

EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

WEEK 6

- (1) State and prove a Kunneth theorem for products of CW pairs $(X, A) \times (Y, B)$. (You can see how to do this by looking in the book.)
- (2) Suppose $i + j = n$. Explain why the choice of $\mathbb{R}P^i, \mathbb{R}P^j$ and p we made in class implies that all arrows in the following diagram make sense (with coefficients in any commutative ring). Then explain why the diagram commutes.

$$\begin{array}{ccc}
 H^i(\mathbb{R}P^n) \times H^j(\mathbb{R}P^n) & \xrightarrow{\smile} & H^{i+j}(\mathbb{R}P^n) \\
 \uparrow & & \uparrow \\
 H^i(\mathbb{R}P^n, \mathbb{R}P^n - \mathbb{R}P^j) \times H^j(\mathbb{R}P^n, \mathbb{R}P^n - \mathbb{R}P^i) & \xrightarrow{\smile} & H^{i+j}(\mathbb{R}P^n, p)
 \end{array}$$

- (3) Show that every covering space of an orientable manifold is orientable. (If no ring is specified, “orientable” means orientable over \mathbb{Z}).