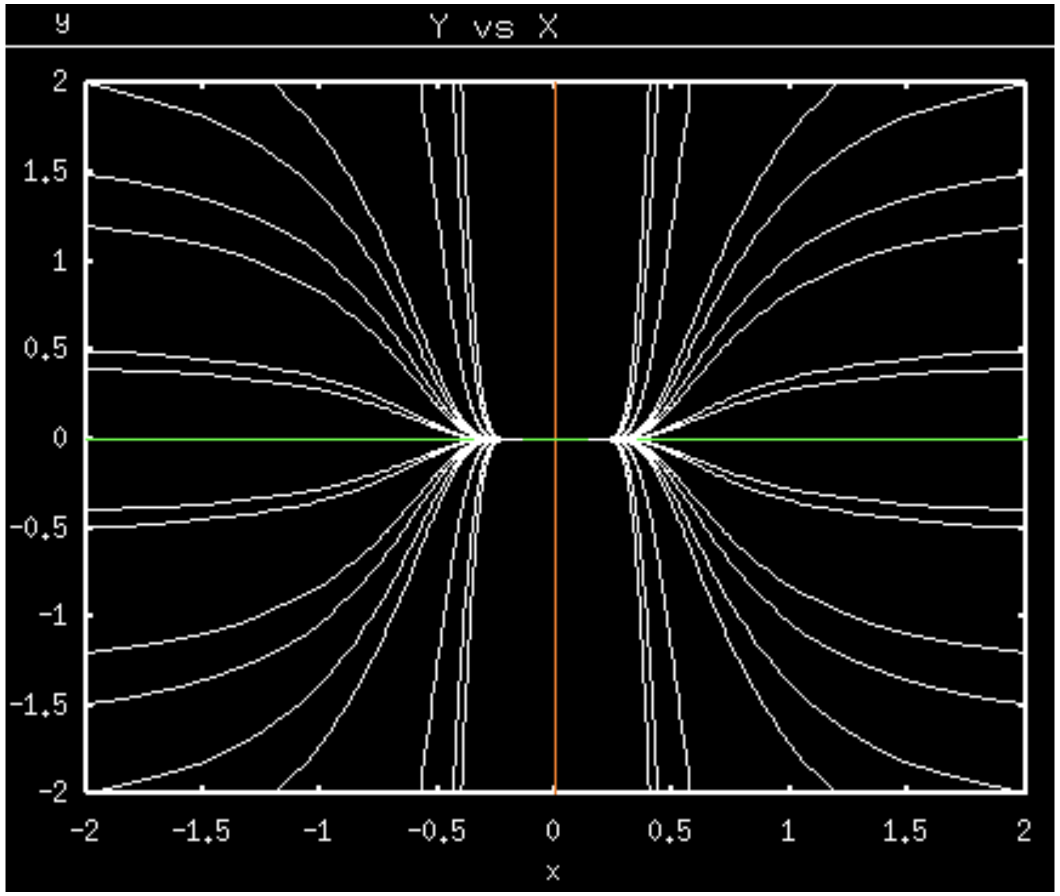
$$\dot{x} = \mu x - x^3$$
$$\dot{y} = -y$$



