

Measuring and Modelling the PDF of Energy Flux in Two-dimensional Turbulence

Project Outline and Objectives

Since turbulence is a dissipative phenomenon, it requires continuous injection of energy from an external source in order to sustain itself. When a turbulent, flow is in a statistically stationary state, the rate of injection of energy balances the rate of dissipation of energy, on average. Typically, however, the injection and dissipation mechanisms act at very different scales, requiring a large range of intermediate scales, known as an *inertial range*, through which the energy is simply transferred by a nonlinear transport process, called an energy cascade. Standard scaling theories of turbulence are usually based on the idea that the energy flux should be constant in the inertial range, and equal to the rate of energy injection. While this is true in mean, the rate of energy injection and consequently the energy flux are themselves fluctuating quantities with interesting PDFs of their own. The objective of this project is to measure and attempt to model these PDFs at different scales.

Recent investigations of the statistics of the energy injection rate [1] reveal an interesting structure. The distribution of the energy injection is peaked at zero with long exponential tails. The fact that the mean energy injection rate is positive is due to an asymmetry of these tails (the negative tail decays more quickly than the positive tail). In practical terms, this means that injecting energy into a turbulent flow is difficult. The flow returns energy to the forcing almost as often as the forcing imparts energy to the flow. We shall investigate how this phenomenon persists in the statistics of the energy flux as one moves away from the forcing scale into the inertial range. We anticipate that the energy cascade flows backwards almost as much as it flows forwards. The project will be based on analysing data from previous numerical simulations of two-dimensional turbulence. Some (Lagrangian) experimental data is also available from the lab of Bob Ecke and Mike Rivera at Los Alamos National Laboratory. More data can probably be obtained if the computational results merit it.

Required Background and Methodology

Some background in numerical simulation of PDEs will be required to understand the origin of the numerical data and to generate more if required. The student will not be required to write their own hydrodynamics code. The project will, nevertheless, have a strong numerical component since we will be extracting, processing and visualising large amounts of data. The student will be expected to be comfortable with basic probability and statistics. Spatial filtering techniques [2] will be used to examine different scales. The project may demonstrate the plausibility of modelling the energy flux as the PDF of a combination of products of normal variables. At this stage, some more advanced mathematics will be required including complex analysis and asymptotics.

Research Outcomes and Outlook

Turbulent power fluctuations feed into many engineering applications involving hydrodynamics since the drag experienced by an object moving through a turbulent flow depends on the rate at which it transfers energy to the surrounding turbulence. In addition, a better understanding of the statistical properties of the energy flux is important from the point of view of turbulence modelling in applications such as ocean/atmosphere models and Large Eddy Simulation where one would like to parameterise the effects of small scale turbulence in order to avoid the large computational cost of computing it directly.

References

- [1] M. Bandi and C. Connaughton. arXiv: 0710.1133v3 [cond-mat.stat-mech], 2007.
- [2] S. Chen et al. *Phys. Rev. Lett.*, 96:084502, 2006.

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