Complexity Science Doctoral Training Centre

CO903 Complexity and Chaos in Dynamical Systems

Introduction to XPPAUT

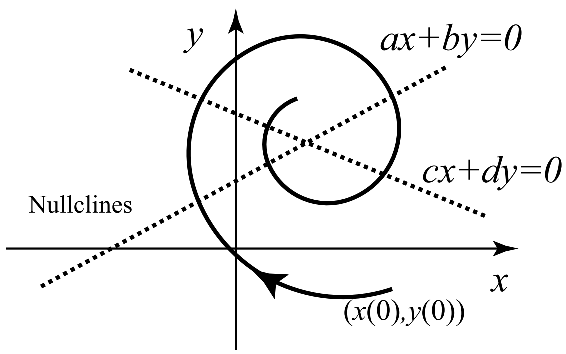
XPPAUT is developed by Bard Ermentrout and his web site contains instructions for the installation of XPPAUT on various platforms, as well as a tutorial.

http://www.math.pitt.edu/∼bard/xpp/xpp.html

1. Consider a system of two linear ODEs

$$\begin{aligned} \mathrm{d}x/\mathrm{d}t &= ax + by \\ \mathrm{d}y/\mathrm{d}t &= cx + dy \end{aligned}$$

The useful way of visualising solutions is to plot a *phase plane* graph, in which x is plotted versus y with time serving only as a parameter:

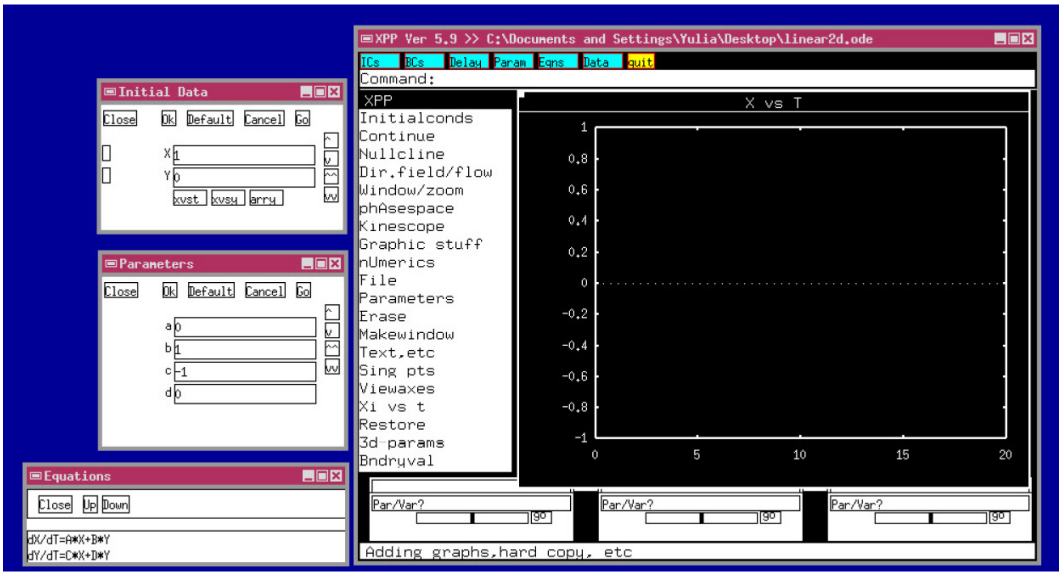


A *nullcline* is a boundary between regions where $\mathrm{d}x/\mathrm{d}t$ or $\mathrm{d}y/\mathrm{d}t$ switch signs. Nullclines (dashed lines) can be found by setting either $\mathrm{d}x/\mathrm{d}t = 0$ or $\mathrm{d}y/\mathrm{d}t = 0$. The arrow in the figure represents the direction of the initial point on the trajectory given by the solid line. The intersection of nullclines is the steady-state.

