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Are particles detrapped by constant B_y in static magnetic reversals?

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Abstract. Single-particle dynamics in simple models for the static reversing magnetic field of the geotail current sheet have been extensively studied in the case where the reversing field $B_x(z)$ varies, the linking field B_z is constant, and the crosstail field B_y is zero (and some generalization to include time dependence has been achieved). More recently, numerically integrated trajectories in static reversals which include a constant shear B_y component have suggested some differences in the nature of the dynamics in this and the $B_y = 0$ case. The invariant of the z cross-sheet motion for the $B_y = 0$ case is well known, here we find its equivalent for systems with constant B_y. Our results hold for reversals with general z dependence and arbitrary constant B_y. The form of this invariant suggests that it is still conserved for trapped particles, but for certain values of energy, B_y and B_z, the invariant is destroyed and particles are detrapped. This corresponds to an increase in the volume in phase space available to current carrying particles that transit the sheet. For typical magnetotail parameters, both protons and electrons in an average 1 R_E thick sheet will be detrapped, but in a thin $\sim$ 100 km sheet protons will not.

Introduction

Single-particle dynamics in the Earth's geotail has been studied in detail in simple magnetic reversal models where a shear (B_y) field is absent, both in the static case [see *Chen*, 1992; *Buchner and Zelenyi*, 1989; *Wang*, 1994 and references therein] and including time dependence [e.g. *Chapman*, 1994]. However, observations suggest that a shear field is present and depends upon the interplanetary magnetic field direction [*Cowley and Hughes*, 1983], and in the substorm growth phase in particular [*Sergeev et al.*, 1993]. Recent results obtained for specific field models suggest that energization and scattering of particles in static reversals is fundamentally different in the presence of a shear field [e.g. *Karimabadi et al.*, 1990; *Zhu and Parks*, 1993; *Buchner and Zelenyi*, 1991; *Baek et al.*, 1995].

In this paper we will consider the dynamics in the general case, that is, for a reversal with constant linking field B_z, constant shear field B_y, and z dependent reversing field $B_x(z)$. We work in the frame of zero electric field but show that this is valid for any $\mathbf{E}.\mathbf{B} = 0$ that can be achieved by De Hoffman Teller frame transformation, which means $\mathbf{E} = -\mathbf{U} \wedge \mathbf{B}$ for any constant $\mathbf{U} = (U_x, U_y, 0)$, that is, transformation velocity $\mathbf{U}$ consistent with MHD. The results hold in the "Speiser" limit [*Speiser*, 1965; *Sonnerup*, 1971] of weak linking field, that is, $B_z \ll B_x$ but are valid for any shear field magnitude B_y. The motion of particles trapped in the reversal is assumed to have periodicity; however, the reversing field $B_x(z)$ is not assumed to be symmetric about the $z = 0$ plane, so that although this study is aimed primarily at the geotail, the results also have application to asymmetric geometries such as at the magnetopause.

We provide the first proof that an adiabatic invariant of the z motion, J, of particles trapped in the reversal exists and is conserved. From the form of the integral for J we conjecture that the particles detrap for certain shear field strength B_y but for certain energies only: this process is illustrated with numerically integrated trajectories in a simple field model with entry and exit regions. The implication is that regions in phase space usually occupied by trapped particles will, for certain energies and shear field, become accessible to particles that transit the sheet. Since trapped particles carry zero net cross-tail or cross-sheet y-directed current averaged over an orbit, whereas transiting particles carry finite net current, this may have implications for the structure and evolution of the current sheet under conditions of nonzero, and slowly evolving, shear field.

System of Equations

We consider a magnetic reversal with general z dependence:

$$\mathbf{B} = B_0(f(z), b_2, b_1) \quad (1)$$

where x, y, z Cartesian coordinates correspond to GSE and $z = Z/L$ is normalized to a convenient length scale of the system L. An electric field consistent with ideal MHD plasma flow $\mathbf{U}$ will satisfy $\mathbf{E} = -\mathbf{U} \wedge \mathbf{B}$ (and hence $\mathbf{E}.\mathbf{B} = 0$). The most general electric field

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amenable to the analysis here is generated from a constant $\mathbf{U} = (U_x, U_y, 0)$ which simply corresponds to a De Hoffman Teller frame transformation velocity in the x, y plane. This gives

$$\mathbf{E} = (-U_y b_1, U_x b_1, -U_x b_2 + U_y f(z)) \quad (2)$$

(the special case $U_y = 0$ was considered by *Zhu and Parks*, [1993]).

