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~~[illegible]~~

Artin Gps:

Def Small type: Finite graph Γ: 

Vertices: v. If v_1, v_2 are not connected by an edge: $v_1 v_2 = v_2 v_1$

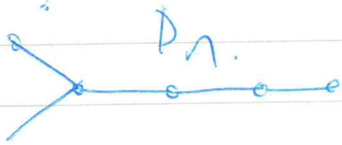
connected by an edge: $v_1 v_2 v_1 = v_2 v_1 v_2$

Exs: Finite type Artin gps (Brieskorn – Saito)

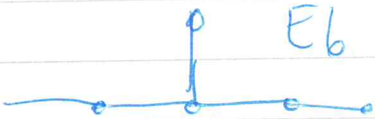
Coxeter gp. defined by Γ is finite

A_n

A_n = Artin braid gp on $n+1$ strands = MCG. of disk with $n+1$ punctures.



D_n.



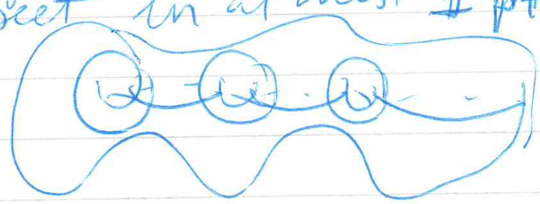
E_6

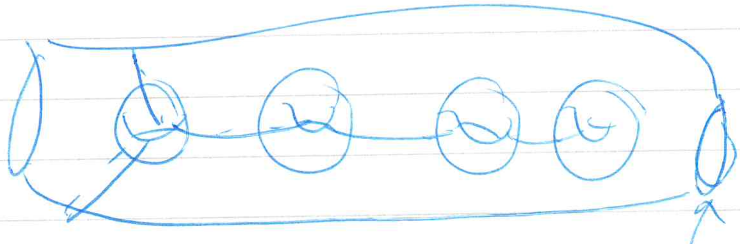
E_7

E_8

Take surface S (closed)

Curve diagram on S: scc $c_1 \ldots c_k$ s.t. c_i, c_{i+1} intersect in at most 1 pt, connected.





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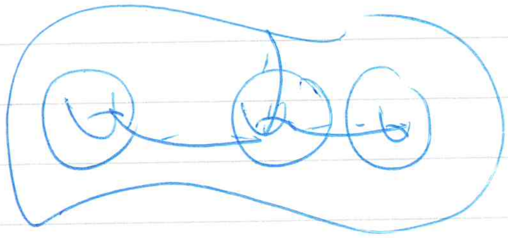


Diagram is minimal $\iff$ Cut S along diagram, get topological disk

Give curves orientation $\Rightarrow$ c_i define basis of $H_1(S;\mathbb{Z})$

<u>Def</u>: Arf invt of minimal curve diagram:

$c_1, \dots, c_{2g}$

$w \in H_1(S; \mathbb{Z}/2\mathbb{Z}) \Rightarrow$ write $w = \sum a_i [c_i]$

take those $a_i = 1$.

$Q(w) :=$ Euler char. of the graph, vertices c_i.

