

Decay of vortex rings in a rotating fluid

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Vortex rings propagating through a rotating fluid, along the axis of rotation, are studied experimentally. The length, x_d , traveled by the rings until they have completely decayed is measured. In agreement with theoretical arguments based on the Taylor–Proudman theorem it is observed that the value of x_d decreases with decreasing Rossby number, Ro . For the parameter range investigated here we determine an overall, global trend for the scaling of the mean decay length that can be roughly summarized by the expression $\overline{(x_d/D)} = 4.77 Ro^{1.06}$, where D is the diameter of the generator nozzle used to produce the vortex rings. © 2009 American Institute of Physics.

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I. INTRODUCTION

Vortices are the basic dynamical structures governing fluid flows. Vortex rings represent one geometrically simple type of vortex that is often employed to study aspects of vortex dynamics—for review articles, see Ref. 1 or 2. Here we investigate a new fundamental problem associated with the dynamics of vortex rings for the first time. The issue dealt with concerns how background system rotation affects the stability of a vortex ring.

Rotating flows are ubiquitous in science and technology. In rotating flow Coriolis forces give rise to phenomena absent in non-rotating flows.^{3–5} These phenomena can appear counterintuitive to anyone not familiar with the theoretical background of rotating flows. One example is, for instance, the Taylor–Proudman theorem derived from the geostrophic approximation of the full momentum equation.^{4,5}

Briefly, the Taylor–Proudman theorem states that when a fluid rotates with a rotational velocity $\vec{\Omega}$ that is aligned with the z -axis of a Cartesian coordinate system, and if geostrophy applies, then the effect of Coriolis forces is such that the derivatives $\partial u/\partial z$ and $\partial v/\partial z$ of the velocity components u and v in the x and y coordinate directions are identically zero. Here we study vortex rings propagating with velocity \vec{U} in a fluid rotating with rotational velocity $\vec{\Omega}$, as illustrated in Fig. 1. When \vec{U} is parallel to $\vec{\Omega}$ and when geostrophy applies one thus anticipates on the basis of the Taylor–Proudman theorem that background rotation will tend to promote the destruction of the ring.

For the Taylor–Proudman theorem to be applicable Coriolis effects must dominate inertial effects, i.e., the Rossby number must be sufficiently low. For a general flow geometry as in Fig. 1 vortices can, in principle, be generated under conditions that violate this requirement. However, viscous effects result in losses of inertia and, thereby, the vortex rings approach the parameter regime where the Taylor–Proudman theorem holds. Therefore, it is expected that the decay of vortex rings will, ultimately, be accelerated even when a vortex ring is generated with high inertia.

One further anticipates that effects of rotation will be-

come stronger as Coriolis effects become increasingly dominant. Hence, one expects that the average length that a vortex ring can travel before it has decayed entirely will decrease with decreasing Rossby number. The arguments leading up to the qualitative conclusion that background rotation will tend to promote the decay of vortex rings are relatively straightforward. However, obviously there exist no simple arguments to establish how the mean decay length will decrease with the Rossby number. There exist no previous studies that have investigated this fundamental question. The main motivation for the present experiments was to determine a statistical expression that relates the mean decay length of a vortex ring in a rotating fluid to its associated Rossby number.

It is no coincidence that no previous studies exist that investigated the decay of vortex rings propagating through a rotating fluid as in Fig. 1. Such experiments require a very high water-filled rotating tank together with a suitable vortex-ring generator mounted within the rotating frame of reference. This type of large-scale facility was not available anywhere in the past. There appears to exist only one single previous study, by Verzicco *et al.*,⁶ for which experiments investigating vortex rings moving through a rotating fluid in the sense of Fig. 1 were conducted. However, for this study only a comparatively small tank with a height of 1 m was available. The main focus of Verzicco *et al.*⁶ was to verify computational results primarily concerned with the velocity field and the vorticity distribution associated with the vortex ring.

II. EXPERIMENTAL SETUP AND TECHNIQUES

We developed a new, unique large-scale experimental facility described here for the first time and the experimental results discussed are the first obtained with the new rig. Figure 2 displays a technical drawing of the new facility. The setup constitutes a large water-filled tank, mounted on top of a computer-controlled rotating turntable. The tank is 2.5 m high and it has an octagonal horizontal cross section with a cross width of 1 m. The overall height of the facility, from

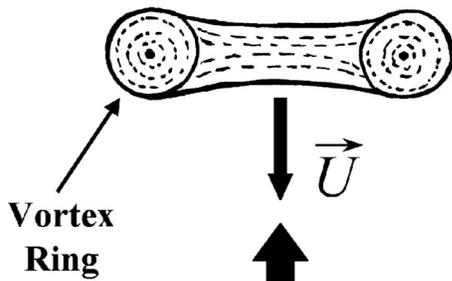


FIG. 1. Sketch illustrating the flow geometry.

the floor to the top of the support structure, is over 5.7 m. Vortex rings are generated using a standard technique^{7,8} where a computer-controlled piston ejects water from a circular nozzle such that a vortex ring forms at the nozzle exit.