Normalizing time to $\tau = t\Omega_0$ where $\Omega_0 = eB_0/m$, we then have equations of motion in the De Hoffman Teller frame:

$$\frac{d\bar{v}_x}{d\tau} = \bar{v}_y b_1 - v_z b_2 \quad (3)$$

$$\frac{d\bar{v}_y}{d\tau} = v_z f(z) - \bar{v}_x b_1 \quad (4)$$

$$\frac{dv_z}{d\tau} = \bar{v}_x b_2 - \bar{v}_y f(z) \quad (5)$$

here x, y, z, v_x, v_y, v_z refer to the particle trajectory in the $\mathbf{E} \neq 0$ frame, and we have used the transformation

$$\bar{v}_x = v_x - U_x \quad (6)$$

$$\bar{v}_y = v_y - U_y \quad (7)$$

The x and y equations then integrate to give

$$\bar{v}_x = \bar{y}b_1 - zb_2 + C_x \quad (8)$$

$$\bar{v}_y = F(z) - \bar{x}b_1 + C_y \quad (9)$$

where $F(z) = \int f(z)dz$ and C_x, C_y are constants and are related to the invariant of the y motion P_y by $C_y = P_y/eB_0 - U_y$. The constant energy in the moving frame is

$$\bar{h} = \left(\frac{dz}{d\tau}\right)^2 + \left(\frac{d\bar{x}}{d\tau}\right)^2 + [F(z) - \bar{x}b_1 + C_y]^2 \quad (10)$$

Substituting $-\phi = -b_1\bar{x} + C_y$ gives the equations of motion in terms of two coordinates:

$$\frac{d^2 z}{d\tau^2} = \frac{b_2}{b_1}\frac{d\phi}{d\tau} - f(z)[F(z) - \phi] \quad (11)$$

$$\frac{d^2\phi}{d\tau^2} = b_1^2[F(z) - \phi] - b_2 b_1 \frac{dz}{d\tau} \quad (12)$$

$$\bar{h} = \left(\frac{dz}{d\tau}\right)^2 + \frac{1}{b_1^2}\left(\frac{d\phi}{d\tau}\right)^2 + (F(z) - \phi)^2 \quad (13)$$

Derivation of Constant of the z Motion

We now wish to obtain an expression for $J = \oint v_z^2 d\tau$ for the z motion. In the limit $b_1 \ll 1$ we can expand coordinates z, ϕ as functions of two well-seperated timescales: τ_s (slow) and τ_f (fast), where $\tau_s = b_1\tau_f$ (and hence $d/d\tau = d/d\tau_f + b_1 d/d\tau_s$). This is an example of the method of multiple scale perturbation theory [see e.g. *Rowlands*, 1990]. The limit $b_1 \rightarrow 0$ corresponds to the integrable limit of vanishing linking field [*Karimabadi et al.*, 1990; *Buchner and Zelenyi*, 1991] and $\tau_s \rightarrow \infty$. The expansion gives

$$z = z_0(\tau_f, \tau_s) + b_1 z_1(\tau_f, \tau_s) + \ldots \quad (14)$$

$$\phi = \phi_0(\tau_s) + b_1\phi_1(\tau_f, \tau_s) + \ldots \quad (15)$$

Equations (11)-(13) to lowest order give

$$\frac{d^2 z_0}{d\tau_f^2} = -f(z_0)[F(z_0) - \phi_0] + b_2\left(\frac{d\phi_0}{d\tau_s} + \frac{d\phi_1}{d\tau_f}\right) \quad (16)$$

$$\frac{d^2\phi_0}{d\tau_f^2} = 0 \quad (17)$$

$$\bar{h}_0 = \left(\frac{dz_0}{d\tau_f}\right)^2 + \left(\frac{d\phi_0}{d\tau_s} + \frac{d\phi_1}{d\tau_f}\right)^2 + [F(z_0) - \phi_0]^2 \quad (18)$$

to first order we have

$$\frac{d^2\phi_1}{d\tau_f^2} = -b_2\frac{dz_0}{d\tau_f} \quad (19)$$

which integrates with respect to τ_f to give

$$\frac{d\phi_1}{d\tau_f} = a(\tau_s) - b_2 z_0 \quad (20)$$

this can be substituted into (16) which then integrates with respect to τ_f:

$$\left(\frac{dz_0}{d\tau_f}\right)^2 = K - (F(z_0) - \phi_0)^2 - \left[\frac{d\phi_0}{d\tau_s} + a - b_2 z_0\right]^2 + \left(\frac{d\phi_0}{d\tau_s} + a\right)^2 \quad (21)$$

where we have assumed that $\phi_0(\tau_s)$ and $a(\tau_s)$ are constants over the integration wrt τ_f.

