Creating an interactive learning environment powered by a JS-EDEN model to aid in the understanding of vectors

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Abstract

This paper presents and discusses "A System for Teaching" (AST), a JS-EDEN model designed to aid in the understanding of vectors, an A-Level Mathematics and Further Mathematics topic. The paper will discuss how Empirical Modelling (EM) techniques and tools can enrich the learning of vectors through interaction and guided discovery, such offering a richer web-like learning structure when compare to other programs written in an imperative language [1]. The paper will also touch upon how these techniques can be applied to other areas of mathematics and indeed, all areas of school education. It will also discuss how AST has gained insight from the methodologies behind sports coaching, as this is often, if not exclusively, an interactive process between the coach and the learner.

1   Scope of AST

AST limits itself to the teaching of vectors. Focussing on the key aspects of how to define a vector; calculating the magnitude of vectors; vector addition; and the scalar product. AST assumes knowledge of the Cartesian coordinate system and Pythagoras's theorem.

2   Acknowledgments

- Mathematics in Education and Industry[1] revision material. Kindly provided by Martyn Parker of the FM Network [2] [3].
- BCU Coaching Qualification, Level 1. Taught by Jenna Saunders [4].
- BaseVector model designed by Joe Butler as part of the EMILIE[2] group [5] [6].

3   Research

This section will look at the need of a new tool for learning, what topics should be taught by this different style of tool, why EM tools and philosophies is well suited to the area of learning and how EM can benefit from an insight into sports coaching.

3.1   A need of AST

A need was identified for a system for teaching that separated itself from "traditional" methods of teaching mathematics that emphasises lectures and note-taking, calculation by hand before using calculators and rote learning [7]. A separation from these methods is needed because, whilst they are tried and true, the question of their worth has been raised [7]. Today's workplace emphasizes flexible and adaptive learners [7], an argument can be made that learning methods should emphasise this as well [7]?

3.2   What AST Covers?

In order to take a decision on what AST would initially cover, an investigation of the topics MEI and CS131 teach was conducted. Full details of this can be seen in Appendix A. The conclusions of this investigation were that AST would initially cover the following topics in the style of MEI [2];

- What a vector is.
- Calculating the magnitude of a vector.
- Vector addition and subtraction.
- Multiplying a vector by a scalar.
- The Scalar Product.

The original plan was for AST to also cover the following, using CS131 slides as the starting point;

- Linear Combinations and Span [8].
- Subspaces [8].
- Linear Independence [9].
- Basis and Dimension [10].

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[1] MEI

[2] The EMILIE project was specifically set up to investigate whether learning could be enhanced through developing EM construals [6].

However, quality of the scaffolding [11], see section 3.3.4, and interaction with the model was prioritised over the quantity, as the aims of this modelling were to discuss the suitability of EM tools for learning. The author believes a better tool can offer a better perspective on this issue than a sub-par tool. As such, the following mathematics topics were not included:

- Vector subtraction.
- Multiplying a vector by a scalar.
- Linear Combinations and Span [8].
- Subspaces [8].
- Linear Independence [9].
- Basis and Dimension [10].

3.3 Why EM?

Education is an area well suited to EM philosophies and tools. This is because the creation of EM construal's mirrors how learners construct an understanding of new ideas [12].

Technology enhanced learning can be said to support three strands: the experimental, the flexible and the meaningful.

There is a close correspondence between the characteristics of EM and these strands of learning. This similarity forms the basis for the argument that EM is a suitable approach for supporting the experimental, flexible and meaningful characteristics of learning [12].

3.3.1 Experimental

An experimental approach to learning, using EM in the form of Open-Ended Model-Building[3] can support individual learning because both learning and OEMB involve an active construction process on the part of the individual. EM has an experimental nature which makes the approach suitable for supporting learning [12].

3.3.2 Flexible

Harfield stressed that learning can be flexible and need not necessarily follow a certain pre-defined path, but that an individual should be able to find their own path for their own learning. He also strengthened the flexible aspect further by relinquishing the idea that

learning should have a prescribed outcome. EM is a particularly flexible attitude to model-building in which the process or outcome can be completely open-ended. The flexible characteristics of EM make it suitable for supporting learning [12].

3.3.3 Meaningful

Harfield states that learning must be meaningful as meaningful learning is the foundation for everyday learning. It is recognised that learning takes place in a particular situation, context and culture which has meaning for the individual. Harfield noted that building models of phenomena and situations using EM tools facilitates the process of sense-making in learners [12].

