Poisson random variables.
Let $X \sim \text{Poi}(\lambda)$ be a Poisson random variable with intensity $\lambda \geq 0$, i.e.
$$
P[X = k] = \frac{\lambda^k}{k!} e^{-\lambda} \quad \text{for all } k \in \mathbb{N}_0.
$$
We have $\mathbb{E}[X] = \lambda$, $\text{Var}[X] = \lambda$ and the characteristic function of $X$ is
$$
\Phi_X(t) = \mathbb{E}[e^{itX}] = \sum_{k=0}^{\infty} \frac{e^{itk} \lambda^k}{k!} e^{-\lambda} = \exp(\lambda(e^{it} - 1)).
$$
Therefore, if $X_i \sim \text{Poi}(\lambda_i)$, $i = 1, \ldots, n$ are independent Poisson, then the sum is also Poisson,
$$
S = \sum_{i=1}^{n} X_i \sim \text{Poi}(\lambda_1 + \ldots + \lambda_n).
$$
For $\alpha \in [0, 1]$, an $\alpha$-thinning $\alpha \circ X$ of an integer random variable $X \in \mathbb{N}_0$ is defined as
$$
\alpha \circ X = \sum_{k=1}^{X} Z_k \quad \text{with} \quad Z_k \sim \text{Be}(\alpha) \in \{0, 1\} \quad \text{iid Bernoulli}.
$$
For Poisson variables we have $X \sim \text{Poi}(\lambda)$, $\alpha \in [0, 1] \Rightarrow \alpha \circ X \sim \text{Poi}(\alpha \lambda)$.
This follows directly from computing the characteristic function
$$
\Phi_{\alpha \circ X}(t) = \mathbb{E}(e^{it\sum_{k=1}^{X} Z_k}) = \sum_{n=0}^{\infty} \frac{(\lambda \alpha e^{it})^n}{n!} e^{-\lambda \alpha e^{it}} = \Phi_X(\Phi_Z(t)) = \exp(\lambda \alpha (e^{it} - 1)).
$$

Poisson processes.
A Poisson process $N = (N_t : t \geq 0) \sim \text{PP}(\lambda)$ with rate $\lambda > 0$ is a Markov chain with independent stationary increments, and $N_t \sim \text{Poi}(\lambda t)$ for all $t \geq 0$. The holding times are independent $\text{Exp}(\lambda)$ variables with mean $1/\lambda$. The above properties for Poisson random variables imply the following:

- **Adding Poisson processes.**
  Let $N^i \sim \text{PP}(\lambda_i)$ be independent Poisson processes, and define their sum $M = (M_t : t \geq 0)$ via $M_t := N^1_t + \ldots + N^n_t$ for all $t \geq 0$. Then $M \sim \text{PP}(\lambda_1 + \ldots + \lambda_n)$ is a Poisson process.

- **Thinning.**
  An $\alpha$-thinning $\alpha \circ N$ of a Poisson process $N \sim \text{PP}(\lambda)$ is defined via $(\alpha \circ N)_t = \alpha \circ N_t$ for all $t \geq 0$, i.e. independently keep jumps with probability $\alpha$ and erase the others. Then $\alpha \circ N \sim \text{PP}(\alpha \lambda)$ is again a Poisson process.