$\mu > 0$

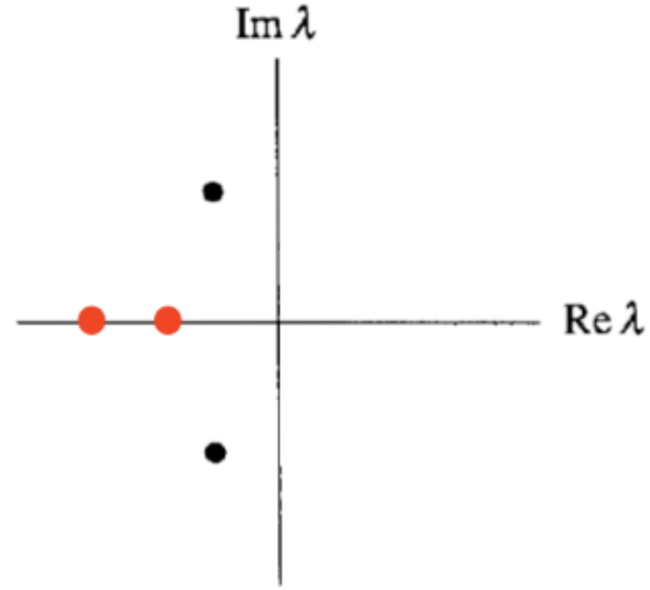


$\mu < 0$



$\mu = 0$

Hopf bifurcation

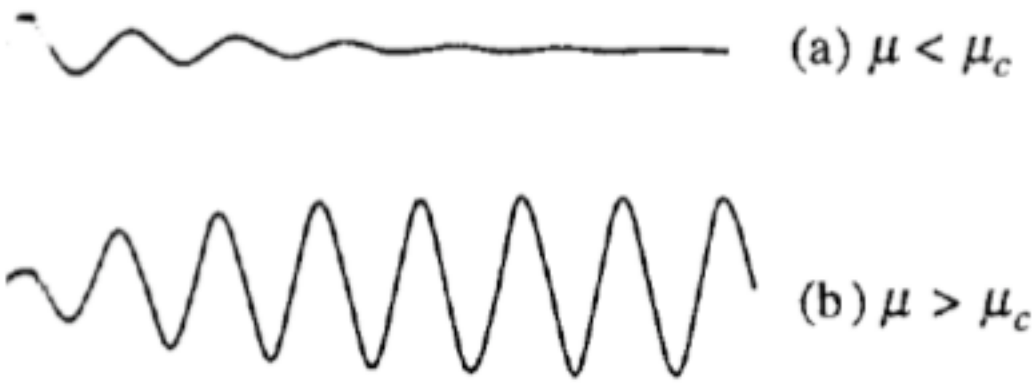


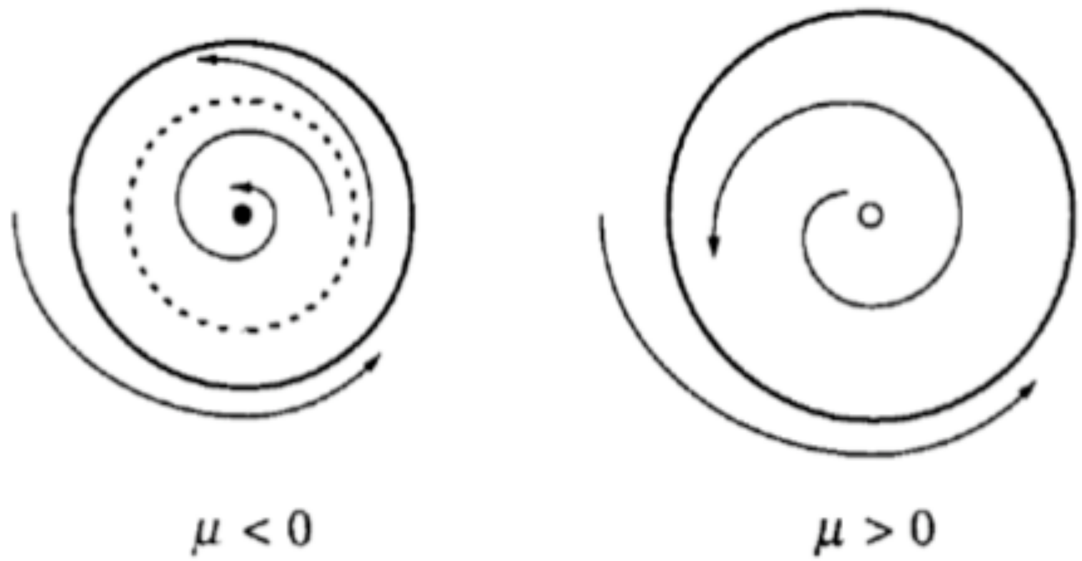
subcritical

$$\dot{r} = \mu r + r^3 - r^5$$