3.3.4 Scaffolding [11] and Microworlds [13]

Roe discusses that a standard approach to developing educational software that can be targeted at different learning scenarios is to expose simple concepts before more complex ones. These concepts, in a learning environment, should be layered and introduced only when the learner has a solid understanding of simpler concepts, in order to maximise knowledge gained. This understanding can be tested directly through the use of exercises or left to the learner. These layers are called microworlds [13] and the exploration of these microworlds is associated with the idea of scaffolding. Scaffolding is defined as a technique for providing support to learners whilst they are learning a new task. Microworlds and Scaffolding can be combined by initially providing a simple, easy to understand, model of a specific referent to the learner. This model is then gradually refined and extended, as the user progresses through the scaffolding, by introducing more advanced concepts associated with the referent. The focus is on providing a computer-based model that accurately reflects the source referent and level-by-level is guided by more precise observation of the referent [1].

The notion of developing increasingly complex microworlds, where each microworld adds more complex ideas or tasks to perform, is a well-established educational strategy proven to deliver learning [1].

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[3] OEMB

3.3.5 Computers for Learning [1]

Roe pointed out that construction of models by end-users may not be a realistic aim as experiments with 17 – 18 year old college students has exposed [1]. Roe's findings were that students without any previous programming experience could not use the tkeden tool to create models because the students lacked essential computing knowledge. However, students with programming experience succeeded in creating models / extending previously created models. The conclusion is that when students do not have a good understanding of basic programming concepts they cannot develop their own models and are reliant on others to produce learning environments for them [1].

This may be problematic. If learners are restricted to using models produced by other users (such as AST) there may be issues as to how their experience can be experimental, flexible and meaningful. The following paragraphs detail the evidence that a restrictive model, designed using EM tools, will still be able to offer an experimental, flexible and meaningful approach to learning.

The intention is that AST will offer an experimental approach to learning through the use of guided discovery, more detail of which can be seen in section 3.4. So whilst the user will not be creating models, or even dealing with the JS-Eden "layer" itself, the user will have agency over a model by progressing through the scaffolding and by interacting with the vector space, such as moving the vector with the mouse. The purpose of this agency is to allow the user to engage in experimental learning. This agency is an example of guided discovery and is similar to the approach of the CS405 early labs at Warwick University. Initially, students were given a (perhaps near) complete model of a lift (for example) and have agency over this model through a series of, almost toy, EDEN commands. In AST these commands have been replaced by the user having agency with the mouse, rather than the EDEN window.

This style of agency immediately leads to another problem. If the agency is achieved through the scaffolding, how can the learning be truly flexible? A user may proceed through the scaffolding and then stop learning /

experimenting? Whilst this is the case, the AST tool is designed to offer the learner a flexible approach to how quickly the progress through the scaffolding. If they are picking up topics quickly they can progress to the next slide almost immediately. If they require more experimentation to familiarize themselves with a concept, they are allowed to do so.

Meaningful learning has proven to be a difficult concept to evidence and deliver within the model. To determine whether AST can deliver meaningful learning, AST would need to be evaluated by someone, or multiple people, who do not yet understand vectors. An evaluation of this type could be achieved through a partnership with the local education authority or training agency offering numeracy support to those seeking employment. The evaluation can also benefit from feedback from students at Warwick or professionals such as mathematics teachers.

3.3.6 Fractions [14]

Fractions was a model that was intended to be an aid to learning about the relationships between different representations of fractions [14]. As such, the author found that a lot of the research and conclusions are useful for AST.

Cronick concluded that to enhance learning it is best to connect the hemispheres of the brain by asking pupils to (for example) describe a picture or diagram and to visualise a written description [14]. As people learn in different ways, it is also necessary to differentiate learning tools. These three ways are visual (learn by seeing), auditory (learn by hearing) and kinaesthetic (learn by doing) [15]. Therefore some learners may desire extension questions, audio instruction or physical activity, possible achieved through a Kinect. Some learners may need larger fonts, whilst others may repeat questions to reinforce learning [14]. The concept of "extension questions" and "repeat questions" is included in AST through the ability to explore one micro-world as much as the learner needs in order to fully understand it. Larger fonts could be achieved through dependency but was not implemented in AST, see section 6.

