Thermal Physics II – Solutions for Problem Sheet 2

1. Number of Specific Macrostates

- ENGINEERING has 11 letters. IF all letters would be different, we would have 11! permutations.
- BUT swapping the three 'E's, three 'N's, two 'G's and two 'I's makes no difference to writing.
- Thus, we have $(3! \times 3! \times 2! \times 2!)$ times the same configurations only due to swapping these four letters.
- The correct number of different permutations of ENGINEERING is thus

$$\frac{11!}{3!\,3!\,2!\,2!} = 277200\,.$$

2. The Meaning of VERY Unlikely

- Choosing the first number, we have 49 possibilities.
- Choosing the second number, we have 48 possibilities.
- ...
- Choosing the sixth number, we have 44 possibilities.
- As the order of the picks does not play a role, we have to take the number of permutations in the picked numbers into account (6!).
- Thus, we find the number of possibilities to be

$$\frac{49 \times 48 \times 47 \times 46 \times 45 \times 44}{6!} = 1398381$$

- This is roughly a chance of 1 : 14 millions. So, why bother?

3. The Meaning of NEVER

- The monkeys have to typo 39 character correctly.
- They have 35 choices for each character (35 key typewriter).
- Each choice is independent.
- This gives for the number of possible character sequences

$$35^{39} \approx 1.6 \times 10^{60}$$

- Compare this number with the outcome of one monkey that types 1 key per second over one year:

$$60 \times 60 \times 24 \times 365 = 3.1 \times 10^{7}$$

- Even a large group of monkeys has no changes at all typing 10^{60} different sequences!

4. Stirling's Formula

- (Riemann) Integral can be written as

$$\int_0^N f(x)dx = \sum_{i=0}^{N-1} f(x_i)\Delta x \qquad \text{for} \qquad x \to 0$$

- Here, we keep $\Delta x = 1$ and investigate the lower and upper bound of the integral.
- The lower bound is given by taking the left side of the interval in $f(x_i)$; the upper bound by taking the right side.
- Thus we have,

$$I_l \stackrel{of}{=} \int_0^N \ln n \; dn = \sum_{i=0}^{N-1} \ln i = \ln \prod_{i=0}^{N} - \ln N = \ln N! - \ln N \,,$$

and

$$I_u \stackrel{of}{=} \int_0^N \ln n \; dn = \sum_{i=1}^{N} \ln i = \ln \prod_{i=1}^{N} = \ln N! \,.$$

- The real value of the integral is between these limits. As we can integrate $\ln n$, we find

$$\ln N! - \ln N < \int_0^N \ln n \; dn < \ln N!$$
$$\ln N! - \ln N < \Big(N \ln N - N\Big) < \ln N!$$

- For $N \to \infty$, we have $\ln N! \gg \ln N$ and thus $I_l \to I_u$.
- As both limiting values agree for large N, we arrive at Stirling's formula

$$\lim_{N\to\infty} \ln N! = N \ln N - N$$