Bolzano's method of variation and Frege's "unsaturated functions"

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At the heart of Bernard Bolzano's method of variation lies the idea that the parts of a *Satz an sich* can be regarded as "changeable" ("*veränderlich*", WL § 147). This means that e.g. in the proposition [Cajus is mortal][1] the idea [mortal] can be replaced by a different idea such as [omniscient] which results in the proposition [Cajus is omniscient]. By using the method of variation the truth-value of a proposition can change (in the given example from true to false) and Bolzano uses this method to define various relations between propositions such as "Ableitbarkeit" (WL § 155). This idea of variation reminds of Gottlob Frege's notion of unsaturated language-functions such as "x is red". These functions contain "changeable" parts as well, which can be substituted. So both Bolzano and Frege develop the idea of systematically exchanging the parts of propositions.

In this paper I am going to compare Bolzano's notion of variation and Frege's notion of unsaturated functions. Although the basic idea behind the two accounts seems to be the same I would like to show that there are fundamental differences between the two methods and explain why this is the case. First of all Bolzano's method of variation needs to be investigated. After that I will discuss two central concepts: Bolzano's concept of the form of a *Satz an sich* and Frege's unsaturated functions. I then will compare Bolzano's and Frege's account. In the end I would like to show that the reasons for the differences between Bolzano's and Frege's approach can be traced back to at least two things: Bolzano's notion of a *Satz an sich* and his notion of a *function*, which shows not to be congruent with Frege's concept of a function.

1. Bolzano's method of variation: *"Veränderliche Satzbestandteile"*

Bolzano himself considers the speaking of "changeable" parts[2] of *Sätze an sich* as a metaphor since they are non-actual, timeless entities and as such do not contain change-

[1] In the following *Sätze an sich* will be marked by square brackets [ ]; linguistic sentences will me marked by quotation marks " ". The same holds for *Vorstellungen an sich* and their linguistic expressions. This notation follows Edgar Morscher's suggestion (cf. Berg 1992, 22). Additionally, the expression "proposition" will be used for "*Satz an sich*" and "idea" for "*Vorstellung an sich*".

[2] Bolzano himself only speaks in very careful terms about the changeability of *Sätze an sich*: "gewisse Vorstellungen in einem gegebenen Satze als veränderlich annehmen" (WL § 147, 136) i.e. "to consider certain Vorstellung in a given proposition as changeable".

able or replaceable parts (cf. WL § 69, 122). But Bolzano gives a hint how to understand the term “veränderlich” by pointing out that also in mathematics it is used in a non-literal way:

> Würden wir das Wort *veränderlich* überall nur in diesem eigentlichen Sinne nehmen; so könnten wir offenbar nur wirkliche und in der Zeit befindliche Dinge *veränderlich* nennen. Allein nach einem schon sehr alten und allgemein angenommenen Sprachgebrauche erlauben wir uns, besonders in den mathematischen Wissenschaften, unter gewissen Umständen auch Dinge, die gar keine Wirklichkeit haben, z.B. bloße Größen- oder Zahlenvorstellungen, Linien u. dgl. *veränderlich* zu nennen; wo wir denn also dieß Wort in einer ganz andern, nämlich einer entlehnten oder uneigentlichen Bedeutung anwenden. [...] An etwas Existierendes also, an welchem eine Veränderung im eigentlichen Sinne des Wortes vorginge, muß man in solchen Fällen nicht denken [...]. (GL Vorkenntnisse § 25, 110 f.)

In the mathematical context there are *variables*, which are “veränderlich” though they are no physical objects (e.g. the x in the mathematical expression x^2). In this case, the x is variable or changeable since it stands as a placeholder for concrete numbers and can be replaced by such.