$Q(c_i) = 1$

<u>Claim</u>: Q is quadratic form for $H_1(S; \mathbb{Z}/2\mathbb{Z}) \iff$

$Q(a+b) = Q(a) + Q(b) + \underbrace{\langle a, b \rangle}_{\text{int \# mod 2}}$

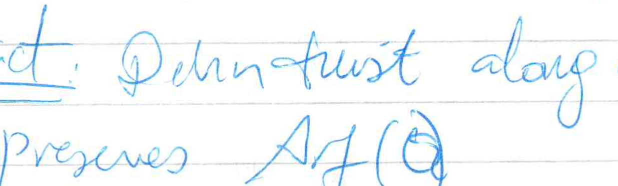
Arf invt of Q

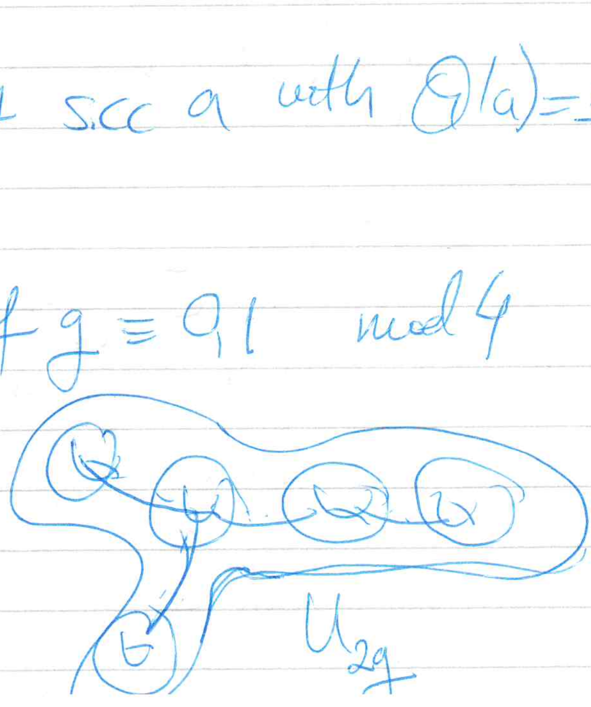
Take symplectic basis $a_1, \dots, a_g, b_1, \dots, b_g$, for $H_1(S, \mathbb{Z}/2\mathbb{Z})$;

$\mathrm{Arf}(Q) = \sum Q(a_i) Q(b_i)$

<u>Fact</u>: Dehn twist along any scc a with $Q(a) = 1$ preserves $\mathrm{Arf}(Q)$

- E_{2g}: Compute $A(E_{2g}) = 0$ iff $g \equiv 0, 1 \mod 4$





$A(U_{2g}) = 0$ if $g \equiv 2, 3 \mod 4$

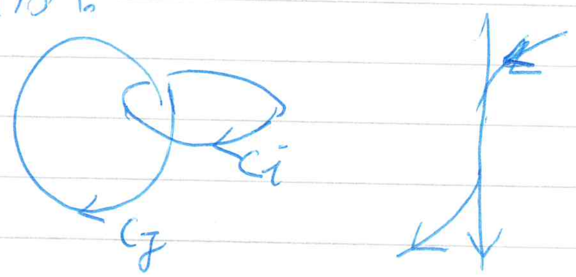
FACT: The Artin gp $A(\Gamma)$ defined by the curve diagram admits a homomorphism $R: A(\Gamma) \to Mod(S_{g,1})$ associates to generator the corresponding Dehn twist

Fact (Perron–Vannier)
For D_n, the map R is injective

Thm (Wajnryb)
For E_6, E_7, E_8, it is not inj. (hence kernel is inf)

Defn. A curve diagram has a consistent orientation iff
let $p \in c_i \cap c_j$, assume that orientation of T_pS equals orientation given by $c_i'(p), c_j'(p)$ then for all ℓ s.t. $c_i \cap c_\ell \ni q$ then $c_i'(q)\ c_\ell'(q)$ is oriented basis of T_qS.

Fact. If the curve diagram is a tree, then $\exists$ a consistent orientation.



replacement with every intersection pt $\Rightarrow$ get a train track $\mathcal{T}$ on S.

Fact: A vertex cycle on $\mathcal{T}$ is an imbedded s.c.c in $\mathcal{T}$ (C^1-embedded)
$\mathcal{T}$ is oriented (consistent choice)

Let $B(\mathcal{T}) < \mathrm{Mod}(S_{g,1})$ gtd by Dehn twists along all vertex cycles in $\mathcal{T}$.

Lemma: $B(\mathcal{T}) = R(A(\Gamma))$

Lemma: If η is obtained from $\mathcal{T}$ by a single split, then $B(\eta) = B(\mathcal{T})$

This implies.

Let us assume $\mathcal{Q}$ is a component of stratum of Abelian diffls of S with single zero, $\Rightarrow$ stabilizer in $\mathrm{Mod}(S_{g,1})$ of a component of preimage in Teich space $= R(A(\Gamma))$
($\neq$ hyperelliptic)

E_6, E_7, E_8:
Via the homom. R, these gps act with unbdd orbit on the curve graph of S.

In particular, these gps act on hyperbolic graph ~~with~~ in non-elementary way.

Huang + H: For every finite type Artin gp, G $\exists$ hyperbolic graph with acylindrical action of G. This graph is a quasi-free.

Bestvina's construction?

Corollary: $A(E_6), A(E_7), A(E_8)$ (Center) embed quasi-isom. embeds into a finite product of hyp. graphs

BBF

Handlebody of genus $g \geq 2$

Disk gr: • Vertices: Disk bounding curves
• Edge: disjointness

$H \subset \mathcal{B}$ quasi convex, not qi emb.

Hole: Realize H as an I bundle over surface w/ 1 bdy cmp
bdy curve γ of F curve in ∂H

Hole: Set of all disk which intersect γ in precisely 2 pts