ABSTRACTS OF TALKS for DYNAMICS OPTIMIZATION AND GAMES,

WARWICK 29th April – 2nd May 2014

**Patrick Saint-Pierre (**Paris-Dauphine)

**Mini-course: Viability Theory**

(Partially taken from J.-P. Aubin, A. M. Bayen, P. Saint-Pierre. Viability Theory, New directions. Birkhauser, Sec Ed, 2011)

Part I: Viability Theory at a Glance

1. Historical background
2. Main Concepts of Viability Theory
3. Examples of simple Viability Problems
4. Viability and Optimality
5. Viability, Games and Uncertainty
6. A Promenade around the Zermelo Navigation Problem

Part II: The Viability Tools for Exploring Complex Dynamical Systems

1. The Viability and Capture Basin Algorithm at a glance
2. The Historical example : Pollution and Tax management
3. Approximation of a fractal set and of its boundary : the Julia Set
4. Approximation of the Lorenz attractor set
5. Three questions arising in Ecology
6. Engineering Applications

Part III: What about the development of Viability Softwares

1. The Set-Valued Numerical Analysis Challenge
2. The actual software dedicated to academic research
3. The main feature of the next generation of numerical tools developed by LASTRE

**Yurii Averboukh**

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**Minimax Approach to First-Order Mean Field Games**

Concept of minimax solutions was proposed by A.I. Subbotin for Hamilton-Jacobi equations. In that case minimax solutions are equivalent to viscosity solutions. We extend the concept of minimax solutions to first-order mean field games. This approach allows one to consider the case when the Hamiltonians is only continuous with respect to all variables, Lipschitz continuous with respect to adjoint variable and measure determining initial condition can contain atoms. We prove the existence theorem for the solution of mean-field game system using Nash equilibrium for the game with infinitely many players. The near-Nash equilibrium in the deterministic differential game with finite number of players is constructed on the basis minimax solution of first-order mean field system.

**Dario Bauso**

**Robust mean-field games and two-point boundary value problems**

Within the realm of mean field games under uncertainty, we study a population of players with individual states driven by a standard Brownian motion and a disturbance term. The contribution is three-fold: First, we establish a mean field system for such robust games. Second, we analyse robust mean field equilibria. Third, we show connections to two-point boundary value problems. Applications to production, opinion dynamics and cyber-physical systems will be discussed.

**Martino Bardi (cancelled)**

**Partial Differential Equations methods for differential games**

(1) Deterministic and stochastic two-person zero-sum differential games and their associated Isaacs Partial Differential Equations. (2) Zero-sum Differential games with multiple scales: convergence of singular perturbations and homogenization problems by viscosity methods, with applications to financial models with stochastic volatility. (3) Systems of elliptic PDEs for N-person stochastic differential games and for Mean-Field games.

**Roman Belavkin**

**Fitness-Distance Information in Stochastic Landscapes**

Many optimisation algorithms are based on computations of the gradient of an objective function (e.g. a fitness function) or its approximations. However, the majority of continuous functions are nowhere differentiable, so that the gradient-based methods are not applicable. Instead, heuristics can be used, such as genetic search, provided that the fitness function communicates some useful information. We study landscapes defined over metric spaces, and show that the values of the fitness function communicate information about the distance to optimum in some non-trivial neighbourhoods. Such information can be used to improve performance of optimization algorithms.

**Pierre Bernhard** (INRIA-Sophia Antipolis)

**Sexual selection: the Handicap Principle and adaptive dynamics.**

The theory of sexual selection is in a large extent dominated by the ``handicap paradox''. Following Amots Zahavi and Alan Grafen, we interpret it as the Bayesian equilibrium of a signalling game. We develop a mechanistic model with explicit mating dynamics ---very similar to Grafen's---, which leads to closed form equilibrium solutions. Then we investigate, beyond local stability, how evolution might have led to that situation, using adaptive dynamics as our model of evolution. Because the trait involved are function-valued, these lead to a partial differential equation. Mathematical analysis and a numerical simulation lead us to the conclusion of global stability of our equilibrium.

**Jacob Engwerda** Tilburg University, Dept. of Econometrics and O.R. The Netherlands: engwerda@uvt.nl

**Hedging in Interval Models**

Trading strategies designed to reduce risk (i.e. hedging strategies) are a widely studied topic of research in finance. Usually this design is based on stochastic models for the underlying assets. However, for a proper assessment of risks associated with the trading of derivatives, the performance of hedging strategies should be evaluated not only in the context of the idealized model that has served as the basis of strategy development, but also in the context of other models. In this talk, we consider in a discrete-time setting a deterministic modelling framework. After a short introduction on hedging we introduce the basic underlying concept of this framework, namely the interval model. The interval model assumes that prices at the next time instant can fluctuate between an upper and lower bound, which are given. We discus pricing of derivatives in interval models and optimal hedging under robust control constraints. Like the well-known binomial tree models, interval models have a certain didactical value in that they allow certain concepts to be explained in a fairly simple context. In particular interval models allow a discussion of incomplete markets and consistent price intervals. They provide an easily understood context in which one can do worst-case analysis and so they may play a role in risk management in addition to the standard tools based on sensitivity analysis and VaR computations. We show that in interval models the price of an option is in general not unique determined. A fair price for an option with a convex payoff may be any price in a compact interval. The lower and upper bound of this interval can be calculated using, e.g., the well-known stop-loss and delta-hedging strategy, respectively. In a simulation study we show that in a discrete-time setting with uncertain volatility this approach suggests that the use of binomial tree models may severely underestimate the involved cost of hedging. This applies in particular when the hedging strategy underestimates the volatility of prices. Finally, we will consider within this framework the question of how to find a hedging strategy for a call option that maximizes potential profits under the restriction that costs must not exceed an a priori given bound.

