Bayesian Inference in Spatial Models with Skew Normal Latent Variables

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The latent Gaussian model is commonly used for spatial data. In the Bayesian framework this entails a Gaussian prior $\pi(x|\theta)$ for latent spatially correlated variable $x = (x_1, \ldots, x_n)$, a likelihood model $\pi(y|x, \theta)$ for observations $y = (y_1, \ldots, y_k)$, and a prior $\pi(\theta)$ for the model parameters. If $\pi(y|x, \theta)$ is Gaussian and linear, i.e. $\pi(y|x, \theta) = N(Ax, T)$, then $\pi(x|y, \theta)$ is also Gaussian. This conjugacy allows fast spatial prediction and estimation of model parameters. When the likelihood is non-Gaussian or non-linear, we can often linearize it in an iterative manner to obtain a Gaussian approximation $\hat{\pi}(x|y, \theta)$. This allows fast approximate spatial prediction and approximate parameter estimation. One could improve the approximations by using them as proposals in MCMC algorithms.

In this talk we will use a skew Normal distribution as prior model $\pi(x|\theta)$. A generalization of the skew normal, referred to as the *closed skew normal*, makes it possible to use the above Gaussian ideas for this skew normal model. For instance, by the properties of the *closed skew Normal*, the posterior $\pi(x|y, \theta)$ is closed skew Normal if the prior $\pi(x|\theta)$ is skew Normal and the likelihood $\pi(y|x, \theta) = N(Ax, T)$. Further, the posterior marginals $\pi(x_i|y, \theta)$ are also closed skew Normal. For non-Gaussian likelihoods one can iterate to construct a closed skew Normal approximation $\hat{\pi}(x|y, \theta)$ to the posterior. This allows approximate prediction and approximate parameter estimation for models such as spatial generalized linear mixed models, with skew Normal prior for the latent spatial variable. Approximations can again be used as proposals in MCMC algorithms.

We present results on two examples with spatial discrete data, and discuss general advantages and challenges of the closed skew normal model.