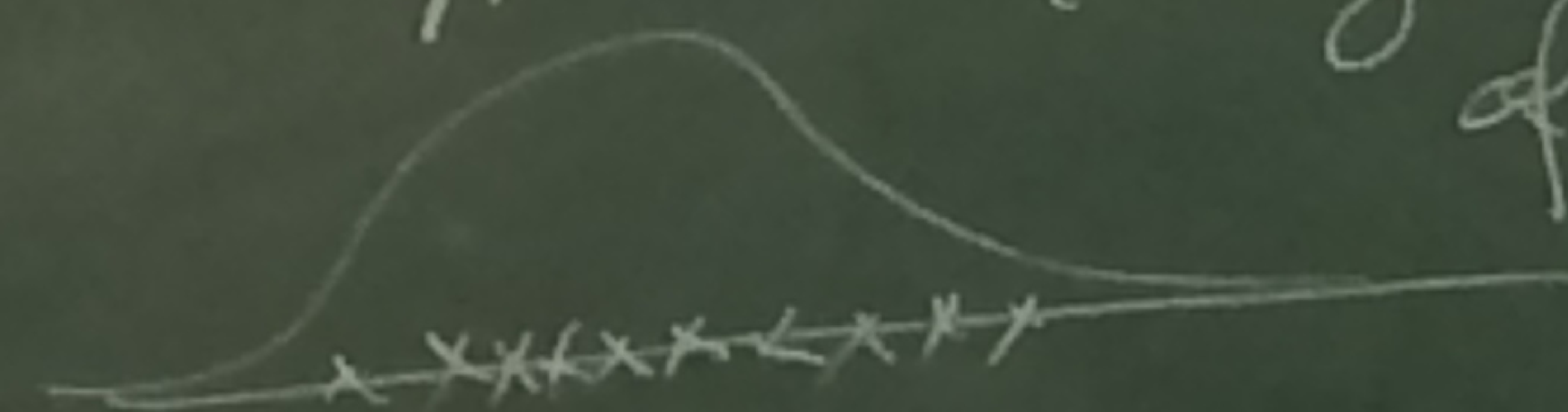
Joint with Eliran Subag

1. REM X_i - iid, unbounded support, exponential tail (+ regularity of tail)

$$\xi_n = \sum_{i=1}^{n} \delta_{X_i - m_n}$$



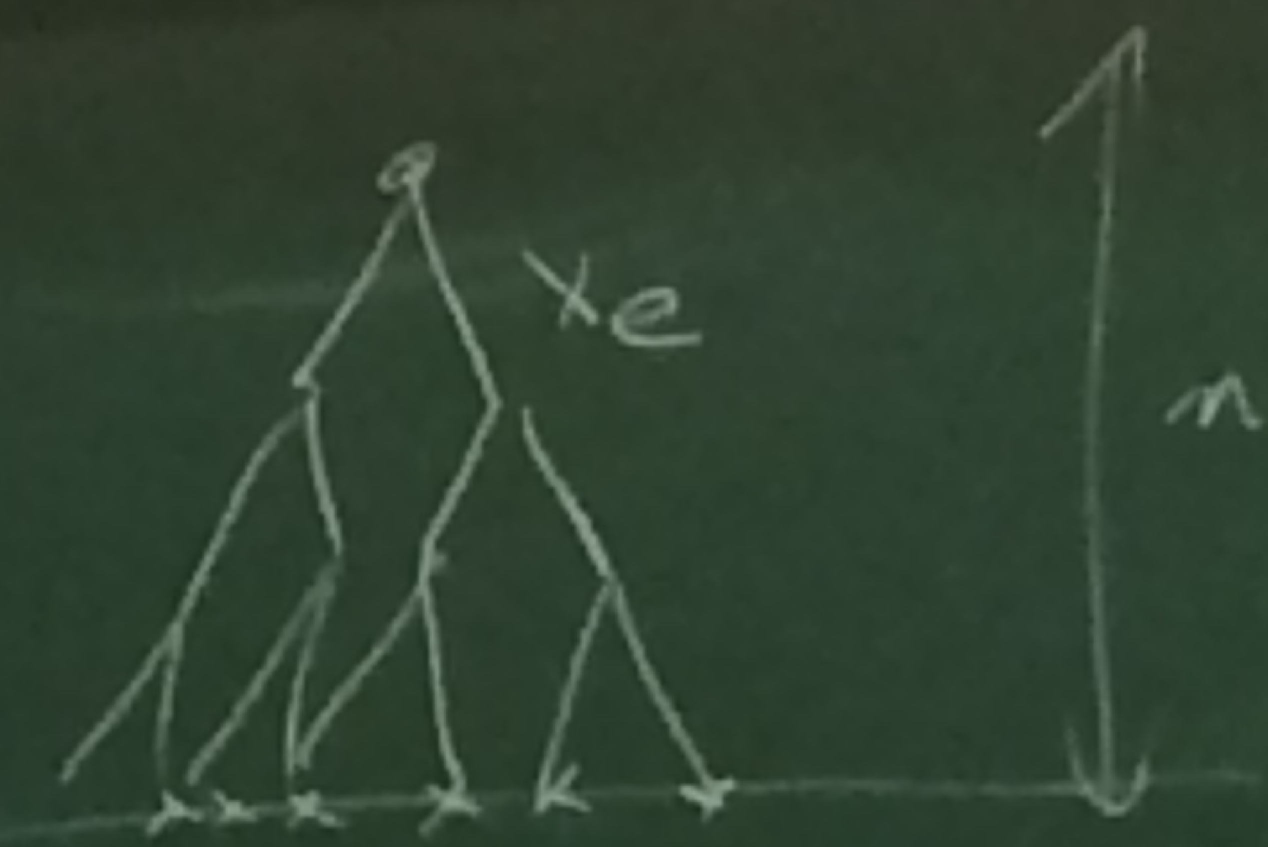
Well known

$\xi_n \longrightarrow \xi$ PPP, intensity $e^{-cx}\,dx$

b. BBM/BRW

(b) BBM/BRW

$$\sum_{v \in D_n} \delta_{S_v - m_n}$$



$$S_v = \sum_{e \in 0 \leftrightarrow v} X_e$$

Thm (ARGUIN - BOVIER - KISTLER
(2010-2013) AIDEKON - BERESTYCKI - BRUNET - SHI)

2013 MADAULE

(a) $\{x_i\}$ - part of PPP e^{-cx}

(b) D - pp, D_i - iid copies

(c) Z - r.v.

$$\xi = \sum \theta_{x_i + Z} D_i$$

Shift

SDPPP(c, Z, D)

