Test 1

- *Write your full name and email on the first sheet*
- *Time:* ***50 minutes***
- *Books, notes and calculators* ***are not allowed***

Problem 1 Prove that for every real x we have

$$\cos\cos x \geq |\sin x|.$$

Problem 2 Let $\mathbb{N} = \{1, 2, 3, \dots\}$ be the set of natural numbers. Let $n, a, b, c \in \mathbb{N}$ satisfy $n = a^2 + b^2 + c^2$. Prove that every natural power of n is also a sum of three non-zero squares, that is, for every $k \in \mathbb{N}$ there are $A, B, C \in \mathbb{N}$ with $n^k = A^2 + B^2 + C^2$.