

# Thermal Physics II – Problem Sheet 5

## Part I: Questions

1. What is called a Boltzmann factor? How is this quantity related to the probability of a state with energy  $E$ ?
2. Is the equipartition theorem invalid in the canonical ensemble?
3. Revise the definitions of the partition function. How is this statistical quantity related to thermodynamics?  
What is the appropriate thermodynamic potential for a system in the canonical ensemble? Which are its natural variables?
4. How do the energy fluctuations in a system with heat contact to a very large system with temperature  $T$  scale with a) the number of particles in the system and b) the temperature of the heat bath?  
Can the micro-canonical description be reached in some limit?

## Part II: Problems

### 1. Partition Function for Combined Systems/Degrees of Freedom

The partition functions of two systems  $A$  and  $B$  are given by  $Z_A$  and  $Z_B$ , respectively. Prove that the partition function of the combined system  $A+B$  is then given by  $Z_A \cdot Z_B$  if the total energy holds the relation  $E_{A+B} = E_A + E_B$ . Convince yourself that the same holds for one system that has more than one degree of freedom (e.g., oscillations in different directions). What follows from this relation for the Helmholtz free energy?

### 2. Magnet with Three Energies

Consider a solid with  $N$  independent atoms that occupy the lattice sites. In a magnetic field of strength  $B$ , the atoms can have energies  $E_1 = \mu B$ ,  $E_2 = 0$  and  $E_3 = -\mu B$ , where  $\mu$  is the magnetic moment of an atom. Calculate the probabilities to find an atom in each of these states when a magnetic field of strength  $\bar{B}$  acts on the atoms. Then calculate the partition function and the average magnetisation. What changes occur if the second state is fourfold degenerate?

What state is reached in the limits of vanishing and infinite magnetic field strengths? What is the related entropy?