$$\boxed{[\phi^2] > 2[\phi]}$$

joint w Ajay Chandra & Gianluca Guadagni

arxiv 1302.5971

I - motivational conjectures

II - Result

III Ex proofs

$$\exp\Big(\sum_{xy} J_{xy}\varphi_x\varphi_y\Big)\prod_x d\rho(\varphi_x)$$

$$\rho \to \cancel{\tfrac{1}{2}(\delta_{-1}+\delta_1)}$$

$$\rho \to e^{-a\phi^4 - b\phi^2}\,d\phi$$

$\underline{d=3}$, (long-range

$$J_{xy} \sim \frac{1}{|x-y|^{d+\sigma}}$$

$$\langle \phi_x \phi_y \rangle_{\text{Gaussian}} \sim \frac{1}{|x-y|^{2[\phi]}}$$

$$[\phi] = \frac{d-\sigma}{2}$$

$$\langle \phi_x \phi_y \rangle_{\text{Gaussian}} \sim \frac{1}{|x-y|^{2[\phi]}}$$

$$[\phi] = \frac{d-\sigma}{2} = \boxed{\frac{3-\varepsilon}{4}}$$

$\underline{d=3}$

$0 < \varepsilon \ll 1$

$\sigma \sim \frac{3}{2}$.

$$\langle \phi_x \phi_y \rangle_{\varphi^4}$$

(n.n. 3d $\quad \dfrac{1}{|x-y|^{2[\phi]+\eta}}$

$\frac{d-2}{2} = \frac{1}{2}$

f test fct. $L > 1$

$$\phi_r(f) = L^{(3-[\phi])r} \sum_{x \in \mathbb{Z}^3} \varphi_x\, f(L^r x)$$

$r \to -\infty$.

- $O.S \geq 0$.

- $\langle \phi(x_1) \cdots \phi(x_n) \rangle \in S'(\mathbb{R}^{3n})$

  sing support $=$ diag.

- OPE,

$$:\phi^2:$$

$$\underbrace{\phi(x_1)\phi(x_2)}_{x_2 \to x_1} = \frac{c_0}{|x_1 - x_2|^{2[\phi]}} \mathbb{1} + \frac{c_1}{|x_1 - x_2|^{2[\phi] - [\phi^2]}} N[\phi^2](x_1) + \cdots$$

$$\langle * \quad \phi(x_3) \cdots \phi(x_4) \rangle$$

$$\langle N[\phi^2](x_1), N[\phi^2](x_2) \rangle^T = \lim_{\substack{x_3 \to x_1 \\ x_4 \to x_2}} \frac{|x_1 - x_3|^{\gamma[\phi^2] - 2[\phi]} \, |x_2 - x_4|^{\gamma[\phi^2] - 2[\phi]}}{c_1^2}$$

$$\times \left\langle \phi(x_1)\phi(x_3) - \frac{c}{|x_1 - x_3|^{\ldots}}, \; \phi(x_2)\phi(x_4) - \frac{c}{|x_2 - x_4|^{\ldots}} \right\rangle^T$$

$$\langle N[\phi^2](\lambda x_1), N[\phi^2](\lambda x_2) \rangle^T = \lambda^{-2[\phi^2]} \langle N[\phi^2](x_1), N[\phi^2](x_2) \rangle^T$$

