Understanding Game Theory Through Empirical Modelling

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Abstract

Game theory is a fundamental concept that governs our interpretation of the world around us. Utilisation's of game theory range from modelling coin flips to analysing financial markets. With such wide ranging applications, an early understanding of its principles is essential. The purpose of this paper is to show how, through the use of empirical modelling, a model can be created that presents these principles and relates them to the real world game.

1 Introduction

Empirical modelling is concerned with the modelling of situations that can be directly linked to real world scenarios. The purpose of this is to create the mental link between the model and the real world. This allows for further thought into the application of models and the principles that surround the state of real world entities. When developing modelling using empirical modelling the focus of thought should be on how the model relates to its real world context and whether the model is a true representation of this scenario. This paper discusses the application of game theory to noughts and crosses which is a simple game with wide ranging game theory applications.

2 Game Theory

2.1 Introduction

The concern of game theory is in the representation of scenarios as individual "games." These games can be represented by a choice between a number of participants or by a probability. To illustrate this point we will initially look at the basic game of flipping a fair coin. This game simply has one step where the coin is flipped and can be represented by the following diagram:

```
       S
      / \
   0.5   0.5
   /       \
  H         T
```

This diagram (called a Game Graph or the Extensive Form) shows the initial state, in this case S, the two states that can follow this, H and T, and the probability of each state occurring, 0.5 and 0.5 (or 50%). As can be seen from this graph, for the case of the coin flip model, there are two possible states that could be entered from the initial state. This is, however, a very trivial example as there is currently no apparent point to this game. We can therefore include payoffs. Say, for example that the game is being played by player A and player B. We can then say that the game offers the the payoff of 1 to the player that wins the game and 0 if they lose. If player A bets on H and B bets on T the resulting game graph would be created:

```
       S
      / \
   0.5   0.5
   /       \
  1         0
  0         1
```

This now shows that there is a 50% chance of each player achieving 1 or 0 payoff.

2.2 Equilibria

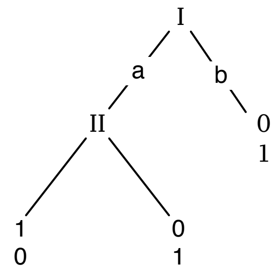
Within games containing more that one player a state may exist where neither player has the incentive to change their move. This is known as a Nash Equilibria. One of the best known examples of this is the prisoners dilemma. There are two prisoners, I and II, who are accused of a crime, they each have the choice of either remaining silent or confessing. If prisoner I chooses to confess and II remains silent, I will be let off for co-operating (shown as a payoff of 5) but II will get 5 years (payoff of 0) and vice-versa for player II confessing. If neither confesses they will receive a lesser sentence (payoff of 3) for lack of evidence and finally if both of them confess they will both receive a sentence of 4 years (payoff of 1). This game can be represented in the following table (known as the strategic form):

I \ II	S	C
S	1 1	4 1
C	1 4	4 4

Using the strategic form we can ascertain the best responses of each player (shown as highlighted payoffs) and from these the equilibria (squares where the payoffs for each player are highlighted). Calculating equilibria for a given game is a fundamental concept and can provide a player with what could be the optimum move for a game. This type of equilibria is known as pure equilibria, meaning that they can be achieved through playing a single strategy when the other player also plays their basic strategy. Pure equilibria, however, are not always present in games leading to the need to find mixed equilibria (equilibria that exists through mixed a number of possible moves) although this is out of the scope of this paper.

2.3 Perfect Information and Information Sets

The choice between using extensive and strategic form is more than simply through preference, each has specific functions. Although equilibria can be shown and calculated in both extensive and strategic form, information sets and whether a game has perfect information can only be shown in extensive form. If a game has perfect information it means that a player can know where they are in the game through knowing what their current moves are.

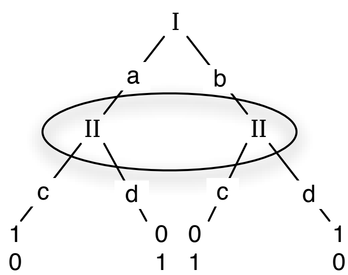


This is an example of a game that has perfect information in that at both player I and player II's turn each player can tell where in the game they are through knowing the future moves.

I \ II	c	d
a	0 1	1 0
b	1 0	1 0

Using solely strategic form, it is not possible to ascertain whether the game has perfect information.

Information sets can also be used in the extensive form to show what information a player has at a point in the game. States that exist within a single information set must have the same number of moves leaving them with the same labels.



This game shows that both moves by player II exist in the same information set and it is therefore not possible for player II to ascertain which point in the game they are (and subsequently what the payoffs would be for a given move.

3 Current Models

3.1 Introduction

In order to understand how TKEden works to model a standard game of Noughts and Crosses (OXO) I will be looking at a number of previously created models. Through understanding these it is possible that I will be able to create a new model, or even directly modify an existing one, to improve its accuracy.

3.2 oxoJoy1994

The model begins by functioning three possible games of standard OXO. Using a command line interface the Player is asked to input their move into the TKEden window, the command line also acts to show the Player where moves have already been played.

If the Player wishes to play their O in square 1, it would require them to type:

$$s1 = O$$

into the TKEden window. Once the move has been played, the computer will commence its turn.

The computer is designed to look for any possible winning moves for the Player and place their X in a location which will block any victory. If there is no such move open to the Player, the Computer will place their X in an arbitrary place. At no point in the game will the Computer actively attempt to win.

This element is part of the reason that the model doesnt represent a true OXO game as the second player would also be aiming for victory. There is also the fact that a Player can overwrite a turn which has already been played, a move which would not happen in a true game.

