MA475 Example Sheet 3

February 21, 2017

1. Show that the unit disc and the complex plane are not conformally equivalent.

2. Show that the unit disc is not a surface of finite type.

3. Show that there is no proper holomorphic map from the unit disk to the complex plane.

4. Let $f : S^2 \to S^2$ be the rational map defined by $f(z) = z^n + 1/z^n$. Compute the critical points, orders of critical points and branch points of π_x. Determine the monodromy of the appropriate cover. (Determine the map from the appropriate fundamental group into the appropriate permutation group.) Compute the monodromy around the point at infinity. Is this a regular cover?

5. Consider the affine curve C defined by $P(x, y) = y^3 - x(x^2 - 1)$. Let $\pi_x = \pi_x|C$ be the restriction of the projection onto the first coordinate to the curve C. Compute the critical points, orders of critical points and branch points of π_x. Explain why C is a curve of finite type. Show that we can add 3 points to create $\bar{C}$ and we can extend π_x to a map from $\bar{C}$ to S^2 which takes the added points to the point at infinity in S^2. Calculate the genus of $\bar{C}$.

6. Let R be a compact Riemann surface which has a meromorphic function of degree 2. Show that R has a nontrivial holomorphic involution where the fixed points of the involution are exactly are the critical points of the meromorphic function.

7. Consider the projective curve corresponding to the affine curve $w^2 = f(z)$ where f is a polynomial of degree d. How many points at ∞ does this curve have (as a function of d)? For which values of d is this curve non-singular at ∞? (Note that the compactification of this affine curve which we constructed in class does not necessarily agree with this compactification.)

8. Consider a torus of the form $T^2 = \mathbb{C}/\Lambda$. We have seen that T^2 admits a holomorphic self map of degree n^2. Are there tori which admit a holomorphic self maps of degrees which are not squares?