But this interpretation does not work for *Sätze an sich* since this would mean that there would be propositions like [x has redness] with x indicating the “changeable” part of the proposition. A proposition of the form [x has redness] however is no valid instance of a *Satz an sich* since for Bolzano every *Satz an sich* must have a fixed truth value (cf. WL § 125, 69; WL § 147, 135) and a proposition of the form [x has redness] has no truth value at all because of its indefinite part [x]. Therefore the assumption has to be rejected that there are *Sätze an sich* of the form [x has redness] with genuine variable parts as Siebel and Sebestik both remark (cf. Siebel 1996, 164ff. and Sebestik 1992, 200). Additionally, the realm of the “an-sich” only contains *Vorstellungen an sich* and *Sätze an sich* but no variable-like entities such as an [x] would be. Since *Sätze an sich* are always composed of *Vorstellungen an sich* there cannot be a *Satz an sich* containing an [x] since Bolzano’s ontology of the “an-sich” does not contain such entities (cf. Siebel 1996, 164).

If there are no variables in the *Sätze an sich* the question arises what else could be “veränderlich” about a *Satz an sich*. Bolzano himself gives a solution by saying that parts of a proposition are “*regarded* as changeable”. This means that we only *pretend* that these propositions contain changeable parts, while during the actual variation we do not look at one and the same proposition, which changes its components. Instead, “changing” the parts of a *Satz an sich* means to examine *different Sätze an sich*, i.e. a *class* of propositions: "Given a proposition S, Bolzano considers the class of propositions obtained from S upon variation of ideas occurring in it." (Berg 1962, 92) Following this interpretation we can give a definition of the variation of a *Satz an sich*:

(Var) n, m ∈ {1,...,k}

The variation of the proposition S_n relative to the ideas **u**, **v**,... at positions *i*, *j*,... is the *class* of propositions S_1,...,S_k for which S_n und S_m are identical except for the ideas **u**, **v**,... at positions *i*, *j*,...

Additionally Bolzano introduces two restrictions for the variation of propositions:

(i) It is not allowed to insert equivocal („gleichgeltend") ideas for **u**, **v**,... at positions *i*, *j*,... in the *Sätze an sich* S_1,...,S_k (WL § 147, 137)

(ii) It is not allowed to replace the ideas **u**, **v**,... at positions *i*, *j*,... in the *Sätze an sich* S_1,...,S_k by ideas which produce a non-objective („ungegenständlich") *Satz an sich*

Bolzano also seems to implicitly suppose that there is always a fixed and finite range of variation such that it is impossible to produce an infinite class of propositions via variation.

This definition of variation is valid on the level of the non-linguistic *an-sich*. But there is a further meaning of "veränderlich" on the linguistic level, which indeed comes near the mathematical variables. To clarify this meaning it is necessary to introduce what Bolzano means by the "form" of a *Satz an sich*.

2. The form of a Satz an sich

For Bolzano, the "form" of a *Satz an sich* can mean two different things. First of all "form of a proposition" denotes a *class* of propositions which are identical except for some ideas **u**,**v**,… at positions *i*,*j*,… (WL § 147, 139). Secondly, it can be a *linguistic expression* which represents a certain class of propositions: „Spreche ich nämlich von Vorstellungen, Sätzen und Schlüssen, die unter dieser oder jener Form enthalten wären:

so verstehe ich unter Form *eine gewisse Verbindung von Worten und Zeichen* überhaupt, durch welche eine gewisse Art von Vorstellungen, Sätzen und Schlüssen dargestellt werden kann“ (WL § 81, 197; italics ASB). Linguistic expressions of this kind can indeed contain variables since an expression like “x has redness” can represent a class of propositions which e.g. contains the propositions [the rose has redness], [the car has redness], [red square has redness] and so forth. The important thing to notice is that these variables can only appear on the linguistic level.

Following this we can distinguish two different meanings of the expression “form of a *Satz an sich*”, one of them on the linguistic level and the other on the level of the an-sich (cf. Siebel 1996, 177 and Rusnock 2000, 131):

($form_{as}$) n, m $\in$ {1,...,k}

The $form_{as}$ of a proposition S_n relative to the *Vorstellungen an sich* **u**, **v**,... at positions *i*, *j*,... is the class of propositions S_1,...,S_k for which S_n and S_m are identical except for the ideas **u**, **v**,... at positions *i*, *j*,...