$$\dot{\theta} = 1$$

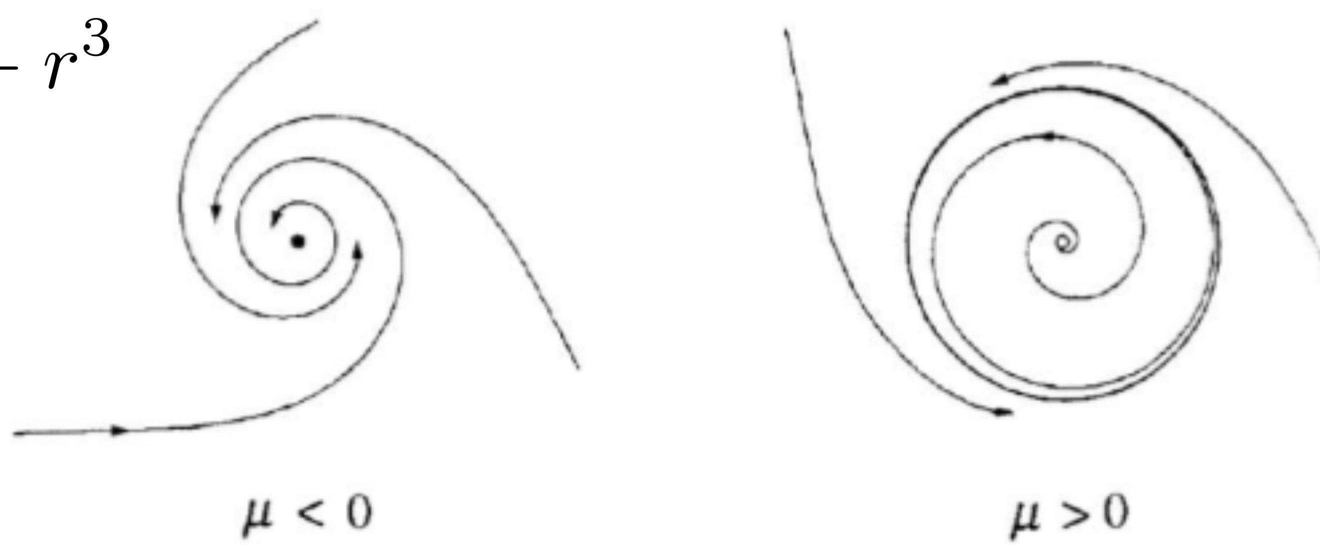
supercritical





$$\dot{r} = \mu r - r^3$$

$$\dot{\theta} = 1$$



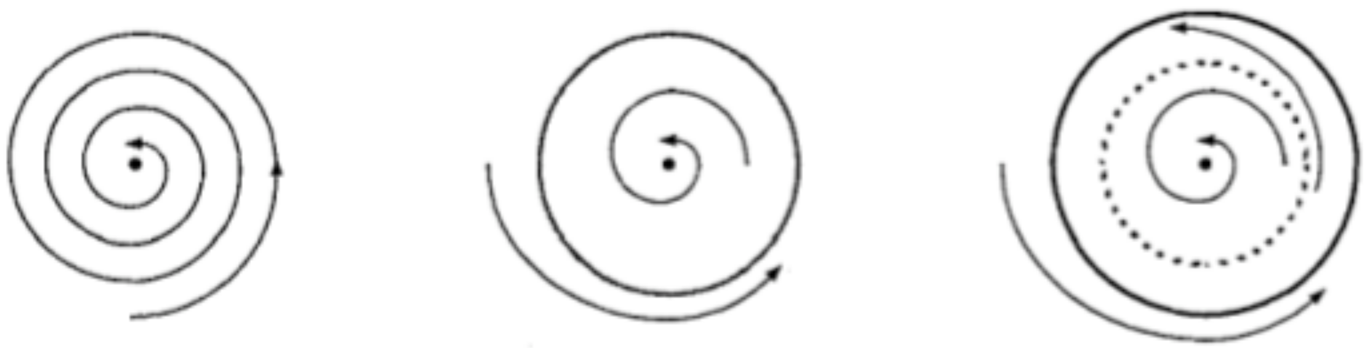
Global bifurcations

Saddle-node bifurcation of cycles