Save the file linear2d.ode from the CO903 module web site

http://www2.warwick.ac.uk/fac/cross_fac/complexity/study/msc_and_phd/co903/co903online/

and run this file in XPPAUT.
Then do Tutorial "A Very Brief Tour of XPPAUT" (provided separately; from the book Simulating, Analysing, and Animating Dynamical Systems (2002) B.Ermentrout) starting from Section 2.2.1 *The main window* and following it until Section 2.7.
Note: Section 2.1 of the tutorial describes the general rules of how to create an ODE file in XPPAUT (useful to read).
Note: To see the additional windows **Initial Data**, **Parameters** and **Equations** click on the corresponding buttons **ICs**, **Param** and **Eqns** located in the top region of the main XPPAUT window (see the figure):



2. **Bifurcations (using AUTO in XPPAUT)**

   (a) Cusp bifurcation

   Consider the ODE

   $$\dot{x} = a + bx - x^3, \qquad a = b = 1.$$

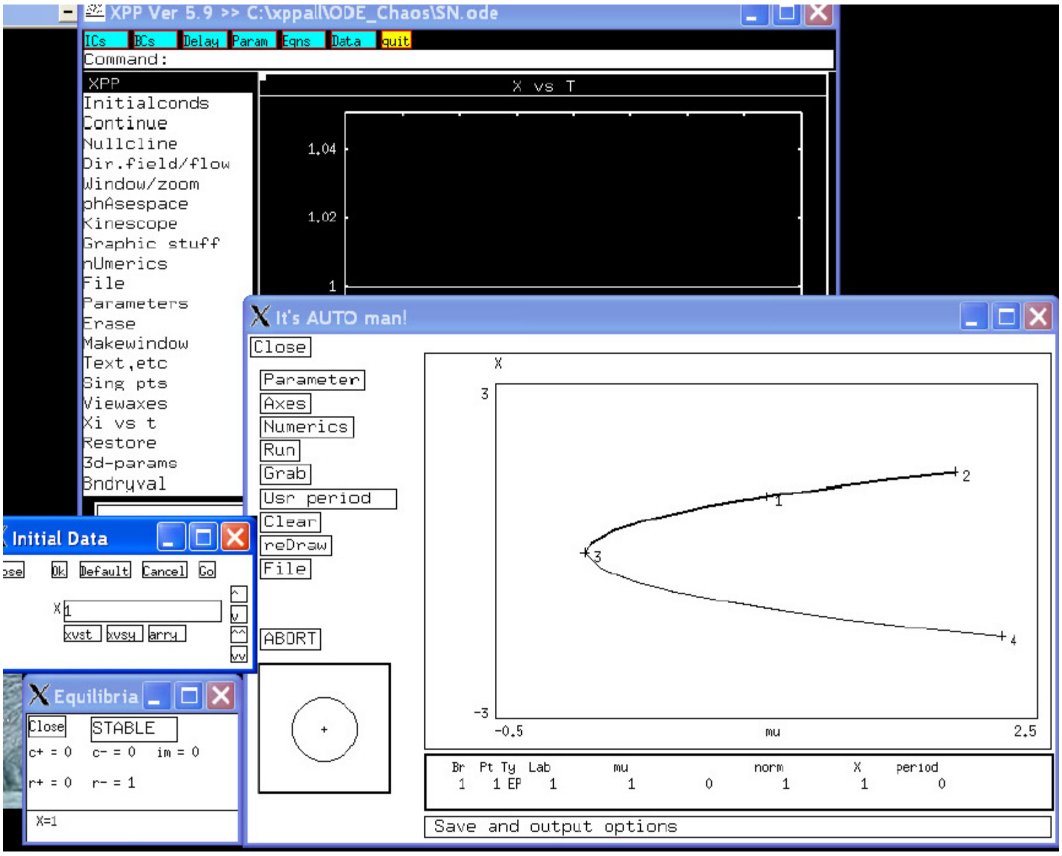
   Save the file cusp.ode for this equation from the CO903 module web site and run this file in XPPAUT. Then do Tutorial "Using AUTO: Bifurcation and Continuation" (provided separately; from the book Simulating, Analyzing, and Animating Dynamical Systems (2002) B. Ermentrout).