($form_{lin}$) The $form_{lin}$ of a proposition S_n is a linguistic expression for a $form_{as}$ of propositions

A class of propositions which builds a $form_{as}$ can be represented by a $form_{lin}$ and a $form_{lin}$ never represents a single *Satz an sich* but always a class of propositions.

What is apparent with the definition of $form_{as}$ is its similarity to the definition of variation given in section 1. Indeed we can reformulate the definition of variation in terms of the form of a proposition:

(Var*) The variation of the *Satz an sich* S_n relative to the *Vorstellungen an sich* **u**, **v**,... at positions *i*, *j*,... is a subset of the $form_{as}$ of the *Satz an sich* S_n relative to **u**, **v**,... at positions *i*, *j*,...

According to these definitions a variation of a *Satz an sich* is a subset of a $form_{as}$ since Bolzano formulates several restrictions for his method of variation (cf. (i) and (ii), section

1). Not all the propositions which are elements of a $form_{as}$ of a *Satz an sich* can be elements of a variation of the same *Satz an sich*. It is e.g. possible that the $form_{as}$ of a *Satz an sich* contains "nonsense" instances of the original proposition or instances which contain equivocal ideas. The $form_{as}$ of a proposition S may even be infinitely large.
Concerning the $form_{lin}$ of a proposition S we can say that since it is an expression for the $form_{as}$ of S it equally is an expression for one of its subsets. So the variation of a proposition S can be linguistically expressed by a $form_{lin}$, an expression containing variables.

3. Frege's unsaturated functions

After the examination of Bolzano's method of variation the next section will deal with Gottlob Frege's concept of an *unsaturated function*. Frege borrows the notion of a function from mathematics and adapts it for the purpose of language analysis. First of all a function is what is denoted by a term like e.g. "$2 \cdot x^3+x$". The function itself is not to be confused with its linguistic expression but is the entity the expression refers to, i.e. its *reference*. Strictly speaking an expression like "$2 \cdot x^3+x$" has no fixed reference because of its undetermined parts indicated by "x". Frege calls this incompleteness of a function expression "unsaturatedness" ("Ungesättigtheit" (GdA, 5)). Function expressions are unsaturated in the sense that they are in need of being completed by a definite expression to refer to a numeric value. But despite their unsaturatedness, function expressions do have a proper reference. It is the function itself, which shares the feature of being unsaturated with the function expression: "Daraus ist zu ersehen, dass in dem Gemeinsamen jener Ausdrücke das eigentliche Wesen der Funktion liegt; d.h. also in dem, was in "$2 \cdot x^3+x$" noch außer dem "x" vorhanden ist, was wir etwa so schreiben könnten "$2 \cdot (\ )^3+(\ )$"." (FB, 5) The feature of being unsaturated is characteristic for functions and distinguishes them from other entities.

Frege applies this mathematical concept of a function to propositions and interprets them as dividable into a functional part and an argument. The functional part is unsaturated as it is a mathematical function and can analogously be indicated by a linguistic expression with empty positions. Frege's example is the proposition "Caesar conquered Gallia" which can be separated e.g. into the unsaturated, functional part "( ) conquered Gallia" and the argument “Caesar”.[3] These parts are genuinely different since “( ) conquered Gallia” is unsaturated and “Caesar” is not: “Der zweite Teil ist ungesättigt, führt eine leere Stelle mit sich und erst dadurch, daß diese Stelle von einem Eigennamen ausgefüllt wird oder von einem Ausdrucke, der einen Eigennamen vertritt, kommt ein abgeschlossener Sinn zum Vorschein. Ich nenne auch hier die Bedeutung dieses ungesättigten Teiles Funktion.” (FB, 12) As in the case of mathematical functions, the incomplete linguistic expressions refer to unsaturated functions, in this case language-functions and proper names serve as arguments. If they are inserted, the proposition gets a truth value (true or wrong). On the level of language the unsaturatedness is marked by variables like “x“ or empty brackets “( )”, and on the non-linguistic level „the detachable function symbol does not itself, to use Frege’s words, have an empty place, but is considered, so to speak, to drag one or more empty places around with it, dangling from the ends of the satisfaction relations which actually link it to its arguments.“ (Simons 1981, 91) Another way of explaining unsaturatedness is by describing its functional role: The feature of unsaturatedness, which separates functions from objects, is the feature which keeps the parts of a proposition together:

> The unsaturatedness thesis was Frege’s answer to what might be called the problem of logical adhesion. The application of a Function to an argument is not a mere juxtaposition of the two. The Function combines with the argument into a self-contained *whole*. »What holds the two together?« Frege asked. Why is it that the whole does not disintegrate to its two ingredients? Frege saw a key to answering these questions in the assumption that a Function by itself is not self-contained. It contains a logical gap which needs filling. This is why the Function latches to its argument, sticking to it as if through a suction effect. This logical gappines is what sets Functions apart from Objects, which are complete and self-contained. (Tichý 1988, 27)

To be able to apply the mathematical concept of a function to propositions, Frege has to adjust it. In a mathematical context, the arguments and resulting values of functions are numerical. But in the case of propositions, Frege must allow every kind of “Gegenstand” as argument or function value. In the example “( ) conquered Gallia” we have to insert a name such as “Caesar” to get a complete proposition. The value of the hereby saturated function is “true”. Different examples are expressions like “the capital of x” which have “Paris” or “Berlin” as function values if we insert something like “France” or “Germany”. These examples show that Frege has to broaden the mathematical concept of a

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[3] There is no „fixed function“ for a proposition but several possibilities to split it up into functional part and argument. Other possibilities would be „( ) conquered ( )“ or „Caesar conquered ( )“ and so forth.

function by expanding the range of allowed arguments and function values. „Es sind nicht mehr bloß Zahlen zuzulassen, sondern Gegenstände überhaupt [...]. Als mögliche Funktionswerte sind schon vorhin die beiden Wahrheitswerte eingeführt. Wir müssen weiter gehen und Gegenstände ohne Beschränkung als Funktionswerte zulassen.“ (FB, 12)

4. Bolzano's method of variation and Frege's unsaturated functions

One way of comparing Bolzano's method of variation and Frege's method of substituting arguments in unsaturated functions is to ask whether Frege's terminology ("function", "unsaturatedness") is congruent in some way with Bolzano's ("variation", "form of a proposition"). The first step is to ask what a "function" is for Bolzano and to examine if this concept fits into his method of variation. In his work *Größenlehre* Bolzano describes what he understands by the term "function":

> Wenn das veränderliche Ding **W**, das von gewissen anderen veränderlichen Dingen **X**, **Y**, **Z**; nach der Erklärung des §. [25] der Vorkenntnisse *abhängt*, eine Größe (gleichviel ob eine wirkliche oder nur eingebildete) ist, und wenn eben so auch die Dinge **X**, **Y**, **Z**. Größen (wirkliche oder blos [sic] eingebildete) sind, so nenne ich die Größe **W** eine Function der Größe **X**, **Y**, **Z**. (GL Erste Begriffe § 16, 227)

For Bolzano a function describes the *dependency* ("Abhängigkeit") between two or more *Größen*. For Bolzano, a *Größe* belongs to a class of things „deren je zwei immer nur eins von folgenden zwei Verhältnissen gegeneinander an den Tag legen können: sie sind entweder einander *gleich*, oder das eine derselben erscheint als ein Ganzes, das einen dem andern *gleichen Teil* in sich fasst.“ (WL § 87, 211) This means that two *Größen* either are equal, or one of them is part of the other. Examples for *Größen* are an amount of coins or a period of time (cf. GL Einleitung § 1, 26) and an important, special kind of *Größen* are numbers (cf. GL Erste Begriffe, 283).