A bifurcation in which two limit cycles coalesce and annihilate

$$\dot{r} = \mu r + r^3 - r^5$$

$$\dot{\theta} = 1$$



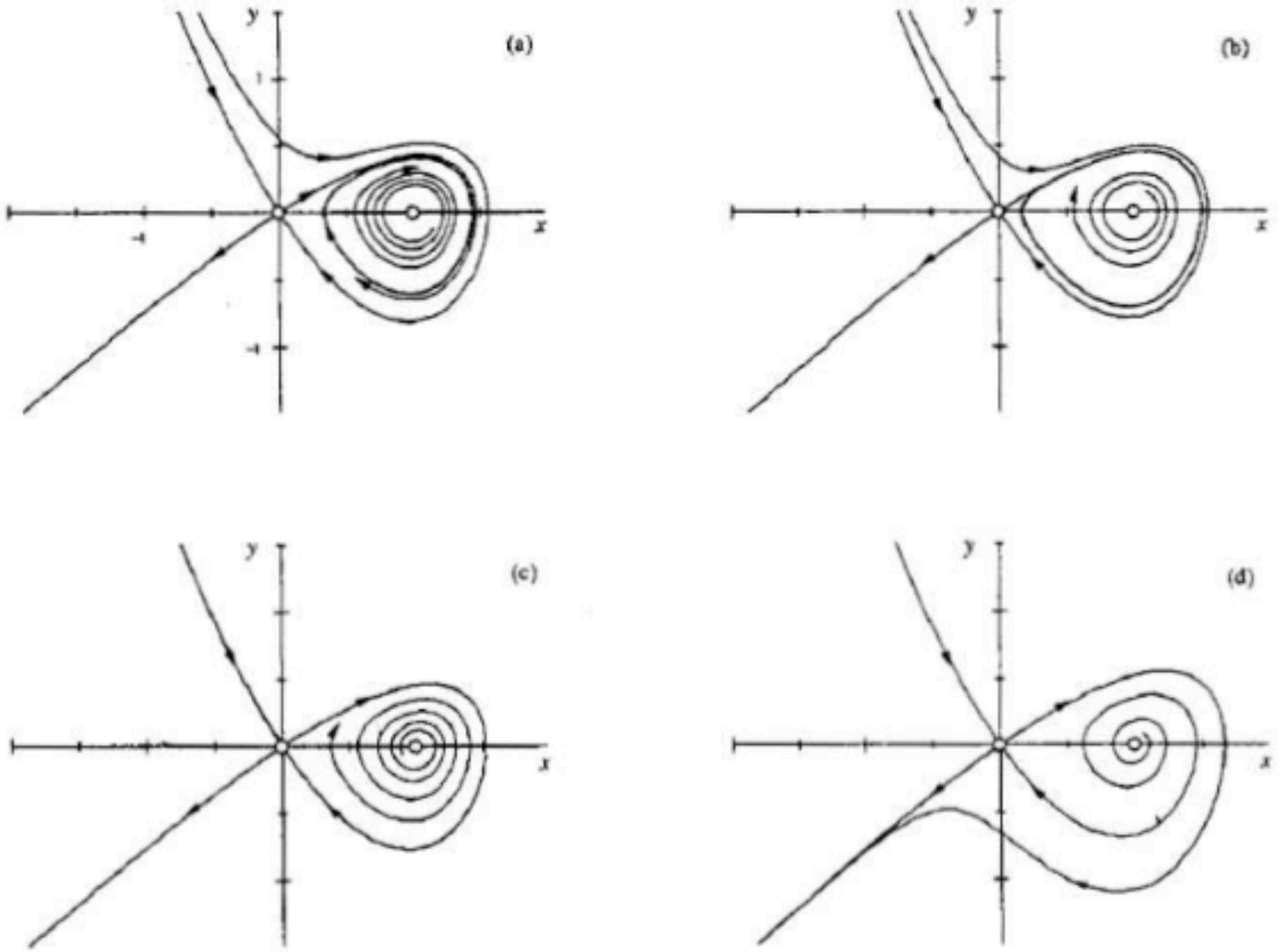
$\mu < \mu_c$ $\qquad$ $\mu = \mu_c$ $\qquad$ $0 > \mu > \mu_c$

