EM and mathematical education by experience

1052900

Abstract

This paper considers issues of Empirical Modelling, which is the research field developed in the University of Warwick, and mathematical learning. There are discussed works of Beynon about Radical Empiricism and Empirical Modelling, also about Empirical Modelling for Educational Tecnologies; James' work about Radical Empiricism and, Tall's work about issues of mathematical learning of derivative.

1 Introduction

This paper consists from discussions about several different issues. First part contains briefly explanation about main concepts of EM and Dewel's point of view about experience. Next section is about effects of computers in education and learning. The third part is about EM for education and analysis of Tall's work about learning of derivative. And the very brief conclusion is in the last section.

- EM

Let us first identify what is Empirical Modelling (EM). Web definition of the word "empirical" is *"relying or based solely on experiment and observation rather than theory"*. According to it, EM is the type of modelling based on observation and experiment.

Because, this work consists of analysis about learning derivative, it would be appropriate to give the most common formal definition of it:

- Derivative

In calculus, Derivative is a measure which finds how much function value changes when its input changes. In geometry it is a line touching circle only in one point and making right angle with its radius, also called tangent line.

2 EM for mathematical education

2.1 EM and Empiricism

As it was mentioned before EM is a new approach to computer-based modelling. EM's important part is construction of artefacts. Artefacts are not models of theory. They are the modeller's source of experiences. Each experience source refers to another independent experience. Experience known as sense-making and learning.

In comparison with standard programming products the results of EM models show a big difference. While using EM models learner can feel and learn the model step-by-step.

Empirical Modelling is not only a new way of solving existing problems in software development, but also a new type of 'alternative computing'. EM is very

helpful to make significant changes in such areas as artificial intelligence, computer graphics, concurrent systems and educational technology. [1]

The idea of EM is very closely related with Radical Empiricism, but not Empiricism.

-Classical Empiricism

In philosophy, Empiricism is called a theory of knowledge. The key point is that this theory asserts that knowledge comes from experience. It stresses aspects, which are closely-related to evidence, such as discovered in experiments. According to empiricism all theories and hypotheses must be checked and tested against observations of the natural world. By nature, science should be methologically empirical.

But William James's opinion about Radical Empiricism which has been developed from 20th century is very interesting and somehow related with Empirical Modelling. As Beynon writes in his work there is *"a potential significance of William James's philosophic attitude of 'Radical Empiricism (RE)' in relation to contemporary problems of knowledge representation in the information sciences."* [2]

Empirical Modelling is based on need to develop methods of modelling rely on experience rather than theory. According to James' 'philosophical attitude' experience is not something that could be knowledge, it is something even more abstract.

As it was mentioned before, empirical models are based on understanding. A lor of applications for EM principles stress both: potential of EM and the challenge of enderstanding its semantics. Modeller's understanding can be illustrated as it is shown in the Figure1.

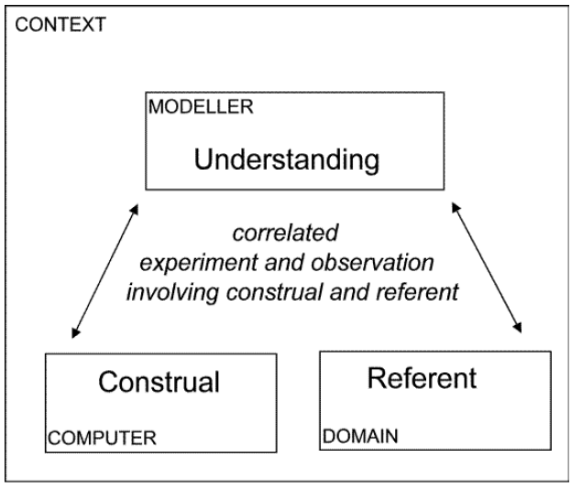

Figure 1. Modeller understanding [3]

Modeller understanding is consists from construal and referent. The main concepts of EM: observation, dependency and agency serve the idea of building models that can be investigated, customised and played with. [3]

The word "experience" was used several times to introduce EM. But what is the meaning of this word and how it could be explained?

In EM experience is gained only through interactions and transactions. So models are constructed such way, that learner have a chance to glean experience using them.