If two or more *Größen* stand in the relation of *dependency* this is called a function wherein "dependency" is defined as follows:

> Wenn sich ein Satz der Form: W hat die Beschaffenheit b, b‘,..., also eine Aussage über die Beschaffenheit eines gewissen Gegenstandes W, aus Sätzen von der Form: X hat die Beschaffenheiten ξ, ξ‘,..., Y hat die Beschaffenheiten η, η‘,..., Z hat die Beschaffenheiten ζ, ζ‘,... u.s.w. also aus Aussagen über die Beschaffenheit einiger *anderer* Dinge X, Y, Z,... (hinsichtlich auf gewisse veränderliche Vorstellungen i, j,...

> *ableiten* läßt, [...] so sagen wir wohl auch, [...] daß W von X, Y, Z,.. *abhängig* sey. (GL Vorkenntnisse § 26, 111 f.)

This means that if W is dependent on X, Y, Z,..., statements about features of W are deducible (in Bolzano's sense, cf. WL § 155) from statements about features of X, Y, Z,.... There can also be dependency without it being specifiable by an expression.

So for something to be a function at least to necessary conditions have to be fulfilled: There has to be a relation of dependency between two or more things and these things have to be *Größen*. A function is thus „jede beliebige von anderen abhängige Größe; gleichviel ob uns die Art ihrer Abhängigkeit, und somit auch ein Ausdruck, der sie darstellt, gegeben ist oder nicht" (GL Erste Begriffe § 17, 231). This notion of function certainly differs from Frege's definition. Additionally it is not possible to apply this concept to *Sätze an sich* and their parts since they are not what Bolzano calls *Größen*. From this follows that (using Bolzano's terminology) the method of variation cannot be classified among the relations which Bolzano calls "functions". According to this Bolzano certainly did not take his method of variation to be functional in character, as did Frege. Bolzano's concept of a function is restricted to the area of mathematics, which was the usual terminology for that time (cf. Currie 1982, 64). He restricts the term to a certain area of application. Frege in contrast defines functions via their feature of being unsaturated, which allows him to extend the term and to include non-mathematical entities such as concepts. To conclude we can say that Frege's concept of a function and his application of this concept to propositions and Bolzano's concept of a function are not congruent because of Frege's broadening of the term. Additionally, Bolzano does not merge his concept of function with his method of variation.

To compare the notion of unsaturatedness with Bolzano's ideas it is necessary to investigate both the linguistic level and the non-linguistic level of Bolzano's approach since according to Frege unsaturatedness occurs on both levels. As seen above, there are linguistic expressions such as "x has redness" in Bolzano's account. These expressions have been characterized as the form_{lin} of a certain class of propositions and they contain undefined parts such as Frege's function expressions. But even if both Bolzano and Frege work with expressions containing variables this does not imply that these constructions bear the same features. For Frege a proposition like "x has redness" is unsaturated and by inserting something for x we construct a complete and saturated proposition with a saturated sense. In Bolzano's case inserting something for "x" means to pick out one of the propositions, which belong to the class of propositions represented by "x has redness". We do not construct anything new but single out a proposition from a class of what could be called saturated propositions.

After considering the linguistic level the question arises if there is a parallel on the non-linguistic level. First of all, we have to notice that the non-linguistic level for Frege can either be the *sense* ("Sinn") or the *reference* ("Bedeutung") of an expression. As Künne remarks, we cannot find a counterpart to Frege's notion of reference in Bolzano's work. Since for Frege the notions of sense and reference are closely connected it follows that the concepts of sense and *Satz an sich* can not be congruent either:

> In Frege's semantic theory the sense of an expression is nothing but a particular way of singling out its reference. In Bolzano there is no concept playing the role the concept of reference plays in Frege. Hence Bolzano's notion of sense cannot possibly be identical with the Fregean notion. But this does not prevent these notions from being close relatives, of course. (Künne 1997, 217)