$$[\phi^2] = 2[\phi] + \frac{1}{2}\eta_{\phi^2}, \quad \eta_{\phi^2} > 0$$

$$(= -\frac{2}{3}\varepsilon + O(\varepsilon))$$

$\Rightarrow \phi^q, \psi^k$; ψ Gaussian

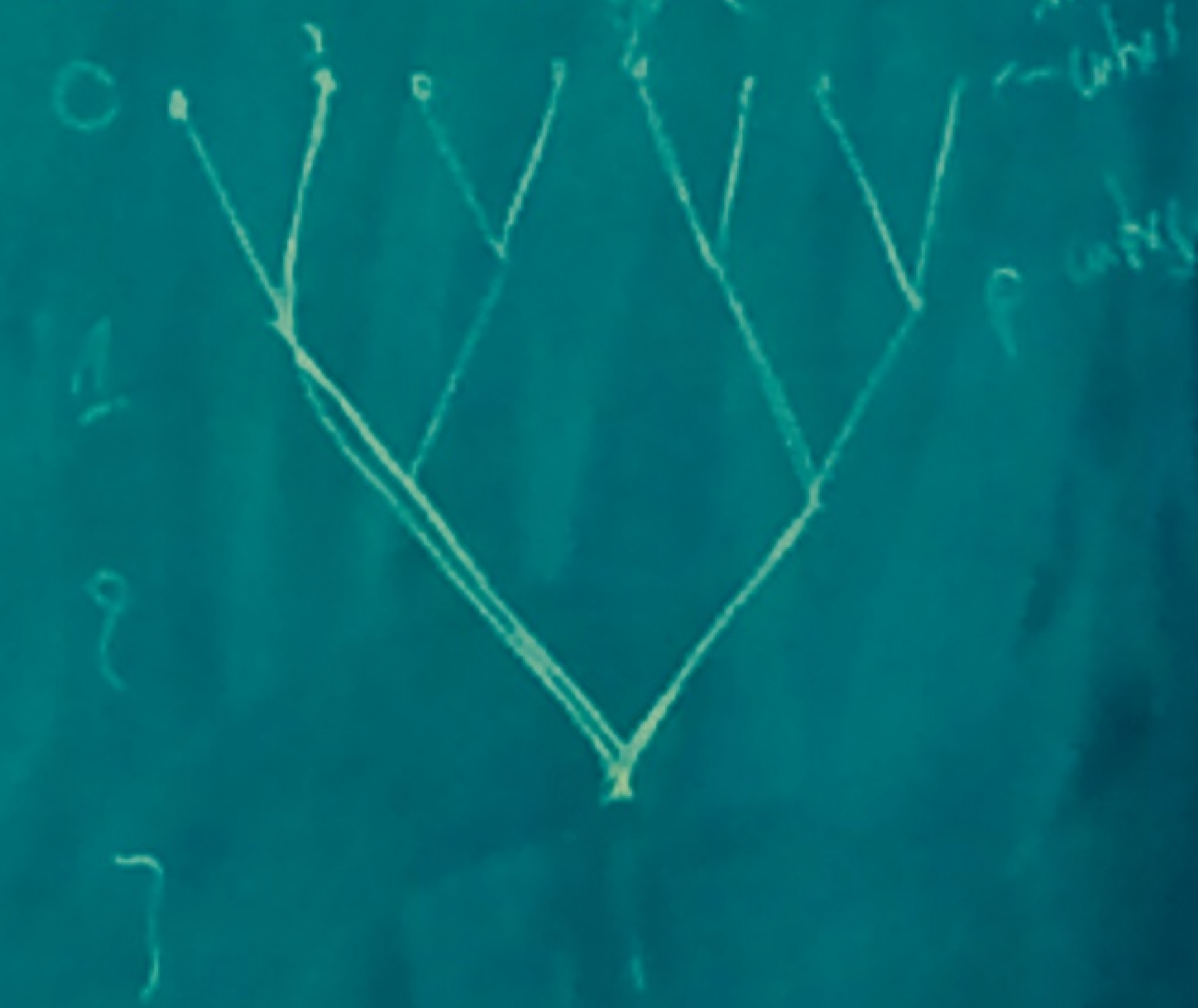
II - Result : proved ... for Hierarchical Model