Two alternative nozzles with diameters $D=50$ mm and 10 mm were used. The vortex-generator nozzle is mounted rigidly at the top of the rotating tank. This means that the nozzle is stationary with respect to the water inside the rotating tank once the water has adopted a state of rigid-body rotation following an initial spin-up phase. Hence, when the piston pushes water out of the nozzle there exists no circumferential shear on the ring as would be the case if the nozzle itself was stationary and only the tank with the fluid was rotating. The vortex rings are ejected vertically downward into the water contained inside the tank. The temperature of

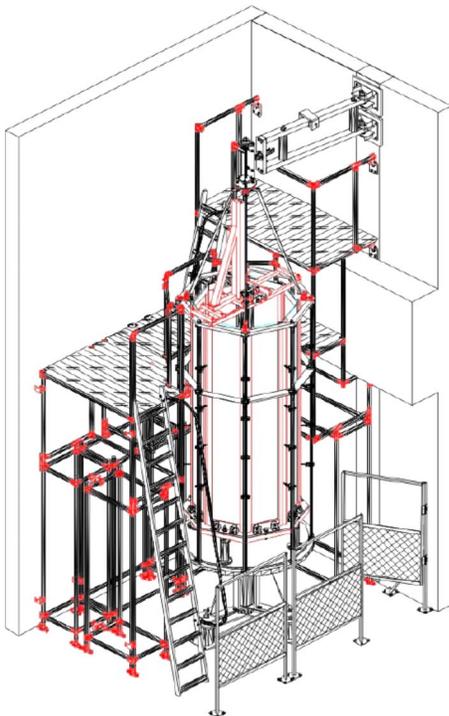


FIG. 2. (Color online) Technical drawing of the large-scale rotating tank facility. Ladder and scaffolding provide access to the top of the rig where the generator nozzle for the vortex rings is mounted.

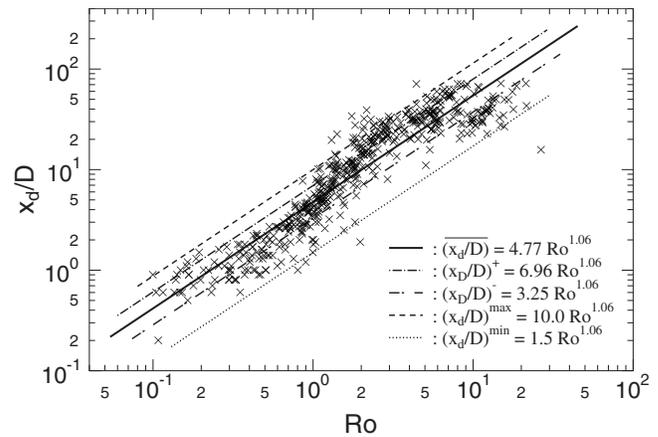


FIG. 3. Vortex-ring decay length, x_d , as a function of the Rossby number, Ro .

the water was in the range of 10–15 °C such that its kinematic viscosity is $\nu \approx 1.30 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$.

The results summarized here were obtained from flow visualizations. The vortex rings were visualized by means of releasing a small amount of neutrally buoyant food coloring through a narrow annular gap on the inside of the nozzle near its exit. Dye was supplied to the gap from a reservoir by a peristaltic pump through a system of thin pipes and tubes contained within the walls of the vortex ring generator nozzle unit.

A Rossby number is defined as $Ro = U_M / \Omega D$. The velocity $U_M = x_d / t_d$ is the mean speed of the vortex rings where x_d represents the decay length. This length is the distance the ring travels away from the nozzle exit until it no longer represents a well defined, coherent structure and t_d is the time interval between the formation of the ring and its decay. The definition of the decay length may appear somewhat qualitative but practice has shown that it is in fact fairly straightforward to judge when a ring has completely decayed. However, the goal is to determine an expression for the overall global trend regarding how rotation affects the vortex ring decay. The distribution of the data to be presented below will reveal that the above definition of the decay length is sufficient for this purpose. For the experiments summarized here the Rossby number covered approximately the interval $0.1 \leq Ro \leq 25$. We further define a Reynolds number $Re = DU / \nu$. The Reynolds number covers values $40 \leq Re \leq 5300$ when U is identified with U_M or $385 \leq Re \leq 19230$ if, alternatively, based on the mean ejection speed of the fluid at the nozzle, as done by Maxworthy.⁸

