

An EM Construal of the Abacus

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Abstract

The abacus is a simple yet powerful tool for calculations, only simple rules are used and yet the same outcome can be derived through various processes depending on the human operator. This paper explores how a construal of the abacus can be used as an educational tool but also how 'human computing' is illustrated. The user can gain understanding regarding how to operate the abacus for addition through a constructivist approach but can also use the more informative material elaborated in the EMPE context.

1 Introduction

1.1 The Abacus

The abacus is a tool engineered to assist with calculations, to improve the accuracy and the speed to complete such calculations at the same time to minimise the mental calculations involved. In a way, an abacus can be thought of as one's pen and paper used to note down the relevant numerals for a calculation or the modern day electronic calculator we are familiar with. However, the latter analogy may be less accurate as an electronic calculator would compute an answer when a series of buttons are pressed and requires less human input to perform the calculation compared to an abacus.

One of the earliest forms of the abacus consisted of bamboo rods each representing a place value of a number and threading these rods with beads. Up to ten beads were allowed on a rod and once ten beads have been threaded on a single rod, these were removed and a single bead would be threaded onto the rod representing the next place value. Beads added or removed from the rods depending on whether addition or subtraction was to be performed.

It was not until the twelfth century AD that the Chinese abacus, or the Suanpan, came to be [1]. This device contained two segments, the upper part or 'Heaven' and the lower part or 'Earth' [2]. A crosspiece, also known as the beam, divides the two parts. There are rods reaching from the top of Heaven all the way to the bottom of the Earth segment, intersecting the beam. On each of these rods there are two Heaven beads and five Earth beads, each Heaven bead is worth five and each Earth bead holds a value of one. The frame

encapsulates all of these components together. The abacus can, not only be used for addition and subtraction, but also for multiplication, division, and even for calculating the square and cube roots of numbers [3].

The Japanese also have a variation of the abacus known as the Soroban where the only difference from the Suanpan is each column contains one less Heaven and Earth bead [4].

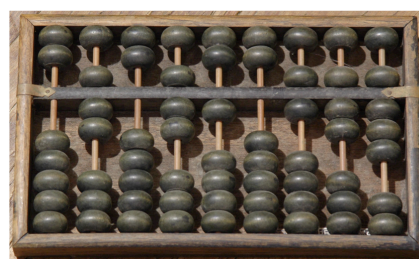


Figure 1 The Chinese abacus or Suanpan

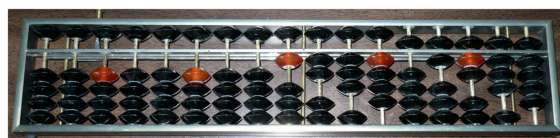


Figure 2 The Japanese abacus or Soroban

1.2 Motivations Using EM Approaches

The abacus is well suited to demonstrate EM concepts. For instance, the agent is able to interact with the model by moving the beads on the abacus and perceive the dependencies that cause the observables to change between states.