ρ^d

$\rho = 2 \quad (\rho^k \ell)^3$

funny metric $|x-y| = \rho^{\text{depth of lowest common ancestor}}$

hierarchical metric



ϕ

artificial

$\dfrac{1}{|x-y|^{\alpha\phi}}$

funny

$\phi = \sum_{k\in\mathbb{Z}} \zeta_k$

~~$\mathbb{R}^3$~~ $\mathbb{Q}_p^3$

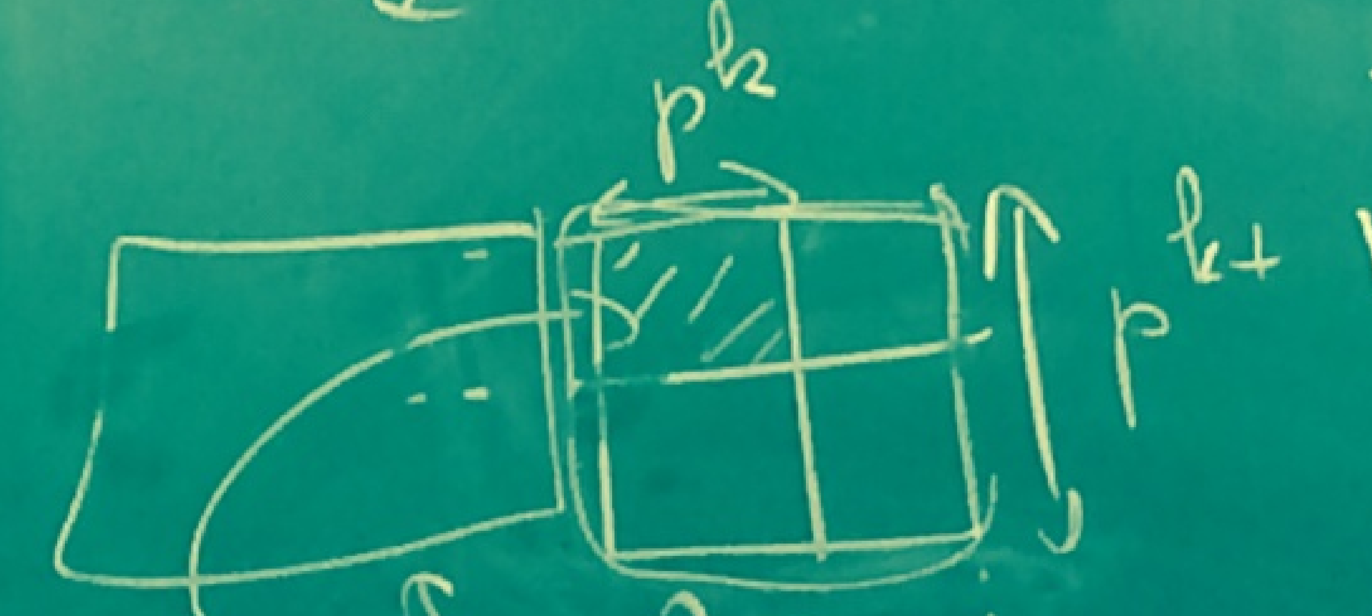
bonus

Fourier cut-off = lattice

$J_{ij} \geq 0$ Ferromagnetic

$OS \geq 0$.

$\zeta_k \sim p^{-k[\phi]}$



ζ_k st

indep

$\int \zeta_k = 0$ on p^{k+1}-cube.

J

$|\cdot| : \mathbb{Q} \to \mathbb{R}_+$

$|x| = 0 \iff x = 0$

$|x+y| \leq |x| + |y|$.

$|xy| = |x||y|$

Ostrowski Thm

$\to$ trivial $\xrightarrow{\text{completion}} \mathbb{Q}$

$\to$ usual $\longrightarrow \mathbb{R}$

$\to |\cdot|_p \longrightarrow \mathbb{Q}_p$.

$|\cdot|_p \to |x|_p = p^{-v_p(x)}$

$$\mathbb{Q}^* \ni x = \pm \prod_{p \text{ prime}} p^{v_p(x)}$$

$x = \cdots + a_{-1}p + a_0 + a_1 p + a_2 p^2 + \cdots$

$x \in \mathbb{Q}_p$. unique $a_i \in \{0, 1, \ldots, p-1\}$

finitely many.

$|p|_p = p^{-1}$

$v_p(x) = \min\{n \mid a_n \neq 0\}$

Haar meas

$\mathbb{Z}_p = \{ x \mid |x|_p \leq 1 \}$

compact subring

$|x+y|_p \leq \max(|x|_p, |y|_p)$

$\{x\}_p = \cdots a_{-2} p^{-1} + a_{-1} p^{-1} \in \mathbb{Q}$

$\mathcal{F}f(k) = \int dx\, e^{-2i\pi\{kx\}_p} f(x)$

$e^{2i\pi\{kx\}_p}, \quad k \in \mathbb{Q}_p$

$\widehat{\mathbb{Q}_p} = \mathbb{Q}_p \qquad S(\mathbb{Q}_p)$

$+$

$\mathbb{Q}_p$

$e^{2i\pi\{k\cdot x\}_p}$

$k_1x_1 + k_2x_2 + k_3x_3$

$|x| = \max(|x_1|_p, |x_2|_p, |x_3|_p)$

$\int_{-\infty} \frac{1}{|x-y|^{2\gamma(\phi)}} \to \frac{1}{|k|^{3-2\gamma(\phi)}}$