If we want to look for the concept of unsaturatedness in Bolzano's philosophy we have to concentrate on the *Sätze an sich* even if for Frege unsaturatedness primarily is located in the area of reference (functions are the references of unsaturated linguistic expressions). Although Frege defines functions as being the references of unsaturated linguistic expressions, he seems to have had the opinion that such expressions also have some kind of an unsaturated *sense* (cf. Wiggins 1984, 311 ff.).
The question now is if we can find unsaturated entities on the level of the an-sich. Such unsaturated, non-linguistic entities certainly are not bearers of truth values since they contain "logical gaps" to follow Tichy's way of speaking. As explained in section 1, for Bolzano there are no incomplete *Sätze an sich* or *Sätze an sich* containing variables. They are always true or false, and propositions with undefined parts, i.e. unsaturated entities, cannot have a fixed truth value.

From these considerations we can conclude that for Bolzano there are no unsaturated, non-linguistic entities. Only on the level of language we can find similar, unsaturated expressions as in Frege's theory but they differ on the level of senses.

5. Conclusion

We found that Bolzano's method of variation is not congruent with Frege's conception of propositions being dividable into unsaturated function-parts and arguments. Neither Frege's concept of functions nor of unsaturatedness has a direct equivalent in Bolzano's terminology. But it is possible to find some parallels, the most obvious one being on the level of language, since both Bolzano and Frege use language expressions with variables. But these expressions have a different equivalent on the non-linguistic level:

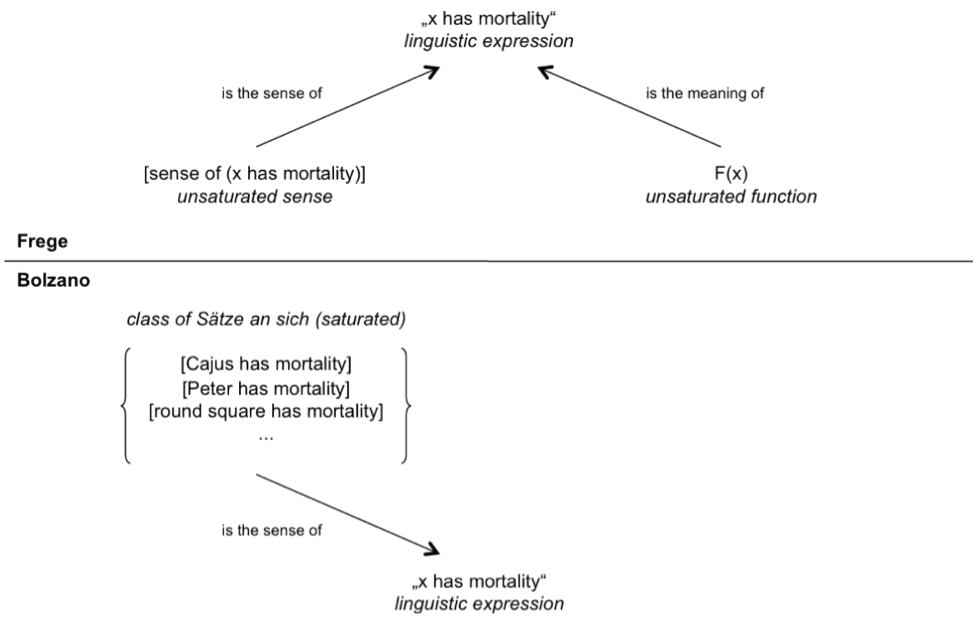

Figure 1

As this figure illustrates, Bolzano's expressions with variables stand for a class of *Sätze an sich* while in Frege's case they represent an unsaturated function. With this system, Frege gains a very flexible instrument to handle propositions and their sense and reference. Bolzano in contrast has nearly the same idea which is systematically changing some parts of a proposition. But the method of variation, which he develops based on this idea is not as flexible as Frege's since it is merely a way of speaking than a real variation (cf. figure 1).