(c) 2 speed BBM/BRW / variable speed BBM

(Bovier-Hartung 13-14)

$\sigma_1 > \sigma_2$ $\sigma_2 > \sigma_1$

Baroque Non Baroque

(d) (2d) DGFF. Consider only local maxima

$$\xi_n = \sum \delta_{X_i^{(n)} - m_n}$$

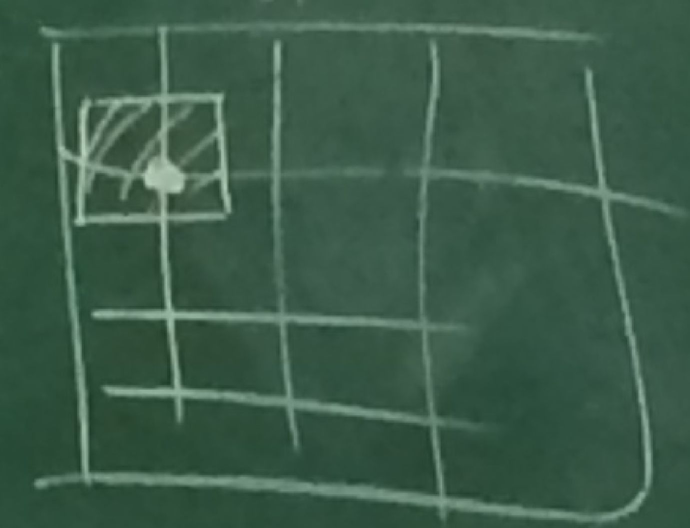
$X_i^{(n)}$

(within boxes of size $n \gg K_n \gg 1$)

Biskup-Louidor ('13) $\xi_n \to \xi$

$\xi \sim \mathrm{DPPP}(c, \delta_0, Z)$

$\xi_n = \sum \delta_{x_i^{(n)}} - m_n$



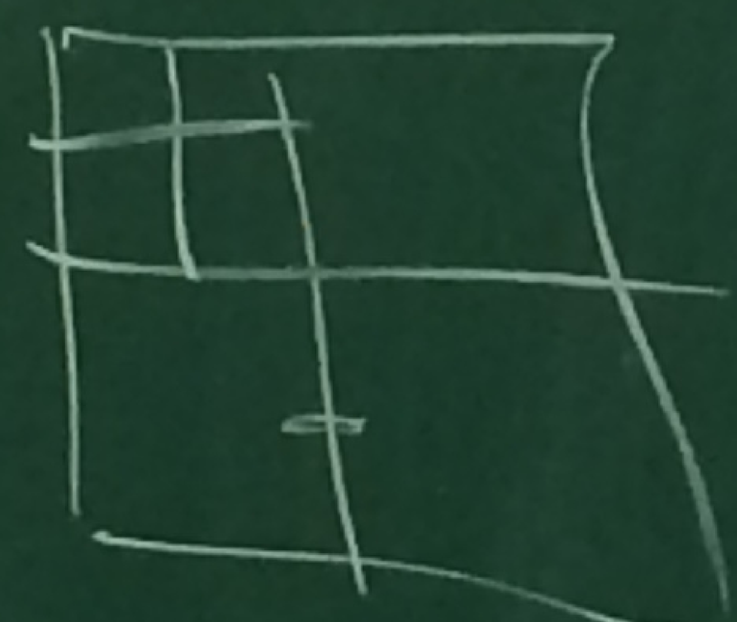
$X_i^{(n)}$

(within boxes of size $n \gg k_n \gg 1$

Biskup-Louidor ('13) $\xi_n \to \xi$

ξ SDPPP$(c; \delta_0, Z)$

— " — (14? 15?) Some full decoration



Basic Q

Features of SDPPP?