$d\mu_{C_a}$ Gaussian on $S'(\mathbb{Q}_p^3)$

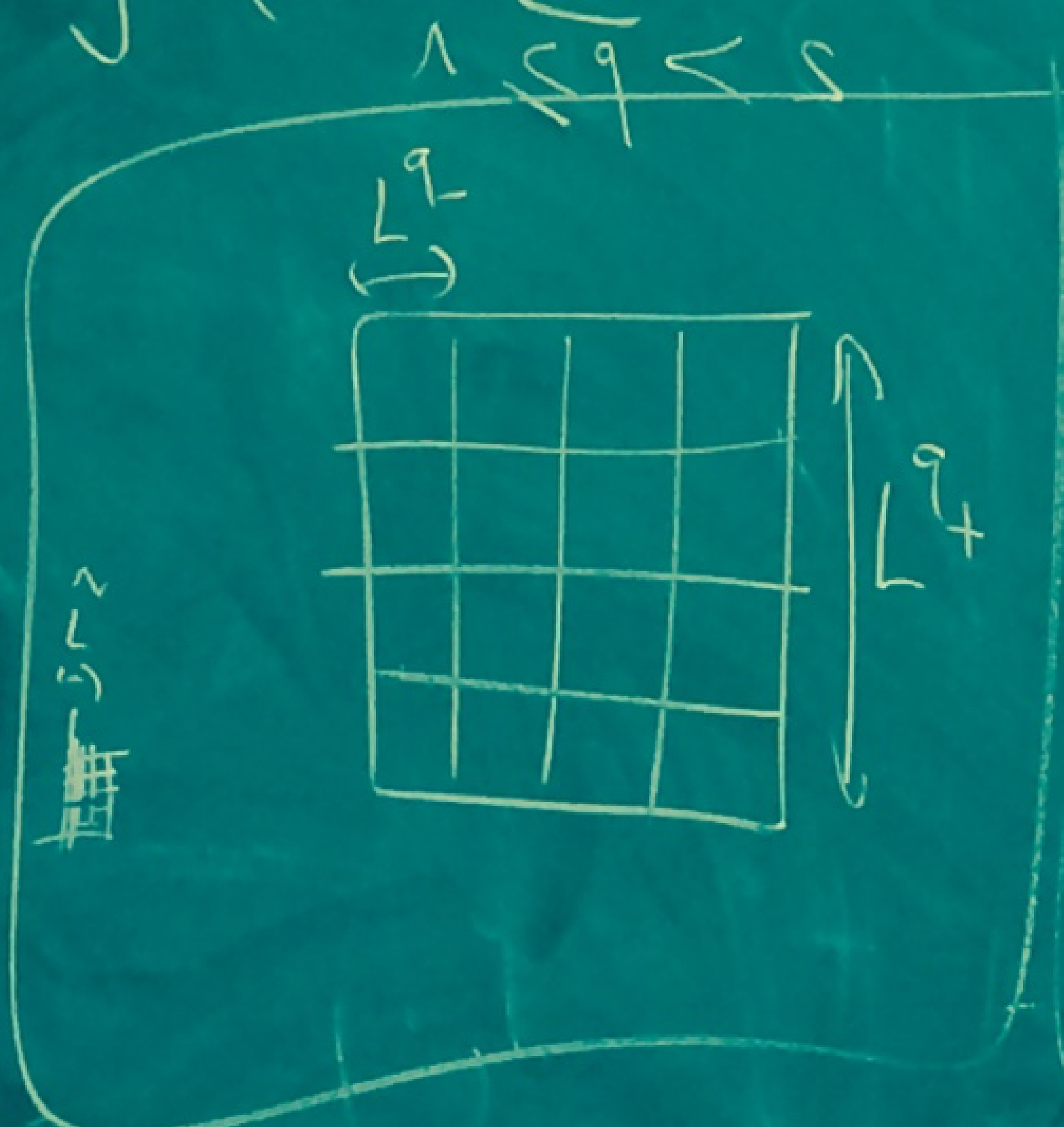
$L = p^{\ell}$ $\ell > 0$, Ur.

