

EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

WEEK 3

- (1) If F is a field, show that $H^k(X; F) \cong H_k(X; F)^*$ for all k .
- (2) Let A and B be subspaces of X with $B \subset A$. Show that there is a long exact sequence
$$\dots \rightarrow H^k(X, A) \rightarrow H^k(X, B) \rightarrow H^k(A, B) \rightarrow H^{k+1}(X, A) \rightarrow \dots$$
and identify the maps explicitly.
- (3) Compute the cohomology and cup product structure of the Kline bottle.