Freezing (Derrida-Spohn '88)

$$Z_\beta(y) = \sum e^{\beta(X_i^{(n)} - y)}$$

$$\frac{1}{\beta n} \log Z_\beta(y) \xrightarrow{?} C_{\beta > \beta_c} \quad (*)$$

$$E\left(e^{-Z_\beta(y)}\right) = G_{\beta, n}(y)$$

Thm

$$G\left(y + m(\beta, n)\right) \longrightarrow g_{\beta_c}(y)$$

(DS)-freezing

Carpentier – Le Doussal
Fyodorov – Bouchaud, LD + F,

DS Freezing pod $*$-scale models

('13) Madaule – Rhodes – Vargas

(DS)-freezing $\Longleftrightarrow$ Gumbel type extremes

??

$\Longleftrightarrow$ DSPPP extremal pr.

Thm (ARGUIN - BOVIER - KISTLER
(2010-2013) AIDEKON - BERESTYCKI - BRUNET
2013 MADAULE

Def. $f \approx g$ if $\exists c_f$ s.t. $f(y - c_f) = g(y)$

- $L_\xi(f \mid y) = E\left(e^{-\sum f(x_i - y)}\right)$, $f \geq 0$ (often, f compactly supported)

DS-freezing $\quad L_\xi(e^{\beta \cdot} \mid y) \approx g(\cdot) \quad \forall \beta > \beta_c$

- L_ξ is uniquely supported on g if

$$L_\xi(f \mid y) \approx g(y), \quad f \in C_c^+(\mathbb{R}), \ f \neq 0$$

- $\mathrm{Gum}(y) = e^{-e^{-y}}$

Thm [SZ '14] ξ PP, loc finite

$P(\xi(\mathbb{R})>0)=1$

g, monotone $\nearrow$, finite $@\infty$

[US] $L_\xi(f|\cdot)\approx g(\cdot)$

[SUS] (US)+ $g(y)= E\,\mathrm{Gum}(c(y-Z))$

some c, Z.

[SDP] $\xi \sim \mathrm{SDPPP}(c, D, Z)$

indep

(A4) $\lim_{x\to\infty} \log \frac{g(x+y)}{g(x)} = e^{-cy}$

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(A4) $\lim_{x\to\infty} \log \frac{g(x+y)}{g(x)} = e^{-cy}$ indep

Direct: SDP $\not\longrightarrow$ SUS

Converse Assume (A4)

(b1) (SUS) $\Rightarrow$ SDP

DPPP

(b2) (US) $\Rightarrow$ $\xi = \Theta_z \psi$, z may depend on ψ

(b3) DS fr... + US $\Rightarrow$ SUS ($\Rightarrow$ SDP)

(b4) (US) + $\exists\, \alpha(\,)$ $\frac{\xi(-y,\infty)}{\alpha(y)} \to c$ $\Rightarrow$ SUS

Conj

(US) $\Rightarrow$ SDP

① If $X_i \stackrel{d}{=} X_i + Z_i$

$\uparrow$ iid

$\to X_i$ PPP, intensity e^{-cx}

Liggett, Aizenman-Ruzmaikina

②

$$\bar{X} \stackrel{d}{=} \Theta_a \bar{X} + \Theta_b \bar{X}'$$

$e^a + e^b = c$

$\Rightarrow$ DPP(c, D)

Then SUS holds, $Z = 0$

$g(x) = \text{Gum}(e^{-x})$

③ SDPPP $\Rightarrow$ max(ξ) is shifted Gumbel

④ <u>Amusing pt</u> Fyodorov - Bouchaud

Max is

$$F = 2e^{-\beta_c x/2} K_1\left(2e^{-\beta_c x/2}\right)$$

Bessel

<u>Gumbel</u> F = dist of max - min

$= G * G$

Direct part: use Poisson

<u>Converse</u> – Construct decoration:

look at Θ_y } conditioned on $\xi(y, o) > 0$

— limit is decoration.

– For (b4) or (b3)!

Compute L_Θ

Direct part : use Poisson
decoration:
conditioned


Direct part : use Poisson
Converse
decoration:
look
conditioned
– For