   (b) Saddle-node bifurcation

   Consider the equation $\dot{x} = \mu - x^2$ with the corresponding ODE file

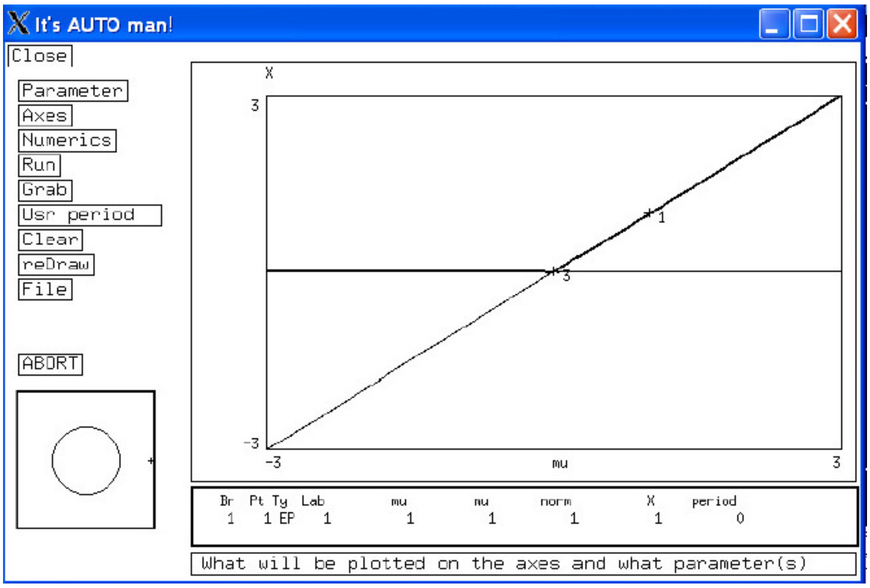
   dx/dt=mu-x∗x
   par mu=1
   init x=1.5
   done

   Create this file using any text editor or download it from the CO903 website (named as SN.ode). Run this file in XPPAUT. Integrate this equation (**I**nitialconds → **G**o). Find the steady-state (**S**ing pts → **G**o). Run the model from its steady-state (one way to do this is to change the initial condition value for X in **Initial Data** window and integrate again; the other way is to choose **I**nitialconds → **N**ew and type the steady-state value for X (here X: 1)). Open AUTO window. Plot the steady-state values (use `Run` → `Steady state`). We have to construct our steady-state curve to the left from 1 as well. To do this, we have to grab point 1 (using `Grab`), open `Numerics` and put the negative sign for the parameter `Ds`, say `Ds`= −0.02. This parameter `Ds` defines the value of the first step (positive - we are moving to the right, negative - we are moving to the left). Now click `Run` again.



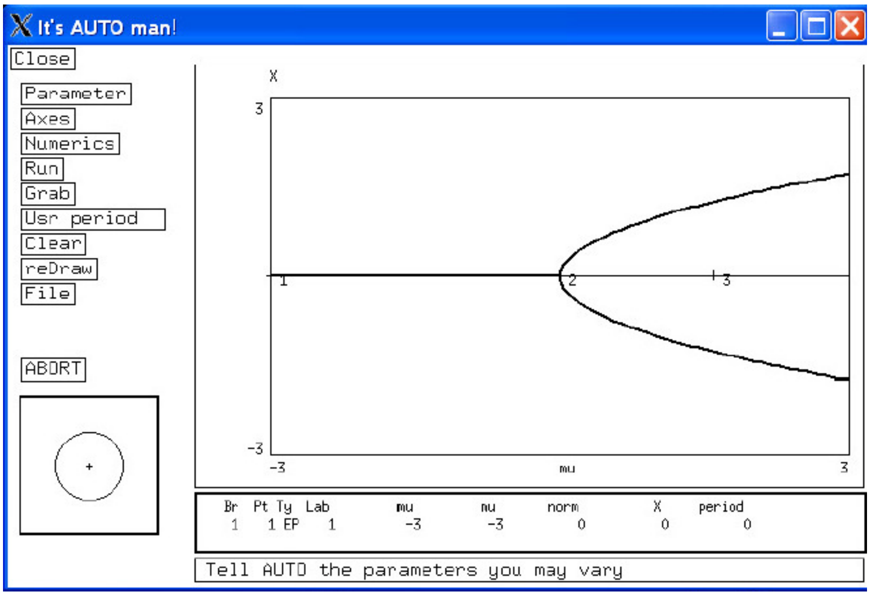
(c) Transcritical bifurcation

Consider the equation $\dot{x} = \mu x - x^2$ and construct the bifurcation diagram



