Thermal Physics II – Problem Sheet 7

Part I: Questions

1. Which of the two quantities, particle number N and energy E, fluctuates in the micro-canonical, the canonical and the grand-canonical ensembles?
2. How is the first law of thermodynamics modified for open systems?
3. How is the Gibbs-factor and the grand partition sum related?
4. How are thermodynamics and statistics connected in the grand-canonical ensemble?
5. What is the chemical potential?

Part II: Problems

1. Relation of Thermodynamics and Statistics

Show that both sides of the relation

$$\Omega_{gr}(T, V, \mu) = -k_B T \ln Z_{gr}(T, V, \mu)$$

are extensive.

2. Relation of Thermodynamics and Statistics

Derive the relation between the average particle number and the grand partition function

$$\langle N \rangle = k_B T \, Z_{gr}^{-1}(T, V, \mu) \, \frac{\partial Z_{gr}(T, V, \mu)}{\partial \mu} \, .$$

3. Gibbs-Duhem Relation

Derive the Gibbs-Duhem relation

$$G(T, p, N) = \sum_j \mu_j N_j \, ,$$

where the sum runs over all species in the system. Thus, it follows that the chemical potential is the Gibbs free energy per particle.

3. More Maxwell- Relations

Derive which sign is correct in the follow Maxwell-relations:

$$\left(\frac{\partial T}{\partial N}\right)_{S,V} = \pm \left(\frac{\partial \mu}{\partial S}\right)_{V,N} \quad \text{and} \quad \left(\frac{\partial S}{\partial N}\right)_{T,V} = \mp \left(\frac{\partial \mu}{\partial T}\right)_{V,N} \, .$$