The most appropriate explanation of 'educational experience' is given by John Dewey. His principal idea is that *"experience arises from the interaction of two principles -- continuity and interaction."* [4]

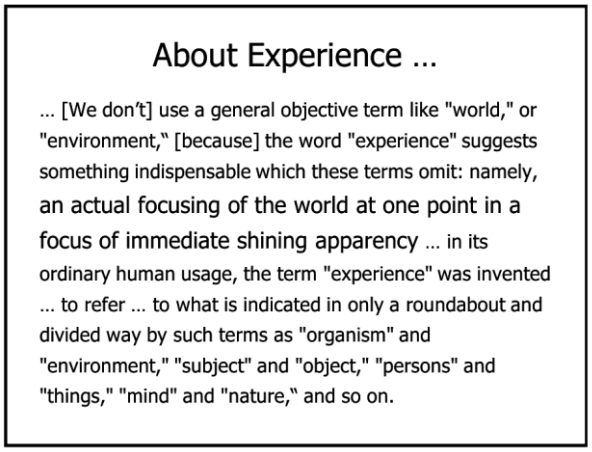

Figure 2. "About Experience" [4]

So according to James and Dewey knowledge is something abstract that is could be called source of experiences.

2.2 Computer Based Education

Annually the population of readers of newspapers and books is decreasing. Contrary, amount of computer and the Internet users is increasing. Relatively various computer programs have being created daily. Because of this growth it is rational that computers smoothly became an important part of educational system. In the history of mathematics the first time computers were used to calculate different simple mathematical equations. In the 1960's and 1970's new programs were developed which gave an ability to create graphical representations and more complicated calculations.

Nowadays education is closely related with computers. It is difficult to imagine any educational institute without computers. Different mathematical computer programs are used not only in the institutes of higher education, but also on schools to teach students the basics of Math.

Computer-based learning is different from standard learning for many reasons. Using special software and mathematical programs changes learner's experience in epistemological level. Because, a learner and computer have an interaction which is based on symbols. In the past, schools were isolated from the world. After computers became widely used in the education it was changed. [5]

People were perceived these kind of changes differently. There were a lot of predictions about computers. For example, in 1995 Clifford Stole said: "**The truth is no online database will replace your daily newspaper, no CD-ROM can take the place of a competent teacher and no computer network will change the way government works**". And all of them have became true.

Of course , it is clear that human factor in education cannot be replaced. However, software effects on both, pedagogical style and a learner perception. [5]

In the past, nobody tried to measure educational outcomes while revolution in computer science were in progress. But now there are a lot of tools to do so. The difference between learning with and without computer programs can be easily differentiated with the help of this tools.

2.3 Effect of EM in Learning

EM take into consideration such things like learner's perception and understanding. Relatively, EM software is kind of model which is developed based on modeller's experience.

Most of the existing educational IT products are restricted and too limited in scope. Such software don't fully engage imagination. These kinds of products work with closed-world models. [6]

According to Beynon's paper "Empirical Modelling

for Educational Technology" experiment and exploration play a very important role in the learning process. Particularly in motivating students and attract their imagination.

Author mentions improvements that can be done by EM:

- improvement of flexible adaptation of models, where they can be used by people who are not familiar with that models at all
- development of machine-independent techniques of IT products
- usage of less analyses of the implementing product, but more creativity of the modeller
- creating software which gives an opportunity to of learning experience without any prepossession

Beynon in the same paper writes that EM does not need any concrete development strategy. Because, it is allowed to represent model details in different order, according to the modeller, which will be seemed the most appropriate.

Issues faced in learning derivative

Software should be oriented for the specific audience. Because understanding of this software depends on background knowledge of the leaner. Tall in his paper *"A Sensible approach to the Calculus"* presents the way of learning and understanding of derivatives. [7]

Like it was mentioned by Tall derivative is explained in a very different ways in schools. He shares his experience and says that he found calculus very difficult when he had studied it in a college, even if he had learned it before and had solved all problems from "*Elementary Calculus"* by Durrel and Robson earlier in school years. What had happened to others, if the person who had a good experience in solving mathematical problems found it very complicated? This paper, written by Tall, is about problems and incomprehension of learning derivatives.

Author starts his paper with explaining perceptual continuity, perceptual limits, and perceptual tangents. According to his explanation, for a student, who is learning mathematical expression "continuity" first time, it is based on idea of drawing curve without taking the pen/pencil off the paper. Also he is explaining this expression by formal and informal definitions. Formal definition is based on finding ε and δ:

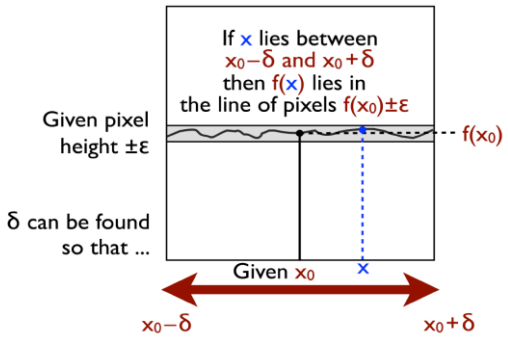

Figure 3. "Perceptual continuity" [7]

Informal definition is explained by scretching the curve, looking only at the marked part.