Homoclinic bifurcation

Part of a limit cycle moves closer and closer to a saddle point. At the bifurcation the cycle touches the saddle point and becomes a homoclinic orbit

$$\dot{x} = y$$

$$\dot{y} = \mu y + x - x^2 + xy$$



$$T \to \infty$$

Heteroclinic bifurcation

A heteroclinic orbit (connection) is a path in phase-space which joins two different equilibrium points.

$$\dot{x} = f(x)$$

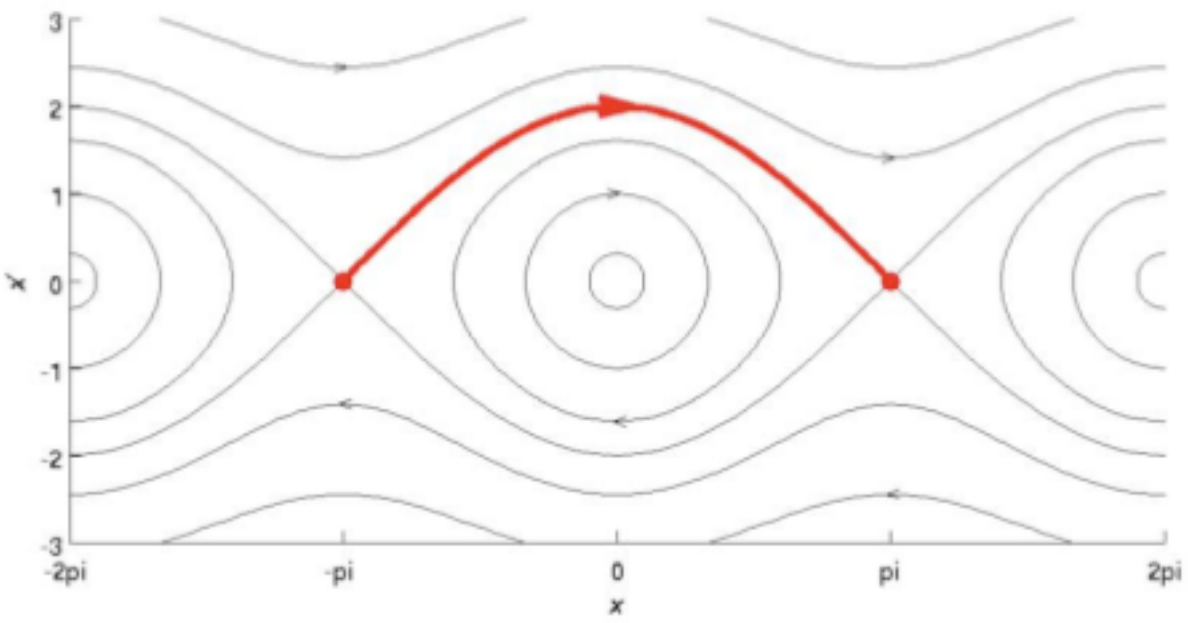
Fixed points x_0 & x_1

$\phi(t)$ - a heteroclinic orbit from x_0 to x_1 if

$$\phi(t) \to x_0, \quad t \to -\infty$$

$$\phi(t) \to x_1, \quad t \to +\infty$$

$$\ddot{x} + \sin x = 0$$



A heteroclinic bifurcation is a global bifurcation involving a heteroclinic cycle.

