EXERCISES FOR MA4J7 ALGEBRAIC TOPOLOGY II

(1) Show that $H_c^0(X;G) = 0$ for a non-compact path-connected space X.

(2) Show that $H_c^n(X \times \mathbb{R};G) \cong H_c^{n-1}(X;G)$ for all n.

(3) Let $\{G_i\}_{i \in \mathbb{N}}$ be the directed system of groups in which each G_i is a copy of $\mathbb{Z}$ and for $i < j$, the homomorphism f_{ij} is multiplication by 2^{j-i}.

$$G_1 \xrightarrow{\times 2} G_2 \xrightarrow{\times 2} G_3 \xrightarrow{\times 2} \ldots$$

   (a) Let $G_1 \to \lim_{\to} G_i$ be the homomorphism which takes k to $[k]$. Is this map injective? Is it surjective? Explain.

   (b) Is there a non-trivial homomorphism $\lim_{\to} G_i \to \mathbb{Z}$? If so, define it. If not, why not?

(4) Show that a direct limit of torsion-free abelian groups G_α is torsion-free.

(5) Show that a direct limit of short exact sequences is short exact.