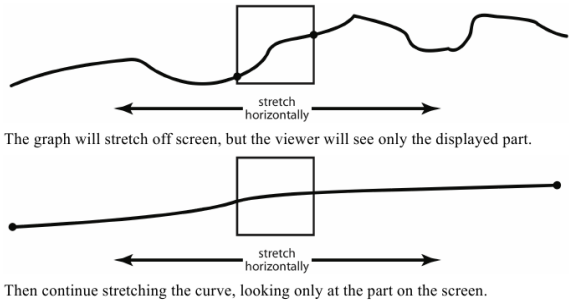

Figure 4. "Scretching curve to formulate informal definition of continuity" [7]

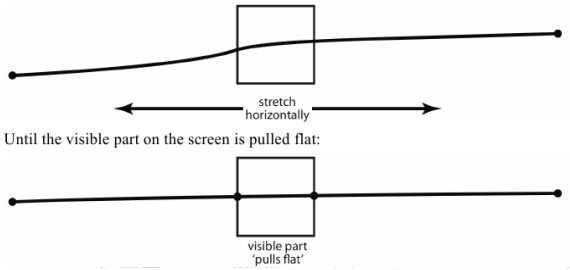

Figure 5. "Scretching curve to formulate informal definition of continuity" – 2 [7]

Also, using a graph in a closed interval points between x=a and x=b, Tall shows a dynamically continuous graph. By choosing values ε and δ more than zero, draw a small rectangle of width 2δ and height 2ε and with the middle point lying on the graph. Each rectangle should be in a distance 2δ.

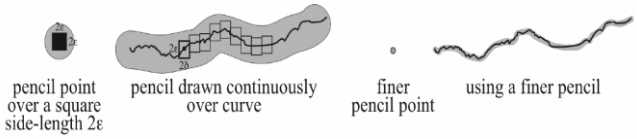

Figure 6. "Dynamically continuous graph" [7]

The important moment in **perceptual limits** "*is to see the limit and then to see the later terms of the sequence become indistinguishable from this limit.*" [7]

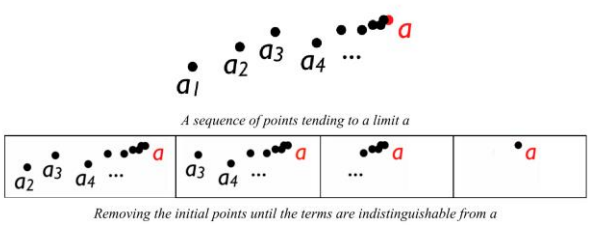

Figure 7. "Perceptual limits" [7]

Since in some definitions derivative is given as the slope of the tangent line, the next section in Tall's work is about tangent and it's different issues.

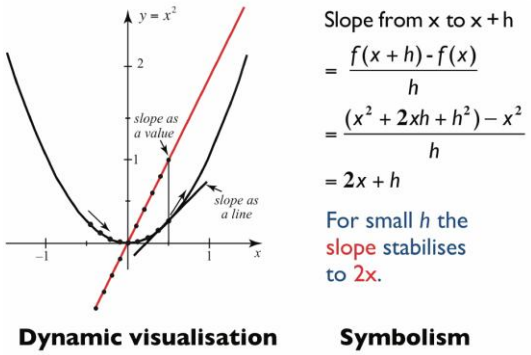

Figure 8. *"Direct links between visualisation and symbolism"*

Step by step, explaining from simple to complex definitions and examples of derivative, author shows the usage of EM. Using "... *personal experience of researching nature of mathematical thinking to produce single framework for its long-term development ...*"

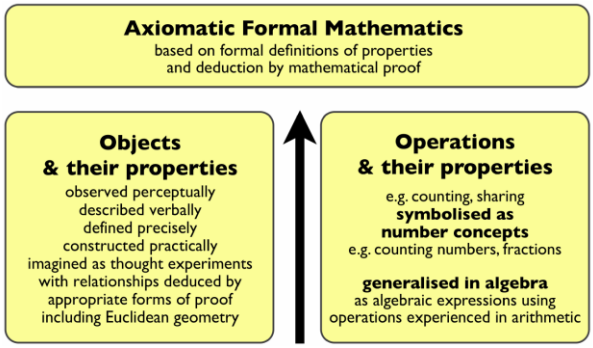

Figure 9. The framework *"three worlds of mathematics"* [7]

Tall's framework of mathematics concentrates on issues of formulating definitions with the help of formal proof.