Bibliography: Bernhard P., J. Engwerda, B. Roorda, J. Schumacher, V. Kolokoltsov, P. Saint-Pierre, J.-P. Aubin, 2013. The Interval Market Model in Mathematical Finance, Birkh\"{a}user, Boston.

**Roland Malhame (**Polytechnique Monreal)

**A Class of Mean Field Control Based Collective Target Tracking Problems in Energy Systems**

 The levels of renewable energy, such as wind and solar, that can be envisaged within the generation mix of electric power systems is strongly limited by their typical intermittent character. Indeed, utilities have in general to mobilize more conventional energy sources to compensate for the variability of their renewables, and past a certain penetration level, such measures become counterproductive. In this context, energy storage can constitute an important asset. We consider the challenge of harnessing the distributed energy storage associated with electric loads such as electric water heaters, electric space heaters, and air conditioners among other loads spread out within the power system, as a tool to carry out renewable generation smoothing and peak system load shaving. The challenges come from the randomness of the individual loads, the very large numbers involved, and the constraint to limit the required communication bandwidth to implement the controls. We show that provided adequate stochastic modelling is carried out at the individual load level, and by viewing loads as agents optimizing an adequate cost function, it is possible to formulate cooperative and non-cooperative versions of a mean field games whose solution provides the required control signals in the form of decentralized control laws. The decentralization allows for scalability and local enforcement of customer safety and comfort constraints, all the while minimizing communications requirements.

This is joint work with Arman Kizilkale.

**Vladimir V. Mazalov** (Institute of Applied Mathematical Research, Karelian Research Center of Russian Academy of Sciences; vmazalov@krc.karelia.ru)

**Anna V. Mazalova** (Saint-Petersburg State University)

 **Competetive services in queueing system with transportation costs**

We consider non-cooperative n-person game related with queueing system on transportation graph. In the game there are n transportation services and flow of customers. The services announce the price for tickets for each route for which they are responsible. The customers are distributed among services depending on transportation costs. The gain of service consists of prices for tickets and the corresponding flow of customers in each link of its list of routes. We derive the equilibrium in this game for different routing schemes and latency functions. The results of computer simulations are presented.

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**Bruno Oliveira**

**Cournot duopolies with R&D investment**
We analyse a duopolistic Cournot competition model, where both firms can invest to reduce their production costs. We study an R&D investment function inspired in the logistic equation and the d’Aspremont and Jacquemin function. We do a full characterization of the associated game with our R&D investment function. Furthermore, we study the short and long term economical effects derived from myopic optimal discrete and continuous R&D dynamics. For high production costs, that can correspond to the production of new technologies, the long term economical effects are very sensitive to small changes in the efficiency of the R&D programs. For some parameter values, namely high production costs, we have the existence of regions with multiple Nash investment equilibria.

**Adam Ostaszewski**  A.J.Ostaszewski@lse.ac.uk

**Filtering and Valuation given intermittent voluntary disclosures**

We impose a filtering framework on investor-valuation of firms, when these are modelled as geometric-Brownian state processes that are privately and partially observed, at random (Poisson) times, by agents (managers). Tasked with disclosing forecast values, the agents are able purposefully to withhold their observations; explicit filtering formulas are derived for downgrading the valuations in the absence of disclosures. The analysis is conducted both for a solitary firm and for m co-dependent firms.

**Alberto Pinto**

 **A Linear Hoteling town with uncertainty**
This paper develops a theoretical framework to study price com-
petition in a Hotelling-type network game, extending the Hotelling
model of price competition with linear transportation costs from a
line to a network. Under explicit conditions on the production costs
and road lengths we show the existence of a pure Nash price equilibrium.
Furthermore, we introduce incomplete information in
the production costs of the firms and we find the Bayesian-Nash price
equilibrium.

**Sebastian van Strien:**

**Some interesting dynamics associated to games: properties of fictitious play**

This talk will focus on dynamics associated to two player repeated games, namely fictitious play. This dynamics was developed in the 1950s as a learning mechanism resulting in both players converging to their Nash equilibrium in a zero-sum game. A few years later, Shapley then showed that in non-zero games, one need not have convergence. In this talk, I will describe some of the fascinating dynamics that occurs in such systems. In the non-zero sum case one can have very unusual chaotic dynamics; on the other hand, the zero-sum setting turns out to be closely related to Hamiltonian dynamics. At the end of the talk, I aim to discuss some recent results (joint with Ostrovski) on the speed of convergence to Nash equilibria in zero-sum games (related to a paper of Shapiro dating back to 1958)

 **Wei Yang** (joint work with Vassili Kolokoltsov)

**Inspection games in a mean field setting**

We present a new development of inspection games in a mean field setting. In our inspection games, there is one forward-looking inspector and a large number of interacting inspectees.The dynamics of inspectees are modelled by controlled Markov Chains on their crime levels. By modifying the standard mean field game model, we prove the existence and local uniqueness of the solution to this type of inspection games.

**Georges Zaccour**

Chair in Game Theory and Management

GERAD and HEC Montréal, Canada

**Node-Consistent Solutions in Cooperative Dynamic Games Played on Event Trees**

I discuss a class of dynamic discrete-time stochastic games, where the uncertainty is described by an event tree. The formalism of these games is useful in dealing with a large array of problems in economics and management science, as it allows quite naturally modelling accumulation processes, uncertainties and competition/cooperation among the players. An important issue in dynamic cooperative games is the sustainability of cooperation overtime, that is, how to insure that an agreement signed by the players at initial date remains in force as time goes by. After recalling the basics, I discuss the construction of Shapley-value and core allocations that are node consistent. I provide some examples to illustrate the theoretical results.