Summary: Number of States and Probabilities

a) Notation

- $\Omega(E, N)$ ... number of all microstates consistent with E, N
- $w_N(N_1)$ ... number of all microstates consistent with E, N that have N_1 particles/events in the state E_1 (2 state system)
- $w_N(n_1, n_2, ..., n_j)$ ... number of all microstates consistent with E, N that have n_1 particles in the state E_1, n_2 in state E_2, ..., n_j in state E_j (general case)
- p_ν ... probability of microstate ν; $p_\nu = \frac{1}{\Omega}$
- $p_N(N_1)$ ... probability to find N_1 particles/events in special state, e.g., E_1 (2 state system)
- $p_N(n_1, n_2, ..., n_j)$ ... probability to find n_1 particles in the state E_1, n_2 in state E_2, ..., n_j in state E_j (general case)

b) Counting the States and Calculating Probabilities

- independent particles that can occupy M states:

$$\Omega = M \times M \times M \times \ldots \times M = M^N$$

- systems with 2 states [N_1 particles/events in state E_1; $(N - N_1)$ in state E_2]

$$w_N(N_1) = \frac{N!}{N_1!\,(N-N_1)!}$$

$$p_N(N_1) = \frac{N!}{N_1!\,(N-N_1)!}\, p_1^{N_1} p_2^{(N-N_1)}$$

- general case [n_1 particles/events in state E_1 with probability p_1 etc]

$$w_N(n_1, n_2, ..., n_j) = \frac{N!}{n_1! n_2! \ldots n_j!} = \frac{N!}{\prod\limits_k n_k!}$$

$$p_N(n_1, n_2, ..., n_j) = \frac{N!}{n_1! n_2! \ldots n_j!}\, p_1^{n_1} p_2^{n_2} \ldots p_j^{n_j} = \frac{N!}{\prod\limits_k n_k!} \prod_k p_k^{n_k}$$

b) Conditions to be Hold

- $$\sum_{\text{all possible } n_k} w_N(n_1, n_2, \ldots, n_j) = \Omega \qquad \text{with} \qquad \sum_k n_k = N$$
- $$\sum_{\text{all possible } n_k} p_N(n_1, n_2, \ldots, n_j) = 1 \qquad \text{and} \qquad \sum_k p_k = 1$$