EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

WEEK 1

(1) Functoriality
   (a) Show that homology is a covariant functor from the category of chain complexes to the category of abelian groups.
   (b) Show that cohomology is a contravariant functor from the category of chain complexes to the category of abelian groups.

(2) Fix an abelian group G and let $0 \to A \to B \to C \to 0$ be a short exact sequence of abelian groups. If C is free, show that
   (a) $B \cong A \oplus C$
   (b) $Hom(B, G) \cong Hom(A, G) \oplus Hom(C, G)$
   (c) $0 \to Hom(C, G) \to Hom(B, G) \to Hom(A, G) \to 0$ is exact.

(3) Let
$$\ldots F_{k+1} \to F_k \to F_{k-1} \to \ldots \to F_2 \to F_1 \to F_0 \to H \to 0$$
and
$$\ldots F_{k+1} \to F'_k \to F'_{k-1} \to \ldots \to F'_2 \to F'_1 \to F'_0 \to H' \to 0$$
be two exact sequences of abelian groups with the F_i and F'_i free and $\alpha\colon H \to H'$ an isomorphism. In class we constructed the first two maps $\alpha_0\colon F_0 \to F'_0$ and $\alpha_1\colon F_1 \to F'_1$ in a chain map $F_\bullet \to F'_\bullet$ extending α. Show how to construct α_k for any k. Read Hatcher's proof that any two chain maps that extend α are chain-homotopic.