

## EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

### WEEK 4

- (1) Let  $A$  and  $B$  be open subsets of  $X$ , let  $C_k(A + B)$  denote the subgroup of  $C_k(X)$  generated by singular simplices with images in  $A$  or in  $B$ , and  $C^k(X, A + B)$  denote cochains on  $X$  which vanish on  $C_k(A + B)$ . Use long exact sequences and the 5-lemma to show that  $H^k(X, A + B) \cong H^k(X, A \cup B)$ .
- (2) Use the relative cup product  $H^k(X, A; R) \times H^\ell(X, B; R) \rightarrow H^{k+\ell}(X, A \cup B; R)$  to show that if  $X$  is the union of contractible open subsets  $A$  and  $B$ , then all cup products of positive-dimensional classes in  $H^*(X; R)$  are zero. (First part of Exercise 2, p. 228)
- (3) Calculate the following tensor products:
  - (a)  $\mathbb{Z} \otimes_{\mathbb{Z}} (\mathbb{Z}/n\mathbb{Z})$
  - (b)  $(\mathbb{Z}/m\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/n\mathbb{Z})$
  - (c)  $(\mathbb{Z}/4\mathbb{Z}) \otimes_{(\mathbb{Z}/2\mathbb{Z})} (\mathbb{Z}/6\mathbb{Z})$
  - (d)  $R[x, y] \otimes_R R[z]$