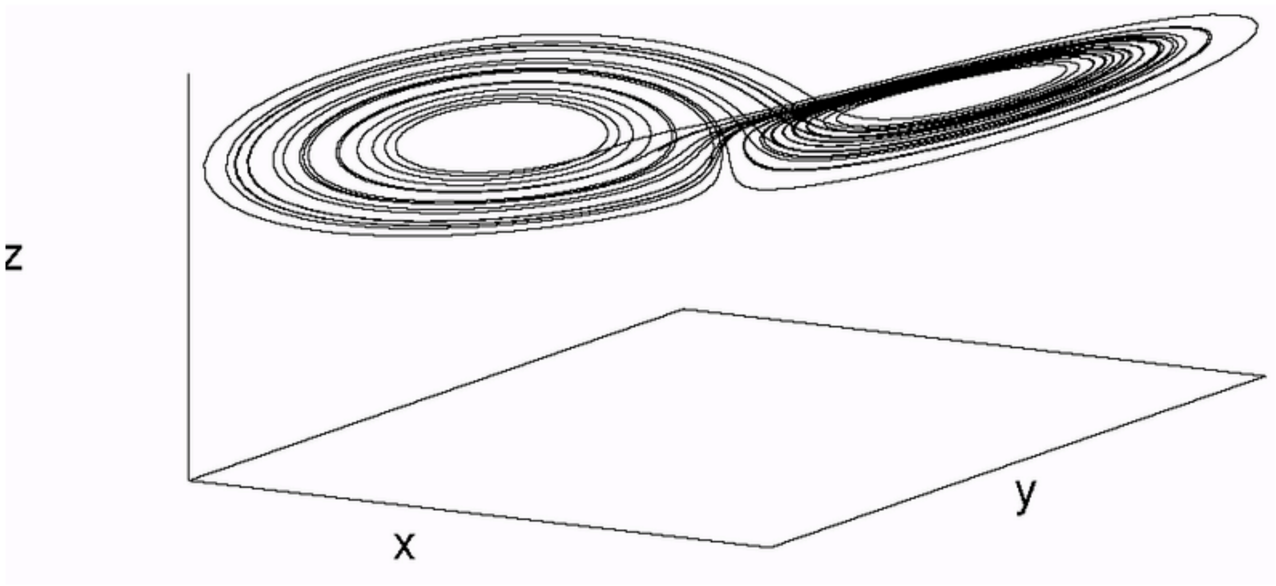
State-space reconstruction

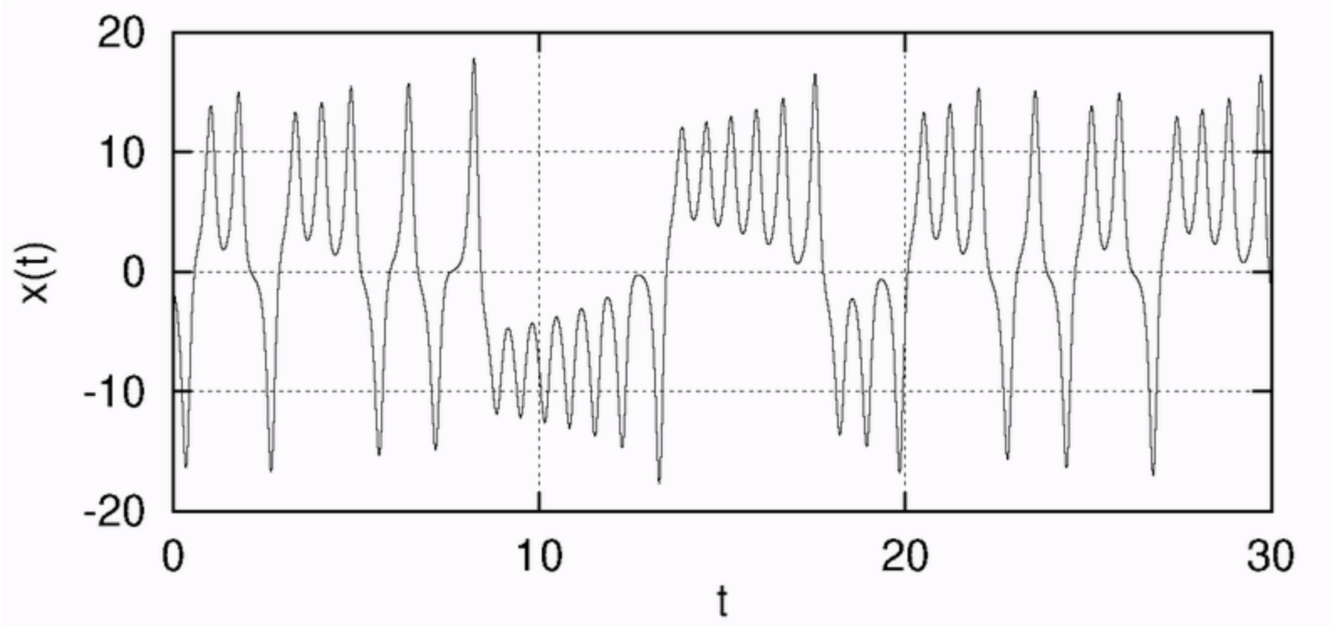
A data analysis technique known as attractor reconstruction (Embedding theorem, Takens 1981).