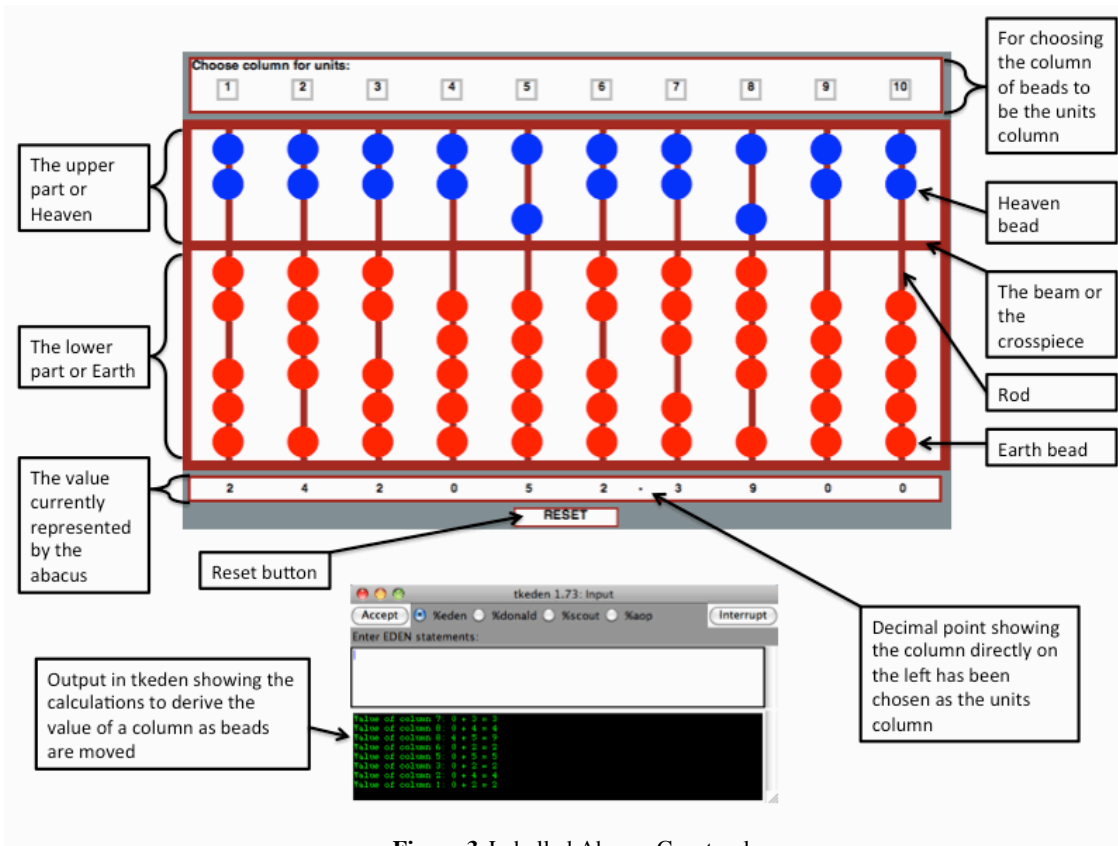


Figure 3 Labeled Abacus Construal

Using EM approaches, the model can be used for educational purposes by determining the rules of the abacus based on the observables and dependencies shown in the model.

The model can also be used to illustrate 'human computing'. Although the abacus is a powerful tool, part of this credit is due to the human operator of the abacus. While the abacus is aimed to reduce the number of mental calculations, it is still required to a degree when using the device. The amount of mental calculation needed depends on the individual human operator.

2 The Abacus Model

2.1 Components of the Construal

The components of the Abacus¹ construal are shown in Figure 3. The observables, dependencies and interaction from the agent will allow for a constructivist approach to understand the state changes in the construal.

¹ From here onwards, 'Abacus' with capitalisation denotes the construal and 'abacus' will be used in reference to abacuses in general.

2.1.1 Unit Selection

There is no standard for a particular column of beads in an abacus to be the units column. The abacus is capable of handling calculations with decimal numbers. In the construal, the agent has the choice to select a unit column of their choice otherwise, by default, the tenth column is used to represent units. The agent has the choice to make a selection again should he change his mind on the choice of units column, however, only one units column can be selected at one time.

2.1.2 The Abacus

Visually, this part of the construal is not too different from the Chinese Suanpan. The Heaven beads and Earth beads are separated by the beam, the rods hold these beads together allowing any bead to move up or down along the rod.

The beads are important observables to be taken into account, only beads that are touching the beam or, in the case of the construal, near the beam are worth any value. Therefore, in the default initial configuration of the Abacus, all Heaven and Earth beads are pushed away from the beam and the Abacus represents a value of zero.

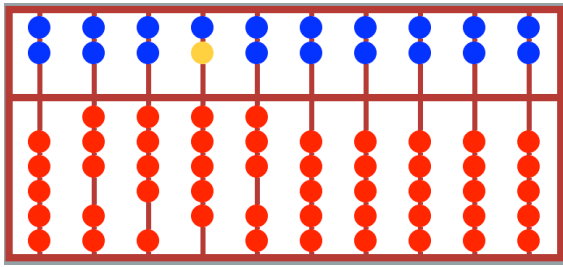


Figure 4 Diagram of construal showing bead dependencies

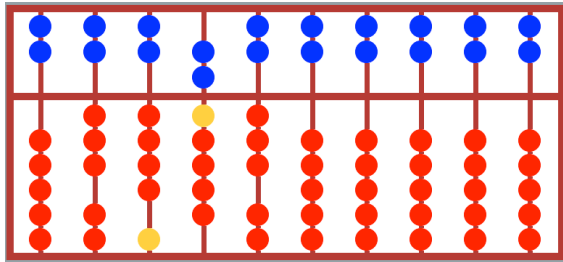


Figure 5 Diagram of construal showing bead dependencies

2.1.2.1 Bead Dependencies

The agent has the option to move one bead or multiple beads at once. Beads will retain their colour until the movements of certain beads trigger other beads to change colour. This dependency occurs when:

- Five Earth beads in a single column have been pushed towards the beam, the closest Heaven bead to the beam in the same column that has yet to be pushed down to the beam will be highlighted, as shown in Figure 4;
- A highlighted Heaven bead that is pushed down, this will trigger the closest Earth bead to the beam to be highlighted;
- Both Heaven beads are pushed down, the first Earth bead that has yet to be pushed towards the beam in the left hand column is highlighted, the Earth bead closest to the beam in the right hand column will also be highlighted, as shown in Figure 5; and
- Pushing up the Earth bead in the left hand column, this will cause the Heaven bead in the right hand column to be highlighted.

The highlighted beads are used to illustrate to the user which beads should ideally be moved next and to show the relation between the different beads.

A single rod is designed to hold a single digit value, that is, any integer up to nine. Like the

bamboo variation of earlier abacuses, once a tenth bead is threaded onto the bamboo it is subsequently removed and a bead is added to the next bamboo instead to signify the value of ten.

However, unlike the bamboo variation, the Suanpan, the Soroban and the Abacus construal have Heaven beads to be taken considered. Effectively, both the Heaven and Earth segments are only designed to hold a value of up to five. Of course, Heaven can temporarily have a value of ten and the two segments combined can have a value of fifteen, but the more traditional way to use the abacus would be to ensure that one rod never holds a value that consists of more than two digits.

It is not necessarily incorrect for a rod to hold a value of more than nine but this is likely to make the calculations more difficult than the abacus is designed for. By allowing a rod to retain a value of more than nine will involve more mental arithmetic by the agent during the calculation but also to interpret the result after the calculation.

After some experimentation on moving beads, the states of the abacus that cause certain beads to be highlighted may become apparent to the agent.

2.1.2 The Value

Each column of bead is representing a value and so the total value the Abacus is represented here in the construal using the decimal numeral system which may be more intuitive to the user. The values for each column will change according to which beads are currently near to the beam.

If the agent chooses a column, other than the tenth one, to be units, a decimal point will be placed in the value at an appropriated position. This appropriate position will be after the value representing units. Selecting different columns to be units will prompt an immediate change to the position of the decimal point.

2.1.2 The tkeden Output

As beads move up and down the rods, the tkeden output aims to provide useful information about the sub-calculations being performed. The output displayed will consist of the origin of the sub-calculation, i.e., which column the sub-calculation is being performed on, the value of the column before the bead move, the change in the value from moving one or more beads and the new value of the column after the move.

Even if the agent has no knowledge of using an abacus, then from this output it can be inferred that moving one Heaven bead increases a column value by five and one Earth bead will increase the value by one.

3 Application of the Construal

3.1 For Educational Purposes

The model serves as an educational tool for even those with none or little knowledge of using the abacus. The value of the Abacus is initialised to be zero with all beads away from the beam, it is then up to the user to move the beads in order to change the values.

The highlighted beads will act as guide to allow the user to follow the simple rules of the abacus and to move certain beads where appropriate.

The users can potentially teach themselves how to add two numbers from the model. Firstly by inputting one of the numbers into the Abacus, the second number can be added onto the first in a step-by-step process and ensuring only the column units of the first number corresponds to the units of the second number when performing addition. A novice user may choose to add the beads one by one experimenting with how it would affect the value represented on the abacus.

For users who experiment with the model, they may learn more about the nature of the abacus and how it calculates. For example, they may discover that the abacus is indifferent to whether calculations are performed from left to right or from right to left, which is quite unlike how we would perform addition on large numbers using a pen and paper method.

3.2 For Illustrating Human Computing

The model can also be used to demonstrate human computing. As we have seen so far, the abacus requires some sort of human operation in order to perform calculations without this human intervention, the abacus will not automatically calculate what the user is thinking without some input. However, this level of input will vary with different users.

The model can be used to recognise the amount of mental calculations used alongside the abacus by considering the beads that were not highlighted but should have done in order to reach from one state to another.

In the addition example mentioned in section 3.1, the step-by-step process described there is likely to use extremely little mental calculations. As the user is performing the calculation moving no more than one bead at one time, the beads would be highlighted when triggered by movements of other beads.

