Thermal Physics II

1st short test – 26 February 2013

given and surname : Lecturer

university number :

course of study : Teaching

marks obtained : 26

total marks : 25 + 1

comments :

1. Order the following thermodynamic quantities into a group of extensive quantities on the left and a group of intensive quantities on the right: internal energy U, energy density $u = U/V$, volume V and entropy S.

U, V, S $\qquad u = U/V$ 2 ½ each

2. State the 1st law of thermodynamics for a gas of atoms.

$dU = \delta Q + \delta W = \delta Q - p\,dV$ 2

3. A small amount of heat, ΔQ, is transferred to a system at temperature T via a quasi-static process. What is the resulting change in entropy?

$\Delta S = \frac{1}{T}\Delta Q$ 2

4. How does the heat capacity of a solid at constant magnetic field behave when the system is cooled close to $T = 0$?

$\lim_{T\to 0} C_B = 0$ 1

5. Which of the following quantities has an exact differential in the variables stated: $U = U(S,V)$, $U = U(T,V)$, $S = S(U,V)$, $Q = Q(S,T)$?

$U = U(S,V)$ and $S = S(U,V)$ 2

6. State the relation that defines the temperature for a isolated system.

$T = \left(\frac{\partial U}{\partial S}\right)_V$ 1

7. A system has 5 microstates with energies $E_1 = 2$, $E_2 = 3$, $E_3 = 4$, $E_4 = 5$ and $E_5 = 1$. Calculate the average energy and its fluctuation σ_E.

$\langle E \rangle = \frac{1}{N}\sum E_i$ $\qquad \sigma_E^2 = \langle (E_i - \langle E \rangle)^2 \rangle$

$= 3$ $\qquad = \frac{1}{5}[1+0+1+4+4]$

$= 2$ $\qquad \sigma_E = \sqrt{2}$ 4

8. At different times, the entropy of an isolated systems has been measured to be i) $S = 3$ J/K, ii) $S = 1.5$ J/K, iii) $S = 3.5$ J/K, iv) $S = 2.5$ J/K. Which of the states probed is closest to equilibrium?

case iii) $S = 3,5$ J/K

1

9. A thermodynamic system is composed of two independent parts having entropies S_1 and S_2 and number of states Ω_1 and Ω_2, respectively. What is the entropy and the number of states for the combined system?

$S_{tot} = S_1 + S_2 \qquad \Omega_{tot} = \Omega_1 \cdot \Omega_2$

2

10. Consider an idealised coin tossing experiment (same probabilities for heads and tails). What is the *probability* to get exactly 2 heads in 5 tries?

$p = \frac{\omega_5(2)}{\Omega}$ ① $\qquad \omega_5(2) = \frac{5!}{2!\,3!} = 10$ ①

$= \frac{10}{2^5} = \frac{5}{16}$ ① $\qquad \Omega = 2^5$ ①

4

11. What is the number of states for a system with 20 entirely independent particles that can each occupy 5 states independent of each other?

5^{20}

1

12. Using the fact that $dU = TdS - pdV + \mu dN$, derive the Maxwell-relation $(\partial T/\partial N)_{S,V} = (\partial \mu/\partial S)_{V,N}$.

$T = \left(\frac{\partial U}{\partial S}\right)_{V,N}$ ① $\qquad -p = \left(\frac{\partial U}{\partial V}\right)_{S,N} \qquad \mu = \left(\frac{\partial U}{\partial N}\right)_{S,V}$ ①

$\left(\frac{\partial}{\partial N}\left(\frac{\partial U}{\partial S}\right)_{V,N}\right)_{S,V} \overset{①}{=} \left(\frac{\partial}{\partial S}\left(\frac{\partial U}{\partial N}\right)_{S,V}\right)_{V,N}$

$\Rightarrow \left(\frac{\partial T}{\partial N}\right)_{S,V} = \left(\frac{\partial \mu}{\partial S}\right)_{V,N}$

3

A+ A family has two children. You are told that one of them is a boy. What is the probability that the family has two boys?

$\frac{1}{3}$

1 extra