Reduction and completeness

Exercise 1. Consider the language of the formulas in conjunctive normal forms with two literals in each clause, which are not satisfiable. Show that this language is $NL$-complete (use exercise 2 from class 8).

Exercise 2. Knowing that SAT is $NP$-complete, show that the following problems are $NP$-complete.

- 3-SAT: Input: a formula in conjunctive normal form in which all the clauses contain three literals, Output: “yes” if the formula is satisfiable, “no” otherwise.
- Clique: (a clique in a graph is a set of vertices that are pairwise connected) Input: a graph $G$ and a positive integer $k$, Output: “yes” if $G$ contains a clique with at least $k$ vertices, “no” otherwise.
- Independent set: (an independent set in a graph is a set of vertices that are pairwise not connected) Input: a graph $G$ and a positive integer $k$, Output: “yes” if $G$ contains an independent set with at least $k$ vertices, “no” otherwise.

Exercise 3. Show that the following problem is $PSPACE$-complete: Input: a non deterministic finite automaton $A$, Output: “yes” if $A$ is universal, that is to say if the language recognised by $A$ is full, “no” otherwise.

$P/poly$

Exercise 4. Prove that $P \subseteq P/poly$.

Exercise 5. Show that the class $P/poly$ is closed under Kleene star.

Exercise 6. Prove that all unary languages (i.e. languages over a one letter alphabet) are in $P/poly$.

Exercise 7. Show that $P/poly$ contains undecidable languages.

Exercise 8. A language $L$ in $\Sigma^*$ is said sparse if $|L \cap \Sigma^n| = n^{O(1)}$. Show that all the sparse languages are in $P/poly$. 