EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

(1) Show that $M \times N$ is orientable if and only if both M and N are orientable.

(2) For a map $f \colon M \to N$ between connected closed orientable n-manifolds with fundamental classes $[M]$ and $[N]$, the *degree* of f is defined to be the integer d such that $f_*([M]) = d[N]$, so the sign of the degree depends on the choice of fundamental class. Show that for any connected closed orientable n-manifold M there is a degree 1 map $M \to S^n$.

(3) For any connected manifold M and ring R we defined a covering space $M_R \to R$. Construct this explicitly for $M = \mathbb{R}P^2$ and $R = \mathbb{Z}/4\mathbb{Z}$, and identify the sections of this covering space.

(4) Show that $(\alpha \frown \phi) \frown \psi = \alpha \frown (\phi \smile \psi)$ for all $\alpha \in C_k(X; R)$, $\phi \in C^i(X; r)$ and $\psi \in C^j(X; R)$. Deduce that cap produce makes $H_*(X; R)$ a right $H^*(X; R)$-module.