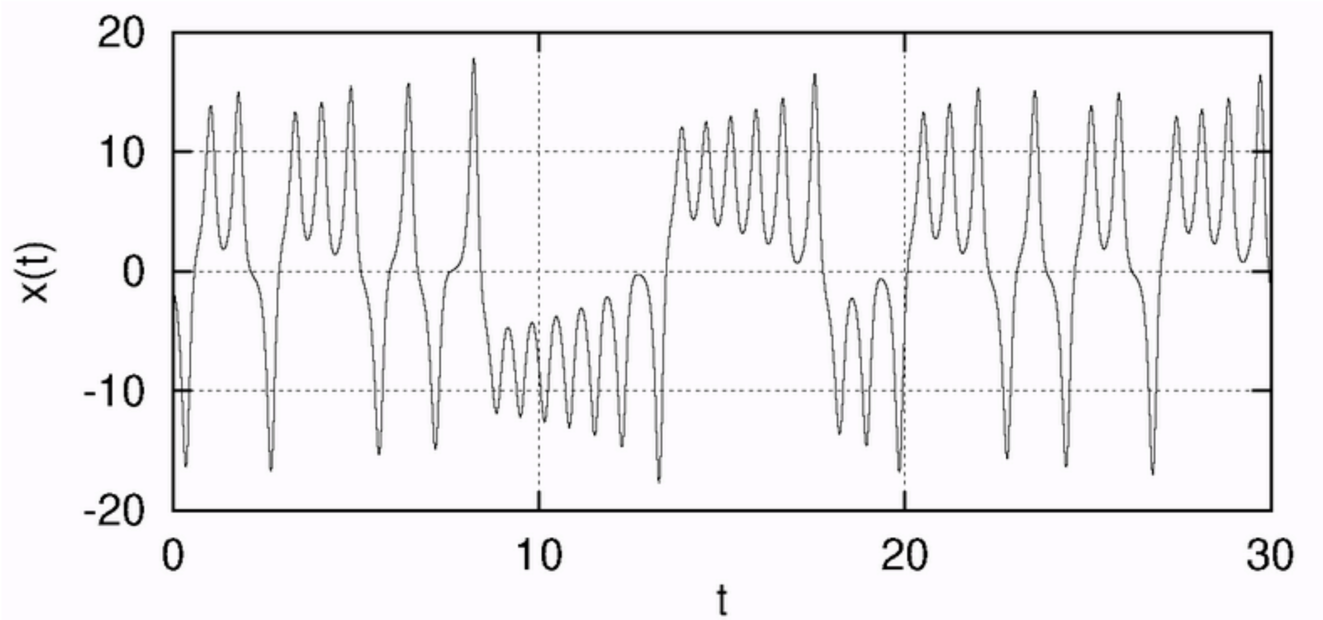
For system governed by an attractor, the dynamics of the full phase-space can be reconstructed from measurements of just a single time series

