Networks and Random Processes

Problem sheet 2 – Part 2

Sheet counts 60/100 homework marks, [x] indicates weight of the question. Please put solutions in my pigeon hole or give them to me by Friday, 28.10.2016, 2pm.

2.4 Dorogovtsev-Mendes-Samukhin model
Consider the following generalization of the Barabási-Albert model. Starting with \( m_0 = 5 \) connected nodes, in each timestep a node \( j \) is added and linked to \( m = 5 \) existing distinct nodes according to the probability (to be adapted to avoid double edges)

\[
\pi_{j \to i} = \frac{k_0 + k_i}{\sum_{i \in V(t)} (k_i + k_0)}, \quad k_0 \in \mathbb{N}_0.
\]

Simulate the model for three different values of \( k_0 = 0, 2, 4 \) to generate graphs of size \( N = |V| = 1000 \), with 20 independent realizations in each case.

(a) Plot the degree distribution in a double logarithmic plot for a single realization and for all 100. For each \( k_0 \) compare to the power law with exponent \(-3 - k_0/m\).

(b) Compute \( k_{nn}(k) = \mathbb{E}\left[ \sum_{i \in V} k_{nn,i} \delta_{k_i,k} / \sum_{i \in V} \delta_{k_i,k} \right] \) where \( k_{nn,i} = \frac{1}{k_i} \sum_{j \in V} a_{ij} k_j \), and decide whether the graphs are typically uncorrelated or (dis-)assortative.

(c) Plot the spectrum of the adjacency matrix \( A \) using all realizations with a kernel density estimate, and compare it to the Wigner semi-circle law.

2.5 Erdős Rényi random graphs
Consider the Erdős Rényi random graph model and simulate at least 20 realizations of \( G_{N,p} \) graphs with \( p = p_N = z/N \), \( z = 0.1, 0.2, \ldots, 3.0 \) for \( N = 100 \) and \( N = 1000 \).

(a) Plot the expected size of the largest two components against \( z \) for both values of \( N \).

(b) For \( N = 1000 \) plot the expected local clustering coefficient \( \mathbb{E}[\langle C_i \rangle] \) against \( z \).

(c) Consider \( z = 0.5, 1.5, 5 \) and 10. Plot the spectrum of the adjacency matrix \( A \) using all realizations with a kernel density estimate, and compare it to the Wigner semi-circle law.