Thermal Physics II

2^{nd} short test – 13 March 2013

given and surname : Lecturer

university number :

course of study : Teaching

marks obtained : 25

total marks : 25

comments :

1. How are the Helmholtz free energy and the internal energy related?

$$F = U - TS$$

2

2. What are the natural variables that allow for the use of the Helmholtz free energy as a thermodynamic potential for a gas?

$$F = F(T, V) \qquad (\text{and } N)$$

2

3. State the relation between the number of states of the system and the number of states of its heat reservoir within the canonical description.

$$\Omega_S \ll \Omega_R$$

1

4. Define the Boltzmann factor for a quantum state with energy E_1 and the partition functions of a system with discrete energies E_j.

BF: $e^{-\beta E_1}$ 1

PF: $Z = \sum_j e^{-\beta E_j}$ 1 $\qquad \beta = \frac{1}{k_B T}$ 1

3

5. Consider an ion with two energy states at $E_1 = -30\,\text{eV}$ and $E_2 = -20\,\text{eV}$ embedded in a gas with a temperature such that $k_B T = 10\,\text{eV}$. Calculate the ratio of the occupation probabilities for these two states.

$$P_1 = \frac{1}{Z} e^{-\beta E_1} = \frac{1}{Z} e^{+3}$$ 1

$$P_2 = \frac{1}{Z} e^{-\beta E_2} = \frac{1}{Z} e^{+2}$$ 1

$$\Rightarrow \frac{P_1}{P_2} = \frac{e^3}{e^2} = e \quad \text{or} \quad \frac{P_2}{P_1} = \frac{1}{e}$$ 1

3

6. Give the basic relation that connects the thermodynamical and statistical descriptions of many-body systems in the canonical ensemble.

$$F = -k_B T \ln Z$$

2

7. Atoms may have magnetic moments $\pm\mu$. What is the average magnetic moment if the probabilities are $P(+\mu) = 0.8$ and $P(-\mu) = 0.2$?

$$\langle \mu \rangle = \sum_j \mu_j P_j = \mu \cdot 0.8 - \mu \cdot 0.2 = 0.6\,\mu$$

1

8. A system is made of two independent subsystems with internal energies, U_1 and U_2, and partition functions, Z_1 and Z_2. State the internal energy, partition function and Helmholtz free energy of the combined system.

$$U_{tot} = U_1 + U_2 \quad (1) \qquad F = -k_B T \ln Z$$

$$Z_{tot} = Z_1 \cdot Z_2 \quad (1) \Rightarrow F_{tot} = F_1 + F_2 \quad (1)$$

3

9. Which quantity in the statistical description of many-body systems is defined by the temperature of the system? What happens to this quantity if the temperature is doubled?

energy fluctuations σ_E ; σ_E doubles

2

10. State the partition function for an ideal gas (N particles in volume V).

$$Z = \left(V \Lambda_m^{-3}\right)^{N} \quad (1)(1) \qquad \Lambda_m = \left(\frac{h^2}{2\pi m k_B T}\right)^{1/2} \quad (1)$$

3

11. Given the fact that the differential of the Gibbs energy (enthalpy) is given by $dH = TdS + Vdp$, derive the Maxwell-relation $(\partial T/\partial p)_S = (\partial V/\partial S)_p$.

$$T = \left(\frac{\partial H}{\partial S}\right)_p \quad (1) \qquad V = \left(\frac{\partial H}{\partial p}\right)_S \quad (1)$$

$$\Rightarrow$$

$$\left(\frac{\partial}{\partial p}\left(\frac{\partial H}{\partial S}\right)_p\right)_S \overset{(1)}{=} \left(\frac{\partial}{\partial S}\left(\frac{\partial H}{\partial p}\right)_S\right)_p$$

$$\Rightarrow \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$$

3