III. EXPERIMENTAL RESULTS

Figure 3 displays the nondimensional decay length, x_d/D , as a function of the Rossby number, Ro . The figure contains almost 520 data points. The results include data points for which $\vec{\Omega}$ and \vec{U} were anti-parallel, as sketched in Fig. 1, as well as data for which both vectors had the same orientation. Figure 3 reveals that the vortex rings can typically travel rather large distances of $x_d/D=50$ at $Ro=10$ where the effects of background rotation are comparatively

weak. Note, for instance, that this value corresponds to $x_d = 2.5$ m for the nozzle with diameter $D = 50$ mm. However, for $Ro = 0.1$, i.e., when effects of rotation are strong, we observed that the rings undergo a rapid and violent destruction process. Here the decay length is reduced to about $x_d/D = 0.4$, as revealed by Fig. 3. Now x_d is only 20 mm for the nozzle with $D = 50$ mm. Hence, the experimental data show that background rotation does lead to a substantially advanced decay of the vortex rings.

In order to obtain a quantitative expression approximating the overall trend of the dependence of the decay length on the Rossby number the data in Fig. 3 have been interpolated. The solid line in Fig. 3 represents a least-squares fit of type $y = kx + c$ in coordinates $y = \ln(x_d/D)$ and $x = \ln(Ro)$ with values of $k = 1.06$ and $c = 1.56$ implying that the mean decay length is

$$\overline{(x_d/D)} = 4.77 Ro^{1.06}. \quad (1)$$

To quantify the errors associated with the least-squares fit we calculated the mean deviation of the data points from Eq. (1). To this end the values $\Delta^i = |\ln((x_d/D)_i^{\text{exp}}) - \ln(\overline{(x_d/D)})|$ for all of the i experimental data points, $(x_d/D)_i^{\text{exp}}$, were determined and then averaged; this yielded $\bar{\Delta} = 0.38 \pm 0.26$. Thus, the average separation of the data points from the mean is $\ln((x_d/D)^{\pm}) = \ln(Ro^{1.06}) + (1.56 \pm 0.38)$. This implies that the data points with an average deviation above Eq. (1) are located at $(x_d/D)^+ = 6.96 Ro^{1.06}$ while those with an average deviation below Eq. (1) are located at $(x_d/D)^- = 3.25 Ro^{1.06}$ (see Fig. 3).

The increase in $(x_d/D) \propto Ro^{1.06}$ means that the decay length increases, on average, approximately linearly with the Rossby number within the Rossby and Reynolds number regimes considered here. To obtain a convenient rule of thumb one can round the factor and the exponent in Eq. (1) to give $\overline{(x_d/D)}_{rd} \approx 5 Ro$. Possibly of more practical interest is the maximum length a vortex ring can travel before it has entirely decayed. This maximum decay length is identified approximately by the dashed line in Fig. 3 given by $(x_d/D)^{\text{max}} = 10 Ro^{1.06}$ such that, rounded, $(x_d/D)^{\text{max}} \approx 10 Ro$. Similarly one can estimate a minimum decay length. This is identified approximately by the dotted line in Fig. 3 given by $(x_d/D)^{\text{min}} = 1.5 Ro^{1.06}$ or, rounded, $(x_d/D)^{\text{min}} \approx 1.5 Ro$.

It is emphasized that the least-squares fit of Eq. (1) interpolating the data (solid line) in Fig. 3 is not intended to imply the existence of a genuine power-law scaling. The interpolation procedure simply averages the data to yield a statistical, quantitative expression for the overall global trend within the parameter regimes considered. It is by no means certain that the data in the figure do collapse onto a single straight line. In fact the overall data distribution in Fig. 3 suggests, as one might expect, that the data level off for $Ro \rightarrow 0$ and $Ro \rightarrow \infty$. Nevertheless, note that the data fit in Fig. 3 extends over approximately two orders of magnitude in each of the two coordinate directions.

The scatter of the data in Fig. 3 is also quite large. However, there are many reasons that can help explaining the magnitude of the data scatter. We typically waited 10–20 min between two successive experiments to ensure that the fluid

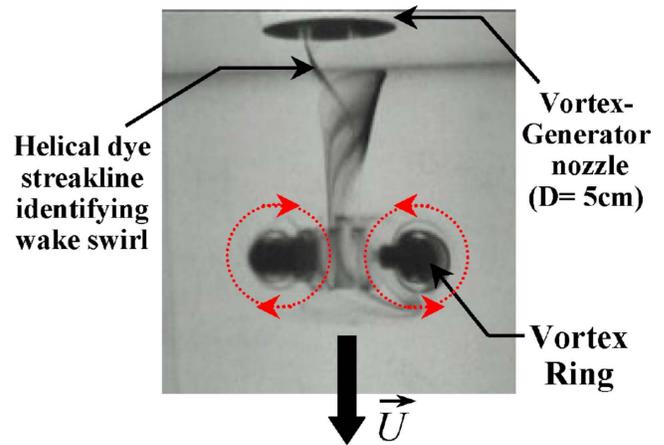


FIG. 4. (Color online) Photograph of vortex ring propagating in rotating system.