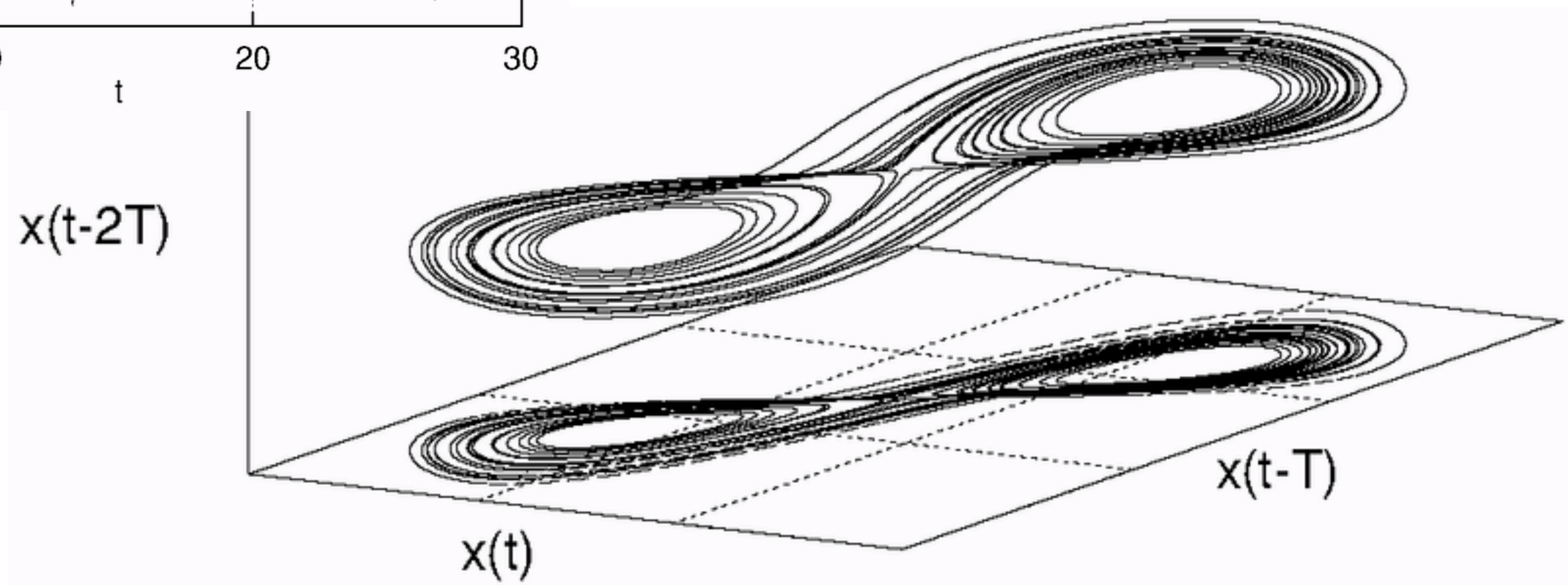
$$(B(t), B(t+\tau), B(t+2\tau), ..., B(t+(m-1)\tau))$$

m - dimensional state-space









τ - arbitrary

m - isn't arbitrary (difficulties!)

reconstructed attractor

A number of methods have been proposed that try to estimate whether the image has been fully unfolded by a given *m*-dimensional map

More details: Alligood et al. *Chaos*