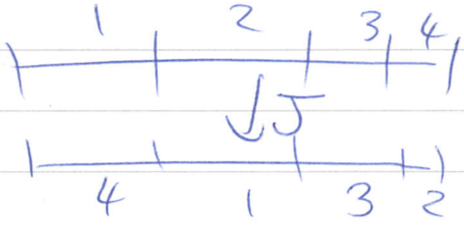
Bufetov

1. Finitely additive inv-measures for IETs



$(\lambda_1, \dots, \lambda_m)$

Masur + Veech
– invariant measure.

$$\mathcal{B} = \{ \Phi : [0,1] \to \mathbb{R},\ \Phi(0) = 0 \}$$

$$\Phi(Tb) - \Phi(Ta) = \Phi(b) - \Phi(a).$$

$d\Phi$ is a finitely additive, T-invariant measure.

For almost all IETs T

1) $\dim \mathcal{B} = \rho$ = genus of corresponding surface
2) $\Phi \in \mathcal{B} \Rightarrow \Phi$ – Hölder
3) $\exists$ positive numbers $\Theta_1 > \Theta_2 > \dots > \Theta_\rho > 0$ depending only on (Rauzy Veech class of) permutation.

$\exists$ basis $\Phi_1(t), \Phi_2, \dots, \Phi_\rho$ s.t.
$\forall \varepsilon > 0$, Φ_i is Hölder w/exponent $\Theta_i - \varepsilon$.

4) $f:[0,1]\to\mathbb{R}$, Lipschitz and i the smallest such index that $\int f\,d\Phi_i \neq 0$

$$\Longrightarrow \forall x\in[0,1],\ \overline{\lim_{N\to\infty}} \frac{\log\left|\sum_{k=0}^{N-1} f\circ T^k(x)\right|}{\log N} = \theta_i$$

Zorich, ETDS '96.

G. Forni '01 (alternative publ^n)

(M,ω) - an abelian differential

M = cpt Riemann surface of genus $g\geq 2$.

ω = holomorphic 1-form on M.

h_t^+ = vertical flow.

$\omega = dz$

h_t^- = horizontal flow.

$\mathcal{B}^+$ = space of cont cocycles Φ^+ over vertical flow satisfying

1) $\Phi^+(x,t+s) = \Phi^+(x,t) + \Phi^+(h_t^+ x, s)$.

2) Φ^+ is continuous in both verticals

3) $\Phi^+(x,t) = \Phi^+(h_s^- x, t)$

provided holonomy admissible.

For almost all pairs (M,ω):

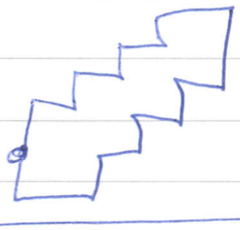
1) $\dim \mathcal{B}^+ = \rho$
2) $\Phi^+ \in \mathcal{B}^+ \Longrightarrow |\Phi^+(x,t)| \le t^{\theta} \quad \forall x$.
3) $\exists$ basis $\Phi_1^+(x,t)=t,\ \Phi_2^+, \dots, \Phi_\rho^+$ s.t. $\forall \varepsilon > 0$ Φ_i^+ is Hölder with exponent $\theta_i - \varepsilon$
4) f - Lipschitz on M $\Longrightarrow \exists \Phi_f^+ \in \mathcal{B}^+$ s.t.

$$\left| \int_0^T f \circ h_t^+(x)\, dt - \Phi_f^+(x,T) \right| \le C_\varepsilon \|f\| T^{\varepsilon}$$

We may consider similar space $\mathcal{B}^-$

$\mathcal{B}^+ \hookrightarrow \mathcal{E}^+(M,\omega)$

$\Phi^+ \hookrightarrow \{\gamma \to \int_\gamma d\Phi^+\}$.



$\mathcal{B}^- \hookrightarrow \mathcal{E}^-(M,\omega)$

$\mathcal{E}^\pm$ - (un)stable space of the Kontsevich-Zorich cocycle.

Natural pairing between $\mathcal{B}^+$, $\mathcal{B}^-$

$$\begin{array}{l}\Phi^+ \in \mathcal{B}^+ \\ \Phi^- \in \mathcal{B}^-\end{array} \Longrightarrow \langle \Phi^+, \Phi^- \rangle = \int_M \Phi^+ \times \Phi^-$$

For $\Phi^- \in \mathcal{B}^-$ denote

$$m_{\Phi^-} = dt^+ \times \Phi^-$$

wrt Lebesgue measure.

$\{m_{\Phi^-}, \Phi^- \in \mathcal{B}^-\}$ – σ-forms (invariant distn)

Description of Φ_f^+.

$\forall\, \Phi^- \in \mathcal{B}^-$ we have

$$\langle \Phi_f^+, \Phi^- \rangle = \int f \, dm_{\Phi^-}$$

$$= m_{\Phi^-}(f)$$

(Needs $\mathcal{B}^+$ Hölder continuous).

$$\Phi_f^+(x,T) = \underbrace{m_{\Phi_1^-}(f)}_{\left(\int_M f \, dLeb\right)} . \underbrace{\Phi_1^+(x,T)}_{T} \quad \left[\Phi_1^-, \ldots, \Phi_p^- = \text{Dual basis in } \mathcal{B}^-\right]$$

$$+ m_{\Phi_2^-}(f) . \Phi_2^+(x,T) + \ldots + m_{\Phi_p^-}(f) \Phi_p^+(x,T)$$

$\mathcal{M}_{\vec{k}}$ – moduli space of abelian differentials w/ prescribed singularities

g_s = Teichmüller flow

$$\underbrace{X'}_{(M,\omega)} \in \mathcal{M}'_{\vec{k}} \hookrightarrow \Phi^+_{2,X'}(x,\tau).$$

ω_Δ – second Lyapunov vector $|v| = 1$

$X' \hookrightarrow D_2^+(X')$ — measure on $C[0,1]$.

Distn of $\dfrac{\Phi^+_{2x'}(x,\tau)}{\sqrt{\mathrm{Var}\,\Phi^+_{2,x}(x,1)}}$

$$\mathfrak{S}[f,s] = \int_0^{\tau e^s} f \circ h_t^+(x)\,dt.$$

$m[f,s]$ – distn of $\dfrac{\mathfrak{S}(f,s)}{\sqrt{\mathrm{Var}(\int_0^{e^s} f \circ h_t x\,dt}}$

(Details in preprint
on archieve)

$$d_{LP}(m[f,s], D_2^+(g_s x')) \leq e^{-\alpha s}.$$

This sequence of probabilities doesn't converge – but does converge to a Teichmüller orbit, in a sense.