On the other hand, a user may already be expecting to see certain values when moving the

beads. As an example, consider a user that is using the abacus to add 237 and 479, if the user knows that $7 + 9 = 16$, then he may then visualise this on the abacus by removing one Earth bead from units column and increasing the tens column by one Earth bead. In this case, the transition between one state of the Abacus to the next does not prompt the beads to be highlighted as will be the case for other calculations if this is the level of mental arithmetic involved in the calculation.

Another process could be to split the 9, as there are not enough Earth beads to represent the 9 on their own. The addition of 9 could be split up by firstly adding 3 (as there are 3 beads in that column which have yet to be pushed to the beam), then 5, then 1 each time moving the appropriate Heaven and Earth beads in both the units and tens column as highlighted by the construal. Here, the user is using more mental calculations than the first example and exploiting more of the functionality of the abacus than the second example. All three processes vary but the outcome should be the same provided the user has moved the correct number of beads, the accuracy of the calculations are highly dependent on the user's input.

4 Evaluation

The output values show in the model and also in the tkeden output window are designed to help the user understand the process behind state changes. The decimal point that appears when a units column is selected is to communicate to the user that addition with decimals is performed in the same way as integer addition.

The agent can follow the dependencies occurring by relating the movement of the beads with the changes in the values below each rod. As mentioned in section 2.1.2.1, it is possible for a rod to hold a value of more than nine and the output value below each rod would reflect this. It is technically correct but it may be undesirable for the abacus' function and be confusing at first to the agent depending on how the values of the construal are interpreted. For instance, if the agent interprets the values to represent the total value, then having a two-digit number of one column would be conflicting with the pattern they were following. However, if the agent considered the values as individual ones for each rod then there would be less confusion.

Confusion can actually have the potential to help with one's learning [5] and may prompt the agent about what is actually being visualised in the construal and the significance of maintain each rod value to have a value no greater than nine.

Without being acquainted with a tutorial to use the Abacus construal, it may not be completely clear to the user as to why beads are suddenly highlighted, having said that, beads that are highlighted follow the same pattern and with experimentation the user should be able to discover this pattern and make the connection with the rules of using the abacus.

4.1 Limitations

The model was based on Jona Gjini's idea of construing the abacus [6]. However, in the Abacus construal, the beads disappear and appear when clicked on to mimic the movement. Whereas Gjini's construal is more true to abacuses in that the beads move relative to the position of the frame.

The bead dependencies for the Abacus construal are only true for addition. The same beads would not necessarily be highlighted for all other arithmetic operations.

5 Conclusion

With Empirical Modelling concepts, such as observables, dependencies and agency, it is possible for a user to understand the simple rules an abacus uses to perform addition, even without being familiar with the abacus beforehand and in the absence of a tutorial. Despite that, included with the Abacus construal is an EMPE that aims to capture the basics of the Abacus construal in a more informative manner and it also contains an example of how addition is calculated using the construal.

Other than being used as an educational tool, it can be inferred that Human computing is the essence of performing calculations on an abacus. It requires the contribution of a human operator to make sense of what is being construed.

5.1 Further Work

Following the idea presented in Gjini's construal, the Abacus construal can be adapted so that the beads move in a more natural way, i.e. to move at all within the abacus frame.

The construal can be improved to cater for the other arithmetic operations. The dependencies of the beads will need to be reconsidered as there are differences in each case of addition, subtraction, multiplication and division.

It will also be interesting to see how the Soroban differed in comparison to Suanpan in terms of the dependencies, the speed of the calculations and how differently the user would operate the two models. With the reduction of a Heaven and Earth bead in each column, the number dependencies also

decrease. The speed of calculations are likely to be more efficient with less steps to follow through when all Heaven or Earth beads are touching the beam but this is dependent on how the user operated the Suanpan.

As part of modelling human computing, the construal can be extended to record the movements of the operator or the changes in states and take into account if any highlighted beads were not triggered that could have been in between the states, this would be an insight into how much mental arithmetic the operator used during the process of altering states.

Finally, in regards to human computing, the notion of automated agency [7] can be incorporated to the Abacus. An example of this is, rather than highlighting beads to prompt the user the beads that would be highlighted would automatically move to the appropriate position when triggered, for instance, when five Earth beads move towards the beam, the first Heaven bead not touching the beam is automatically moved down and subsequently the five Earth beads are pushed away from the beam. At the same time, this change may reduce the effectiveness of the construal as a educational tool. It will become less obvious to the user if the beads are moving automatically and too quickly for the human eye to distinguish the change clearly.

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