$\hat{C}_a(k) = \frac{1\!\!1\{|k| \le L^{a}\}}{|k|^{3-2\gamma(\phi)}}$

$a \to -\infty$

$\Lambda_s = \{|x| \leq L^s\}$
$s \to +\infty$

$$\log S(\ ) = \sum_{s \geq q > s} \left(\delta b(\nabla^{q}(f,j)) - \delta b(\nabla^{q}_{0}, 0) \right)$$



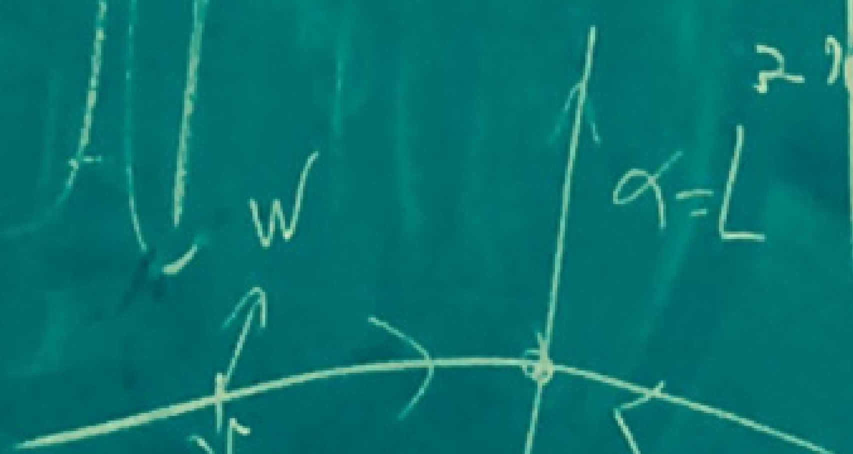
$g_\Delta, \mu_\Delta \quad \int d\mu_{C_0}(\phi)\, \mathcal{J}(\phi) = e^{\delta b[\mathcal{J}]} \int d\mu_0(\phi)\, \mathcal{J}'(\phi)$

K_Δ

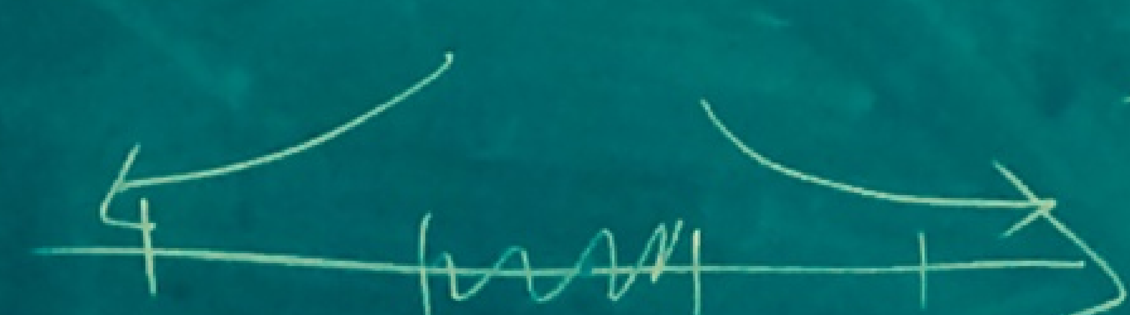
$$\mathcal{J}'(\phi) = \int d\mu_\Gamma(\zeta)\, \mathcal{J}\left(\zeta + L^{-[\phi]}\phi(L\,\cdot)\right)$$

$\alpha = L^{2-2[\phi]-\frac{1}{2}[\phi^2]} \quad \{I_\Delta\}_\Delta \quad \{1_\Delta\} \longleftarrow$

$$\mathcal{J}(\phi) = \prod_\Delta I_\Delta(\phi_\Delta)$$



$$\lim_{n\to\infty} RG^{n}(\nu + \alpha^{-n} w) \sim \mathcal{J} \xrightarrow{RG} \mathcal{J}$$



$\longleftarrow q \quad q_- \quad q_+ \quad s \longrightarrow$