The reasons for these differences between Bolzano and Frege are manifold. One reason for this divergent interpretation of propositions is that Bolzano and Frege have different concepts of what a function is. Frege broadens the narrow mathematical concept, which allows him to apply it to propositions and their parts while for Bolzano a „function" remains a purely mathematical term which belongs to the realm of „*Größen*" or numbers. Truth values, which according to Frege's interpretation can be the function value of a proposition-function are properties of *Sätze an sich* for Bolzano. Thus Bolzano does not transfer this concept from the area of mathematics to the area of language analysis.

A second reason is that Bolzano and Frege have a different understanding of how propositions are structured. Bolzano attributes a linear structure to them while Frege attributes a hierarchical structure to them. This becomes manifest in the detachment of function and argument, which belong to different categories. Simons remarks:

> Frege's revolt in notation was, however, less significant than his revolt in attitude, for he introduced clearly for the first time a way of thinking of sentence structure in terms of hierarchical rather than linear principles, and it is to this hierarchical arrangement, which can also be discerned in natural languages, that the considerations marshaled in categorical grammar find their clearest application. (Simons 1981, 89)

This thinking in hierarchic structures causes Frege to interpret propositions as functions and to introduce at the end a detachment of syntax and semantics: In separating the form of a proposition from its content and regarding certain structures independently from their content Frege constructs a far more flexible system than Bolzano. For Bolzano propositions (*Sätze an sich*) are linearly built up from *Vorstellungen an sich* which are all from the same category. The formal aspects of a proposition are not operationalized as they are in Frege's system and from a formal point of view the parts of a *Satz an sich* are not distinguishable. He neglects the role the form of a proposition can play and does not make it a logically self-contained category. For these reasons Sebestik claims that Bolzano's logic is to be located in the field of semantics whereas the field of syntax is missing:

> La logique de Bolzano est conçue dans une perspective sémantique. La séparation rigoureuse de la syntaxe et de la sémantique suppose, il est vrai, la construction d'une langue formelle qui peut ensuite recevoir une ou plusieurs interprétations. Dans ce sens, la distinction syntaxe – sémantique ne saurait être pertinente avant la construction de la première langue formelle digne de ce nom, celle de Frege. (Sebestik 1992, 201)

After summarizing the differences between Bolzano's and Frege's account I would like to suggest an alternative possibility of interpretation. Above it was indicated that Bolzano misses Frege's understanding of a hierarchical structure of propositions and therefore is not able to introduce an equally systematic account as Frege's. But in Bolzano's work there can be found a thesis, which often is neglected because it is considered as being implausible. It is Bolzano's thesis of the "canonical form of a *Satz an sich*" which claims that every *Satz an sich* is of the fixed form [A has b] (cf. WL § 127, 71). This means that every sentence in spoken language actually represents a *Satz an sich* of the form [A has b] and is reducible to a sentence of that form. In this form of a *Satz an sich*, the copula [has] plays a special role since it is contained in every *Satz an sich* and is not eliminable. It is a special kind of *Vorstellung an sich*. This thesis of a canonical propositional form can be interpreted as containing the idea of a hierarchical organization of propositions. The form [A has b] is unchangeable, while the parts [A] and [b] can have different structures. So the [has] seems to play a different role than the parts [A] and [b] since it is not among the *Vorstellungen an sich* which can be changed by variation. Additionally, we could say that the [has] plays a similar functional role as the property of unsaturatedness in Frege's account since the [has] is the part of the proposition, which keeps alle the parts of a *Satz an sich* together.

These concluding remarks show that a different interpretation might be possible using some of Bolzano's points of view, which often are not taken into account as being important. Certainly Bolzano himself did not notice this systematic character of his canonical form and did not develop it into an instrument, but also in this point he nevertheless may be said to come close to Frege's thinking.

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