The program should also include a design feature that ties in the idea that by using the computer program the learner would not get

things wrong, but instead have a more positive experience from using the program. This is where Empirical Modelling enhances learning software; the software would not need to ask the learner questions which they would either get right or wrong, but the program will encourage learners to "play" or experiment with the program [14], such as asking them to repeatedly move a vector and try an predict features such as the vector's magnitude.

3.3.7 The "BaseVector" Model [5]

Whilst no part of Joe Butler's "BaseVector" model [5] was directly used in AST, this particular model was reviewed, as it was both part of the EMILE project [6] and a Vector based construal. The "BaseVector" model did influence the later design and functionality of AST, such as the button control for multiple vectors.

However, both the author and Butler created similar construals of 2D vector space before the BaseVector model was viewed, a theory could be hypothesised that these similarities are due to both undertaking a similar Mathematics module, CS131, in the first year at Warwick University.

3.4 An Insight from Canoe and Kayak Coaching [4]

As per the abstract, this section details how AST has benefited from the sport coaches perspective on learning. Canoe and Kayak coaching teaches that a learner can only "take in" three items, plus or minus two, based on conditions. For example, if coaching "on the water", where conditions are cold and wet, the learner may only be able to take one item at time before their knowledge needs to be tested. As user of AST is likely to be unaffected by these adverse external factors, it can be assumed that they can "take in" at least three points without being tested, or encouraged to explore. However, there should never be more than five points to learn being made in one set of scaffolding before encouraging exploring, as this would exceed three plus two.

Guided discovery is a concept first met by the author during sports coaching but is also the style of the EM lectures at Warwick University. In a sports setting, the learner can be given a problem and asked to go away and try possible solutions, i.e. "Try and move your canoe sideways". After a period of discovery, the learner returns to the coach and a discussion is held as to what worked well, what worked badly and after corrections from the coach, with the aim that the student leaves with an understanding of proper technique. This is similar to the EM lectures, however, in the author's experience, all the discussion tends to happen before the discovery either in labs or outside of scheduled time. This similarity furthers the concept that EM may benefit from more interaction with sports coaching techniques.

4 AST, the Model

This section explores the model and details the EM tools used to develop AST. It also makes reference to EM principles such as experimental, flexible, meaningful and the observable, dependency, agency[4] framework.

4.1 Model Platform

AST is a JS-EDEN model, primarily because the environment offers one syntax for both dependency modelling and drawing unlike tkeden which has multiple notations such as %eden for basic dependency modelling and %donald for line drawings. JS-EDEN is not without its drawbacks, as discussed in section 4.5.

4.2 Description of the Model

This section describes what the author considers to be the "key" features of the forty slides of AST.

AST uses the JS-Eden Presentation environment to provide the scaffolding to the developing vector space microworld. The "model" starts out as simply an image. This image could be considered the "logo" for AST, although the current image is more of a place holder, as it's copyright status is currently unknown. Slide 5 (Defining a Vector.) demonstrates how transparent images can be placed in the scaffolding to create a more visually appealing model. Slide 9 (Defining a Vector.) demonstrates a dependency between the microworld and the

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[4] ODA

scaffolding. The scaffolding displays the x and y components of the vector.

As the learner moves the vector through the agency they have over the model (via an interaction using mouse clicks) the components displayed in the scaffolding update accordingly. The format of the model is also dependant on the *currentSlide* observable, for example, when the learner is on Slide 13 (Calculating Magnitude.) the axis are 5 units long but on and after Slide 14 (Calculating Magnitude.) the axis are 1 unit long. This feature was implemented because the author believed they "cluttered up" the model and because a simpler visual improves the learning experience. Slide 16 (Calculating Magnitude.) demonstrates another dependency between the microworld and the scaffolding. As the learner moves the vector in the microworld, the magnitude displayed in the scaffolding is updated through a dependency. Slide 19 (Vector Addition.) demonstrates a dependency being broken. This break occurs by using the command in Figure 1.

vectorR_x = mouseClickX;
Figure 1: Assigning vectorR_x.

This command (and the equivalent 'y' command[5]) renders the vector immovable as the dependency created in Figure 2 is broken. Note the use of "is" in Figure 2; "is" creates a dependency, linking the LHS[6] value to the RHS[7] value, whereas "=" gives the LHS the static RHS value, a.k.a the current value, now and forever (unless another assignment or dependency is created) .

vectorR_x is mouseClickX;
Figure 2: Creating a dependency.

