(+ C Liverani)
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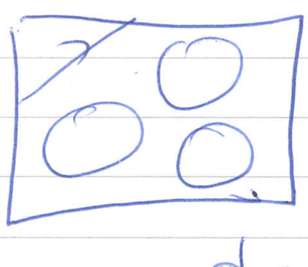
Exponential decay of correlations for piecewise hyperbolic contact flows. [In progress]

1. Motivation
2. Results
3. Ideas in proof

1. Motivation: Prove exponential decay of correlations for 2-dim Sinai billiard (flow)



strictly convex scatters, smooth boundaries, finite horizons.

dx Liouville measure (Lebesgue) is invariant and mixing.

$\boxed{\forall t>0.}$ $\exists\, \sigma_\alpha > 0$

α Hölder observables: ψ_1, ψ_2;

$$\left| \int \psi_1 (\psi_2 \circ F_t)\, dx - \int \psi_1 dx \int \psi_2 dx \right| < C_{\psi_1,\psi_2} e^{-\sigma_\alpha t}.$$

Lai-Sang Young '98: Exponential decay for Poincaré map (discrete) – Uses Towers ≡ Markov partition.

• 2007 Chernov: Stretched exponential upper bounds.

For Anosov dynamics (SRB-measure)

- diffeos (exp 70's Sinai-Ruelle-Bowen)
- flows (dim 3) — Dolgopyat '98

Used ordering symbolic dynamics.

<u>Naive idea</u>: Adapt Dolgopyat's bds to Infinite Markov Partitions (Young towers) — Did not work - lose too much information when constructing tower.

<u>New strategy</u>: "functional" approach (no symbolic dynamics / Markov partition)

Transfer operator:

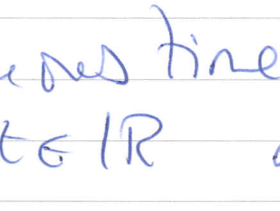
For discrete time: $\mathcal{L}\phi = \dfrac{\phi \circ F^{-1}}{|\det DF \circ F^{-1}|}$

Continuous time:

$t \in \mathbb{R}$ $\quad \mathcal{L}_t \phi = \dfrac{\phi \circ F^{-t}}{|\det DF_t \circ F_{-t}|}$

Banach space of distributions (anisotropic)

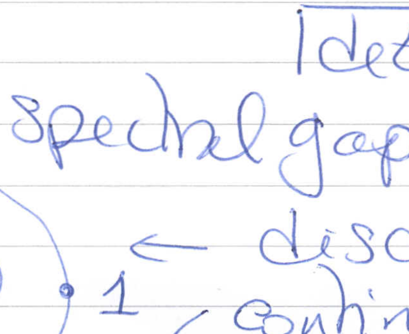
Can get a "spectral gap" if choose well the Banach space



← discrete time

1

↙ continuous time



Discrete time [Smooth, uniformly hyperbolic — Anosov or Ax. A.]

Blank–Keller–Liverani 2002
Gouëzel–Liverani 2006, 2008 [Geometric approach]
B 2005
B–Tsujii 2007, 2008. [Analytic]

(Harmonic analysis) = Fourier transform / Generalized Sobolev space $1<p<\infty$ ($p=2$, Hilbert space)

Smooth hyperbolic contact flows | Liverani 2004
Butterley–Liverani
Tsujii — semiflows (BTDS, 2008)

Start from the Banach space which works for discrete-time.

— Hyperbolic flow (2010, Non-linearity + WIP)
Spectral gap for the transfer operator for time-one map

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2 dim Sinai pw hyperbolic contact
subexponential complexity growth.

Problem A: $M = \bigcup_{\text{finite}} B_i$, pw C^2

Problem B: Blow-up of derivatives $\to \partial B_i$

4 steps: I. Spectral gap → ST, Chernov (*) for pw hyp map.
(new proof of (*))
Demers–Liverani 2008
B – Gouëzel 2009 + 2010

Complexity control
Transversality
Bunching – two dimensions.

II Spectral gap for pw hyperbolic contact flows
(Complexity + Transversality + Low dim)
3-dim flow.

III Sp gap for pw maps with blow up (including Poincaré map of Sinai Billiard)
WIP - with Balint + Gouezel
(use homogeneity layer)

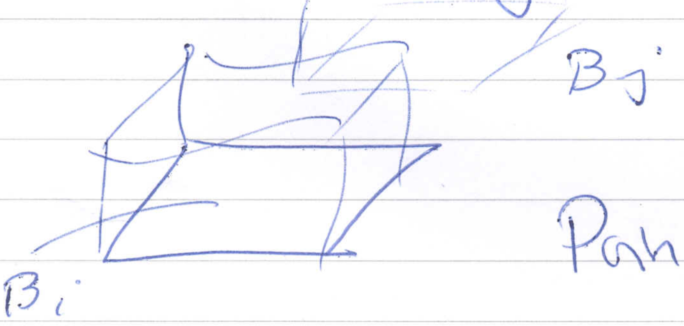
IV ?? Sp gap for billiard flow.

2. Result.

Theorem (B-Liverani): M cpt 3-d manifold
$F_t : M \to M$ pw C^2 cone hyperbolic flow
such that a) F_t preserves a contact structure ($\Rightarrow$ Lebesgue is preserved)
b) (F_t, dx) mixing
c) Hyperbolicity dominates complexity
d) Transversality between stable cones, bdy of domains
Then exponential mixing.

Contact $\exists$ smooth 1-form ω, such that invariant, $\omega \wedge d\omega$ is volume.

pw C^2 conehyperbolic: $\bigcup_{\text{finitely}} \underbrace{B_i}_{\text{flowboxes (closed)}}$



$\tau_i : O_i \to (0, \infty)$, C^2

Poincare map $P_i : O_i \to O_j$
C^2.

Cones: $\underbrace{C_i^u, C_i^s}_{1\text{-dim}^e}$ invariant

$\lambda_u > 1$, $\lambda_s < 1$

transversal

Hyperbolicity dominates complexity

$P^n_{\vec{i}}$ $\quad O_{\vec{i}}$ $\quad i_0\, i_1, \dots, i_{n-1}$

$D_n^b = \max \# \{ \vec{i},\ \text{s.t. } z \in \overline{O_{\vec{i}}} \}$

$D_n^e = \max \# \{ z \in \overline{P^n(O_{\vec{i}})} \}$

Assumption is $\overline{\lim}_{n \to \infty} (D_n^e D_n^b)^{1/2n} < \lambda$

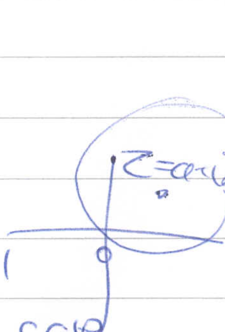
$\lambda = \min(\lambda_u, 1/\lambda_s)$

Transverse assumption.

$\partial O_i = \bigcup_e K_{i,e} \qquad K_{i,e} \pitchfork C_i^s$

C^3 curves

<u>Main tool</u>: Liverani: $X = \frac{d}{dt}(\mathcal{L}_t)$



$R(z) = (z - X)^{-1}$

$R(z) = \int_0^\infty e^{-zt} \mathcal{L}_t \, dt$ $\quad$ BGZ - mollifiers / C^∞, C^1

Easy step — Lasota Yorke (ess sprad $R(a+ib) \le 1/(a+\varepsilon)$

Hard step — Dolgopyat. $\| R(a+ib)^{c \log b} \| \le \frac{C}{(a+\varepsilon)^{c \log b}}$