in the tank had readopted solid-body rotation prior to ejecting the next vortex ring. However, we cannot exclude the possibility that, in some cases, there may still have been some residual fluid motion in the tank. This would evidently be detrimental to the vortex ring stability implying an increased likelihood for a premature decay of the vortex ring. Furthermore, due to the nature of the problem it is evidently impossible to define a quantifiable, unique criterion that characterizes when a ring has completely decayed. Judging this necessarily involves a degree of uncertainty that cannot be resolved even if sophisticated measurement techniques, such as particle-image velocimetry (PIV), were used. Finally, Fig. 3 does not resolve any potentially existing Reynolds number effects. All discussed errors are likely to tend to result in under estimates for the decay length. However, the large number of data points contained in Fig. 3 ensures that, within the Rossby and Reynolds number regimes considered, Eq. (1) should represent a robust statistical expressions for the overall global trend for the mean decay length with associated mean deviations and estimated maximum and minimum decay lengths as discussed above.

IV. DISCUSSION AND CONCLUSION

We include with a brief comment concerning the possible physical mechanism that results in the reduced decay length of vortex rings in rotating fluids. Our observations have revealed the existence of a strong secondary Coriolis-induced cyclonic swirling flow behind the rings. The existence of the wake swirl can be inferred in Fig. 4 from the helical dye streakline identified in the photo and it has previously been briefly commented on in Verzicco *et al.*⁶ The necessity for the existence of this cyclonic wake swirl follows from considering the local flow field around the core of the vortex ring as identified by the dotted lines with superposed arrow heads in Fig. 4. The ring propagates with velocity \vec{U} downward. Just upstream behind the ring the local flow velocity, \vec{v} , of the fluid swirling around the vortex ring core is directed inward, i.e., toward the axis of symmetry as indicated in Fig. 4. The vector of the rotational velocity, $\vec{\Omega}$, points upward, i.e., it points in the opposite direction of \vec{U}

(compare Fig. 1). The Coriolis force associated with the local flow velocity, \vec{v} , is given by $F_C = -2\vec{\Omega} \times \vec{v}$. Hence, Coriolis forces are oriented such that $\vec{\Omega}$ and \vec{v} induce a wake swirl that has the same sense of rotation as the turntable. Thus, in agreement with the observations the wake swirl is expected to be cyclonic. For reasons analogous to those establishing the cyclonic wake swirl there must exist a Coriolis-induced anticyclonic swirl ahead of the rings. The cyclonic wake swirl together with the anticyclonic swirl ahead of the rings will result in strong net torsional shear forces acting on the vortex rings in the horizontal x - y plane. It can be anticipated that this torsional shear will be detrimental to the stability of the vortex rings. The intensity of the swirling flows increases with the rotation rate $\vec{\Omega}$. Hence, the torsional shear forces increase with, $\vec{\Omega}$ and, thus, with decreasing Rossby number. It appears plausible to conclude that the torsional shear represents one of the main physical mechanism advancing the vortex ring decay. Verzicco *et al.*⁶ remark (see pp. 229 and 232) that the anticyclone ahead of the ring is associated with a high pressure region while the cyclone at its rear results in a low-pressure region. They observed that the high pressure ahead of the ring establishes a strong depression at its front. This may result in further detrimental effects as regards the stability of the vortex ring. However, in order to determine the consequences of the ring distortion and the exact effects of the torsional shear, with the ultimate goal to establish the details of the overall decay process of the rings, it is evidently required to perform detailed PIV measurements, computational studies and a theoretical stability analysis in the future.

In conclusion we emphasize once again that the interpolation of the data in Fig. 3 is not intended to suggest the

existence of a genuine power-law scaling with its associated implications. The data interpolation of power-law type in the figure simply averages the data to yield a quantitative expression that roughly summarizes the overall global trend. This trend undoubtedly exists for Fig. 3 where the data extend over more than two orders of magnitude in both coordinate directions. Despite the fact that a proper power-law scaling may not exist an interpolation of the data by means of a power-law function nevertheless remains a valid averaging technique to reveal a mean statistical trend. For the data in Fig. 3 the interpolation of the data has yielded a result that can be of practical use to obtain rough estimates of the minimum, the mean and the maximum distances that nozzle-generated vortex rings can propagate through the rotating fluid before they have completely decayed. This represents an entirely new result. However, the expressions stated should not be used to draw any further reaching inferences.

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