Slide 20 (Vector Addition.) introduces the "Blue Vector", so called because it is coloured blue. This differs from the above vector, which is coloured red and will henceforth be called the "Red Vector". Slide 26 (Vector Addition.) demonstrates how assignment and dependency relations can be defined in terms of other observable and dependences. The slide introduces two buttons and each button has a *proc* associated with it. Each button assigns

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[5] *vectorR_y = mouseClickY;*
[6] Left Hand Side
[7] Right Hand Side

the position of one vector, breaking the dependency, whilst creating a dependency between mouse clicks and positions for the other vector. The buttons also affect each other, when one is pressed the button deactivates itself and activates the other button, by assigning a boolean value to the *enabled* attribute of the button. Slide 37 (The Scalar Product.) shows how the scaffolding, which is powered by java script (in turn powered by HTML), can include special characters such as Θ. These special characters cannot be used to name dependencies in the EMILE flavour of JS-Eden, although research is being conducted into using Greek characters to define "template" definitions to mass create observables, such as the dash_xα[8] variables.

4.3   Testing and Evaluating

This section details the pros and cons of a two-prong potential testing and evaluation strategy that could be used in the evaluation of AST and includes a summary of the expected results.

4.3.1   Strategy

AST could first be tested on and evaluated by the author's peers and the academics involved with WEB-EM 09. This informed feedback could be used to produce a first stage improvement to AST. The benefit of testing on this group is that AST will be evaluated by people familiar with the principals and capabilities of EM. The downside of this approach is that the vast majority, if not all, of those people will already understand vectors and will not be learning as they progress. However, those involved with WEB-EM 09 will have some experience of teaching so would be able to give objective insight into whether AST can deliver meaningful learning.

After this educated input, AST could then be shown to select teenagers in schools as well as adult learners. The benefit of testing AST on and having the model evaluating by a this different group is that they will not have learned about vectors yet, so could better analyse the meaningfulness of the learning provided by AST. The downside is that the group will have no appreciation for the currently technical limitations of EM tools or of computing in general, as such some of their

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[8] α here refers to a number between 0 and 9

feedback may be infeasible to implement at this time.

AST could also be tested and evaluated on mathematics teachers. The benefit of using this group of education specialists is that the author assumes they will have the most experience of learning tools aimed at A-Level topics and hence their informed feedback is expected to bring about the most meaningful change. Again, the downside is that teachers will not be aware of the technical limitations of the EM tools.

4.3.2   Expected Results
The author expects that such an evaluation strategy would not only improve the quality of the scaffolding used in AST, but also improve the visual impact of AST.

4.4   Pilot Deployment Strategy.
This section details a pilot deployment strategy for AST. In order to maximize the value of the feedback, as discussed in section 4.3, AST could be piloted in a panel of participating local schools. This involvement would be a useful case as feedback would be received from teachers and teenagers. Any initial deployment could also include night schools and other adult centred learning environments, such as community colleges, in order to receive feedback from them.

4.5   Problems caused by JS-EDEN
JS-EDEN is still in development and as such is not bug free. Below is a summary of the bugs that directly affect AST and why.

JS-EDEN can only be used correctly through the Google Chrome browser, possibly only on specific versions of Chrome, such as those installed in the Department of Computer Science[9]. This is not a major issue as Chrome is a free to use browser, the main problem will be ensuring a correct (possibly outdated) version is installed.

The canvas is only defined once, when JS-EDEN is loaded, necessitating loading the JS-EDEN in either a browser the same size as the desired window or loading it then resizing and then refreshing the browser. This was noted when designing the model, as the origin is

defined to be in the centre of the canvas, not the centre of the window.

Also, the canvas width is prone to change for unknown reasons. It is not known if this bug is present when using the JS presentation environment but the bug was not spotted during development on a laptop, using the latest version of Chrome[10]

The string "Blue Vector is moveable" and it Red Vector counterpart are displayed in their own shared (invisible) canvas with dimensions, and this prevents certain mouse clicks from being registered. In general, you need to experiment to find out how much space a Div component needs, and ensure that its canvas is large enough without being so large that it is obstructive [16]. This was done with the status div.

The *round* function in JS-Eden appears not to be 100% effective, as occasionally it does not round figures at all, such as the Scalar Product value on slide 38 (The Scalar Product.).

4.6   Author Opinion of AST
The author believes the model is well constructed, using many of the features of JS-Eden such as the use of images, to enrich the scaffolding.

