Nonasymptotic bounds on the estimation error for regenerative MCMC*

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Let $0 = T_0 < T_1 < \cdots < T_r < \cdots$ be consecutive moments at which Markov chain X_n regenerates. Let π be the stationary distribution. Regenerative estimators of $\theta := \pi(f) = \int \pi(\mathrm{d}x) f(x)$ are of the form

$$\hat{\theta}_r := \frac{\sum_{i=0}^{T_r-1} f(X_i)}{T_r}.$$

We consider a sequential/regenerative estimator $\hat{\theta}_{R(n)}$, where n is fixed and $R(n) := \min\{r : T_r \geq n\}$. We prove bounds on the mean square error of the estimator and the expected value of the stopping time $T_{R(n)}$ (length of simulations). The main assumption is a geometric drift condition towards a small set. The result holds for a general state space and possibly unbounded function f.

Theorem. *We have*

$$(i) \quad \mathbb{E}\,(\hat{\theta}_{R(n)} - \theta)^2 \leq \frac{A_1}{n}\left(1 + \frac{A_2}{n}\right)$$

and

$$(ii) \quad \mathbb{E}\,T_{R(n)} \leq n + A_2.$$

Constants A_1 and A_2 are explicitly expressed in terms of the quantities which appear in the drift condition.

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