3.3 oxoGardner1999

The oxoGardner model initially starts by showing the winning lines for an OXO game in diagrammatic form. The conceptual layer then follows allowing the player to place noughts and crosses onto the board. This allows the user to understand how the game functions.

The next layer involves the numeric values the computer will allocate to each square being shown on the screen allowing the user to see how the computer calculates its move.

The final layer shows the entirety of the game and implements a turn based system. It also sets the governing rules of the game thus preventing the overwriting of previous turns as found in the OXO model described previously.

This model works to accurately model the OXO game as there can be no bending of the rules, however, it is still possible to beat the computer. This creates scope for improvement within the AI of the model.

4 Modelling

4.1 Game Graph Complexity

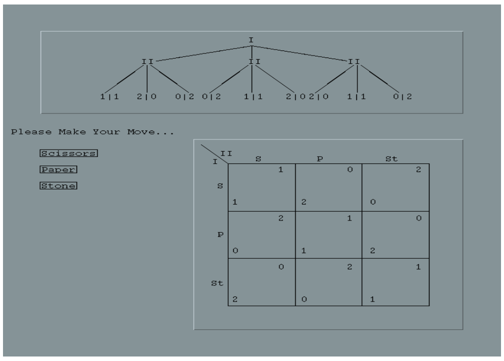
Through evaluation of the OXO model it is apparent that the creation of a game graph would not be feasible due to the number of possible paths that a single game may take. Therefore, it has been chosen that a simpler model shall be used to demonstrate game theory through empirical modelling. The model that has been chosen is the scissors-paper-stone (SPS) game. This game involves each player choosing either scissors, paper or stone. Each combination of choices results in either a player being declared the winner (scissors beats paper, stone beats scissors and paper beats stone) or, if both players choose the same option, in a draw. Unlike the OXO model, the SPS model presents no single equilibria:

I \ II	S	P	St
S	1, 1	2, 0	0, 2
P	0, 2	1, 1	2, 0
St	2, 0	0, 2	1, 1

Rather, the game demonstrates the importance of perfect information and the effect this can have on the game.

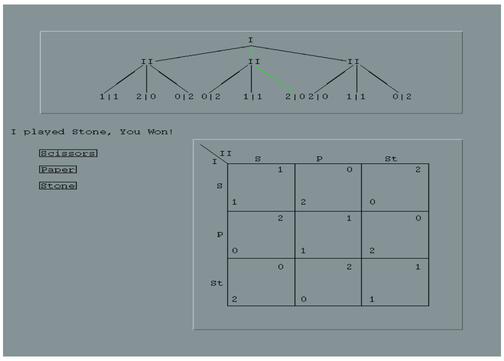
4.2 Interface Modelling

The first stage of modelling the SPS game involves the creation an interface to show the current state of the game, the extensive game graph and the strategic form. The user of the model must also have the ability to interact with the model via a set of buttons. The extensive game graph will show all possible options for the entire game to allow the user to fully comprehend the choices available and their consequences. Showing the strategic form to the user can allow them to realise the lack of equilibria and the seemingly stochastic nature of the game. Through playing the game by visualising both extensive and strategic forms, the player can develop and test any possible strategies for playing the game and therefore understand how their actions effect the game.



4.3 Game Modelling

SPS is a game with imperfect information where the move of each play cannot be determined through analysis of either the extensive or strategic form. There are two reasons for the behaviour; the game is a 'one shot' game (where each player plays simultaneously) and the game has no pure equilibria. This presents both an ideal model to allow basic understanding of game theory whilst also providing a suitable basis for further study. The game has been modelled in such a way that the user cannot directly affect the computers move as this is governed by a random variable. This provides an accurate model of SPS although it does prevent some of the calculations capable as part of a game theoretic approach. The model also allows for the user to view the path of the game through the game graph to further understand how the opponent is playing the game.



5 Conclusion

5.1 Evaluation

As previously stated, the model that has been created accurately models the SPS game. This does however, limit the models usefulness in terms of understanding the applications of game theory.

Approaching this subject through empirical modelling has allowed the understanding of not simply how a tool can be created to show how the SPS game works but also understanding how the SPS game functions and how it could be approached.

The purpose of this model was to allow users new to game theory to understand its principles and how they can be applied to real-world situations. To a large extent this aim has been achieved as the model does allow users to visualise how both the extensive and strategic form applied to the real-world states. For this to be fully achieved, however, the user must approach the model from an empirical perspective. This implies that the user will use the model with the understanding that the model is intended to apply to the real-world situation of playing a SPS game. Approaching this model from this perspective allows the user to develop a deep understanding of the fundamentals of game theory.

5.2 Future Applications

The OXO model, could still provide the basis for developing useful model to aid the understanding of game theory. The application of such a model would require high levels of computation to ensure the model is both feasible and accessible. This could be achieved through a simplified extensive form where the user is simply show the most likely (or possibly average) payoff for each possible move. This would only show the user the next couple of layers in the extensive form whilst still providing enough information to make a informed decision. It would however, be more advantageous to simply model many simple games that could provide the user with a number of straightforward, scenarios that cover commonly faced game theoretic problems.

Further applications could also be made to the current model by allowing manipulation both the extensive and strategic form. This could allow the user to actively choose their next move directly from these views. Finally, the model could be extended to make both the extensive and strategic form dynamic, allowing multiple games to be quickly modelled and even providing tools for automatic finding of equilibria. This could also allow the computer player to make plays based on game theoretic concepts further increasing the sophistication of the AI and providing a more accurate model.

Acknowledgements

This work has relied on the research undertaken by Mike Joy (1994) and Simon Gardner (1999) and could not have been completed without these original models.