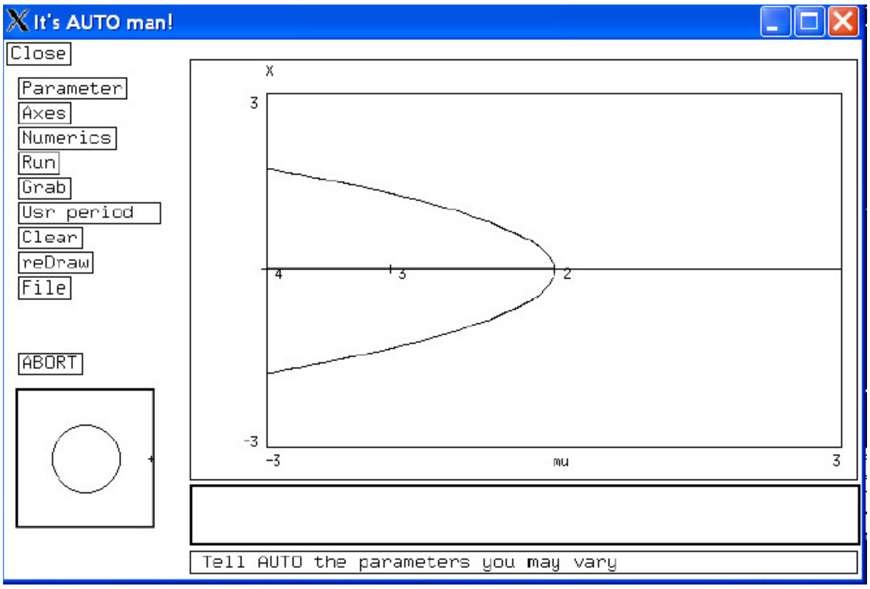
(d) Pitchfork bifurcation: supercritical

Consider the equation $\dot{x} = \mu x - x^3$ and construct the bifurcation diagram (start to construct it from the negative μ)



(e) <u>Pitchfork bifurcation: subrcritical</u>

Consider the equation $\dot{x} = \mu x + x^3$ and construct the bifurcation diagram (start to construct it from the positive μ)



Some XPPAUT commands

- **Initialcond** → **Go** (**IG**) computes a trajectory with the initial conditions specified in the Initial Data Window.
- **Window/zoom** → **Fit** (**WF**) fits the window to include the entire solution.
- **nUmerics** → **Total** (**UT**) allows you to change the total time of integration.
- **nUmerics** → **Dt** (**UD**) allows you to change the discretisation interval Δt.
- **nUmerics** → **Ncline ctrl** (**UN**) allows you to change the mesh for computing nullclines.
- **nUmerics** → **Method** (**UM**) allows you to change a numerical scheme for your integration.
- **Viewaxes** → **2D** (**V2**) lets you define a new two-dimensional view.
- **Nullcline** → **New** (**NN**) draws nullclines for a two-dimentsional system.
- **Initialcond** → **Mouse** (**IM**) computes a trajectory with the initial conditions specified by the mouse (it only works for a phase plane diagramm when you plot one variable against the other).
- **Dir.field/flow** → **Direct Field** (**DD**) drwas direction fields for a two-dimensional system.
- **Sing pts** → **Go** (**SG**) computes steady-states (fixed points) for a system with initial guess specified by the current initial conditions.
- **Sing pts** → **Mouse** (**SM**) computes steady-states (fixed points) for a system with initial guess specified by the mouse.
- **Graphic stuff** → **Postscript** (**GP**) allows you to create a PostScript file of the current graphics.

See this website for more information http://www.math.pitt.edu/∼bard/bardware/tut/xppmain.html

Some parameters in AUTO

Diagram axes

- **(H)i** This plots the maximum of the chosen variable.
- **h(I)-lo** This plots both the max and min of the chosen variable (convenient for periodic orbits).
- **(P)eriod** Plot the period versus a parameter.
- **(T)wo par** Plot the second parameter versus the primary parameter for two-parameter continuations.

Numerical parameters

- **Ntst** This is the number of mesh intervals for discretisation of periodic orbits. If you are getting bad results or not converging, it helps to increase this.
- **Nmax** The maximum number of steps taken along any branch. If you max out, make this bigger.
- **Npr** Give complete info every Npr steps. Set this to a big number if you want to speed things along.
- **Ds** This is the initial step size for the bifurcation calculation. The sign of Ds tells AUTO the direction to change the parameter. Since stepsize is adaptive, (Ds) is just a "suggestion".
- **Dsmin** The minimum stepsize (positive).
- **Dsmax** The maximum step size. If this is too big, AUTO will sometimes miss important points so if it seems to miss a stability transition, or if the diagram is jagged, decrease this.
- **Par Min** This is the left-hand limit of the diagram for the principle parameter. The calculation will stop if the parameter is less than this.
- **Par Max** This is the right-hand limit of the diagram for the principle parameter. The calculation will stop if the parameter is greater than this.

See this website for more information http://www.math.pitt.edu/~bard/xpp/help/xppauto.html