5   Conclusions
EM proposes that scaffolding, as well as learning in general, should be experimental, flexibly and meaningful. This proposal is not the only one, others exist. McKenzie [17] suggests that learning must meet the following criteria:

1. Provides clear directions, offering step-by-step directions throughout.
2. Clarify why the learner is doing.
3. Keep users on task.
4. Offer short tests and other assessments.
5. Deliver efficiency & create momentum.

An argument could be made that criteria 1) 2) and 3) are polar opposites to the principles of EM. EM teaches that free exploration, often for no reason other than gaining personal

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[9] Version 14.0.835.186

[10] Version 23.0.1271.97

understanding, whereas McKenzie is advocating stricter learning. From the author's experience of modelling AST, progressing through ever more complex microworlds can be likened to providing direction, clarification and strict learning. Whilst AST has tried to move away from this by offering opportunities to explore, learning from such exploration cannot be as rich as the learning from modelling at the JS-Eden "layer". The author believes the strength of EM as a learning tool is derived from the creation of models from personal understanding and whilst learning tools such as AST can be created with EM tools, such tools may be no better than tools created in other paradigms, such as object orientation. The author feels AST is an example of this fact. A more effective learning exercise may be to create the construal of vector space "from scratch", rather than progress through AST, as the author has found creating AST more beneficial to refreshing personal understanding than using AST. From the modelling activity undertaken by the author, a conclusion is drawn that similar tools could be created for any area of A-Level Mathematics.

6 Further Work

This section details further work that could be conducted to improve AST, focusing on further testing, different versions for teachers, sports coaching, different learning styles and accessibility adaption.

6.1 Testing

Testing of AST is crucial by both learners, teachers and EM academics, whilst at this stage if development AST has been tested from a technical stand point i.e. you can go back to a slide and it display the same model as the first time you go to the slide, the model has not been subject to sufficient levels of testing, evaluation and design changes from the point of view of AST as a learning tool mainly due to a lack of opportunity. It would be informative to enhancements to see how AST performs in schools and sixth forms.

6.2 Teacher Version of AST

One potential extension of the work would be to have a teacher version of AST, through which a teacher could have agency over the student's instantiation of AST, such as limiting

the progress through the scaffolding or moving the vectors. This could be achieved through a new extension tool that would extend JS-EDEN in a similar way to how dtkeden extends tkeden. Such a tool does not yet currently exist, nor is dtkeden fully functional at this time. Such a feature would also raise questions as to the models flexibility; a student will be on a more fixed path as they can only proceed to a certain point in the scaffolding without teacher intervention. However a teacher being able to move the vectors is a good idea as it would allow for more interaction between student and teacher, with AST providing the springboard for this.

6.3 Further Sports Insight.

AST gained insight from sport coaching and this paper has drawn conclusions between sports coaching and the EM lectures at Warwick. Because of this insight and the conclusions, an area of future work would be to look at higher levels of sports coaching, such as BCU level 2 and upwards. These levels may provide more features of learning that would suit the styles and beliefs of EM.

6.4 Different Learning Styles

Learners learn using different styles, but AST focuses primarily on visual (an argument could be made that the learning is also kinaesthetic as the learner moves the vector). An extension for AST would be to include audio phrases to reinforce the learning. A Microsoft Kinect's (or other camera's) visual feed could be interpreted into JS-Eden so that observables could change based on the learners physical activity, allowing the user to learn kinaesthetically.

6.5 Accessibility Adaptation

Another extension of the work would be, through dependency, allow AST to adapt to the learner. One such adaptation to increase accessibility could be to have the font, both in the scaffolding and the vector, be dependent on some variable which the learner can access though the EDEN interpreter. This adaptability gives the learner greater agency over the model. This is just one example of how the model can adapt to the user, there are others, such as audio description of the model, but such adaptation was ultimately decided to be outside the scope of AST as it stands for CS404.

7 Appendices

Appendix A:

Research into current MEI and Warwick Computer Science Mathematics courses.
http://www.dcs.warwick.ac.uk/~csujaw/CS405/appendices/A.pdf

Appendix B:

The "Train of Thought" Model for AST. This is the Model Script as created, before being reordered (for example, the multiple definitions of the slideTrigger *proc* were condensed into one) and refined (several new slides were introduced).
http://www.dcs.warwick.ac.uk/~csujaw/CS405/appendices/Train_of_thought.js-e

Appendix C:

Model README
http://www.dcs.warwick.ac.uk/~csujaw/CS405/appendices/README.txt

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