Thermal Physics II – Problem Sheet 1

Part I: Questions

1. What are the three laws of thermodynamics? What is sometimes called the 0^{th} law?
2. Which consequences arise from the third law?
3. What are the natural variables that make U a thermodynamic potential?
4. Please revise the characteristics of an ideal gas.

Part II: Problems

1. Consequences of the Third Law of Thermodynamics

Consider a gas near $T = 0$ of the absolute temperature scale. Show that the heat capacities at constant pressure and constant volume, that is c_p and c_V, hold the relation

$$\lim_{T\to 0} \frac{c_p - c_V}{T} = 0$$

You may find it useful to start from the expression (Thermal Physics I)

$$c_p - c_V = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial V}{\partial T}\right)_p .$$

Given the upper result and the fact $\lim_{T\to 0} c_p = 0$ shown in the lecture, what follows for the heat capacity c_V in the limit $T \to 0$?

2. Maxwell Relation

For systems with variable particle number, the first law of thermodynamics is given by $dU = TdS - pdV + \mu dN$, where $U = U(S, V, N)$ is a thermodynamic potential. μ is called the chemical potential and will become important later (quantum statistics). Use these facts to prove the following Maxwell relation:

$$\left(\frac{\partial T}{\partial N}\right)_{S,V} = \left(\frac{\partial \mu}{\partial S}\right)_{V,N} .$$

3. Stability of the Equilibrium

Consider a gas consisting of N particles that fill a volume V. The internal energy of the gas is U. Split the system by an imaginary border into two parts with V_1, N_1 and U_1 in the left part and V_2, N_2 and U_2 in the right part. Show that from the stability relation

$$S = S_1 + S_2 \to \max$$

follows that $T_1 = T_2$ and $p_1 = p_2$. (Hint: apply the first law!)