By sharing his experience he makes a conclusion that, *"elementary calculus belongs in the parallel worlds of embodiment and symbolism"*, and *"mathematical analysis belongs in the ... world of axiomatic..."*

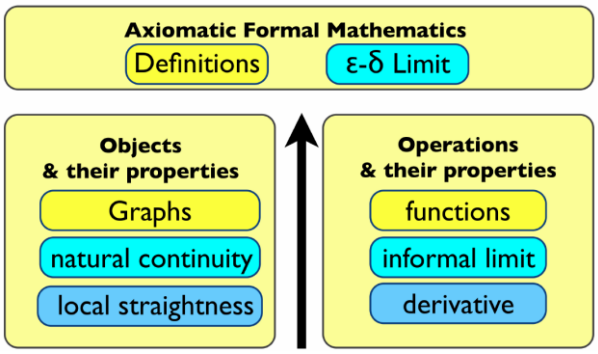

Figure 10. *"Calculus and Mathematical Analysis in the Three World framework"* [7]

Learning by experience

One example model of learning by experience is created by the author. Model is related with mathematical learning of graphics of linear functions and, showing its dependency on gradient and intercept.

The initial view of function is given as

$ax + by = c$

Modifying if we have

$y = (c – ax) / b = c/b – ax/b$

Let's mark

$c/b = M$ and $(-a/b) = B$

Then we have a function dependent on M and B, where M is called gradient and B is called intercept.

$f(x) = Mx + B$

Working on this model learner can easily experience the dependence of function's graph on these two variables.

3 Conclusion

In this paper different issues and approaches of EM and mathematical education were discussed. The main problem considered in paper is mathematical perception of learners. The issues are:

- What tools do learners need according to their experiences?
- What kind of experience learners need?

Considering sections mentioned above such conclusions can be done:

- Modeller understanding is consists from construal and referent. The main concepts of EM: observation, dependency and agency serve the idea of building models that can be investigated, customised and played with.

- According to Dewel "*...in its ordinary human usage, the term "experience" was invented ... to refer... to what is indicated in only a roundabout and divided way by such terms as "organism" and "environment"...*"

- After understanding what is EM and what is experience we refer Tall's work about *"A sensible Approach to the Calculus"*, where using his personal experience he makes a conclusion that *"elementary calculus belongs in the parallel worlds of embodiment and symbolism"*, and *"mathematical analysis belongs in the ... world of axiomatic..."*

Analysing materials above the next conclusion can be done:

Knowledge is practice, which consists of experiential thread. There is no need to separate given things to explain them. Radical Empiricism is different experiencing of dependency. If learner has no experience in a particular field, words used to explain something in this field would be only words, but not knowledge.

References

1. Introduction to Empirical Modelling. Online: http://www2.warwick.ac.uk/fac/sci/dcs/research/em Accessed: Jan 2011

2. M. Beynon, Radical Empiricism, Empirical Modelling and the nature of knowing, Online: http://www2.warwick.ac.uk/fac/sci/dcs/research/em/publications/papers/downloads/078.pdf, Accessed: Jan 2011

3. Online lecture notes, Online: http://www2.warwick.ac.uk/fac/sci/dcs/research/em/teaching/cs405/emsysdev_cs2008wk910bw.pdf. Accessed: Jan 2011

4. Online lecture notes, Online: http://www2.warwick.ac.uk/fac/sci/dcs/research/em/teaching/cs405/statedewey.pdf. Accessed: Jan 2011

5. A. J. Bishop. "International handbook of mathematics education". *Kluwer Academic Publishers, Netherlands,* 469-501, 1996.

6. W. M. Beynon. "Empirical Modelling for Educational Technology". *Proc. Cognitive Technology '97, University of Aizu, Japan, IEEE,* 54-68, 1997.

7. D. Tall, A Sensible approach to the Calculus, Online: http://www.warwick.ac.uk/staff/David.Tall/pdfs/dot2010asensible-calculus.pdf, Accessed: Jan 2011