Comparing (21) with (18) and using (20) we have

$$K + \left(\frac{d\phi_0}{d\tau_s} + a\right)^2 = \bar{h}_0 \quad (22)$$

The equation for the zero-order z motion is then

$$\left(\frac{dz_0}{d\tau_f}\right)^2 = \bar{h}_0 - (F(z_0) - \phi_0)^2 - \left[\frac{d\phi_0}{d\tau_s} + a - b_2 z_0\right]^2 \quad (23)$$

In addition, if to lowest order we assume periodic motion on the fast timescale τ_f with period $T = T(\bar{h}_0, \tau_s)$ from (23) then $z_0(\tau_f) = z_0(\tau_f + T)$. If ϕ_1 has the same property then (20) can be integrated over period T to give

$$a(\tau_s) = b_2 < z_0(\tau_f) > \quad (24)$$

where

$$< z_0 > = \frac{1}{T}\int_0^T z_0(\tau_f)d\tau_f$$

This periodic motion has an associated constant J which to lowest order is given by

$$J_0 = \int_0^T \left(\frac{dz_0}{d\tau_f}\right)^2 d\tau_f = 2\int_{z_1}^{z_2} \frac{dz_0}{d\tau_f} dz_0$$
$$= 2\int_{z_1}^{z_2} \left[\bar{h}_0 - (F(z_0) - \phi_0)^2 - \left(\frac{d\phi_0}{d\tau_s} + b_2 < z_0 > -z_0 b_2\right)^2\right]^{\frac{1}{2}} dz_0 \quad (25)$$

where $J = J_0 + b_1 J_1 + \ldots$. This reduces to the result obtained by *Buchner and Zelenyi*, [1989] for $b_2 = 0$ for the more restricted case of zero shear field. The conservation of J_0 is shown in the appendix.

It is important to note that we can insist that $\bar{h}_0 \equiv \bar{h}$. This imposes constraints on the solution for z_r with $r \geq 1$ which can always be satisfied by choosing constants of integration which naturally occur in solving the higher-order equations.

The oscillation period of the fast z_0 motion is then

$$T = 2\int_{z_1}^{z_2} \left[\bar{h}_0 - (F(z_0) - \phi_0)^2 - \left(\frac{d\phi_0}{d\tau_s} + b_2 < z_0 > -z_0 b_2\right)^2\right]^{-\frac{1}{2}} dz_0 \quad (26)$$

The introduction of the shear field b_2 then implies that for certain $\bar{h}$, b_2 and implicitly b_1 (since $z_0 = z_0(\tau_f, \tau_s)$, $\phi_0 = \phi_0(\tau_s)$ and $\tau_s = b_1\tau_f$) the denominator of (26) may vanish. Hence particles of a given energy will "detrap", that is, $T \to \infty$ for a particular range of shear field strength. By analogy with the simple nonlinear pendulum, chaotic motion first appears near the separatrices of the motion defined by $J_0 =$ const. These correspond to the case of orbits with infinite period [*Lichtenberg and Lieberman,* 1992].

The implication is that a region of phase space occupied by trapped J conserving particles which over an orbit do not carry net cross-tail current becomes accessible to particles which enter and exit the reversal and hence can carry net cross-tail current.

We can demonstrate this with numerically integrated trajectories in a simple model for the magnetic reversal, here we use B_x =tanh(z). This model allows particles that are sufficiently field aligned to enter and exit the reversal and will therefore permit three classes of particle orbit (see, e.g. the review by *Chen*, [1992]): transient (or Speiser) trajectories that transit the reversal once, stochastic trajectories that after entry interact with the reversal many times but will ultimately exit the reversal and regular trajectories that remain on KAM surfaces and are trapped indefinitely within the reversal. Since trajectories cannot cross in phase space, the KAM surfaces of regular motion partition phase space, as can be seen on a Poincare surface of section (SOS) plot.

Each of the surface of section plots in Figures 1-5 were generated for a given $\bar{h}, b_2$ and b_1. The plots show the $\dot{x}, x$ coordinates of trajectories as they intersect the $z = 0$ plane. The many trajectories shown in each plot are initialized at $x = y = 0$ and over the full range of pitch angle and gyrophase so that the phase space for a given $\bar{h}$ is well explored (the initial $\bar{x}$ is such that all trajectories have the same P_y from (9)).

The Figures compare surface of sections for $b_1 = 0.1$ and $v = \sqrt{\bar{h}} = 1, 0.1$ and a range of shear field $0 \leq b_2 \leq 1$. The $v = \sqrt{\bar{h}} = 0.1$ phase space has a range of values of b_2 for which no KAM surfaces are apparent, that is, no regular trapped orbits can be found. This is in contrast to the $v = \sqrt{\bar{h}} = 1$ case where although the region occupied by trapped orbits is reduced in the vicinity of $b_2 \sim 0.1$, the KAM surfaces persist (it should be stressed that this does not exclude the possibility that the KAM surfaces do vanish for some infinitesimally small range of b_2). The tendency for "maximal

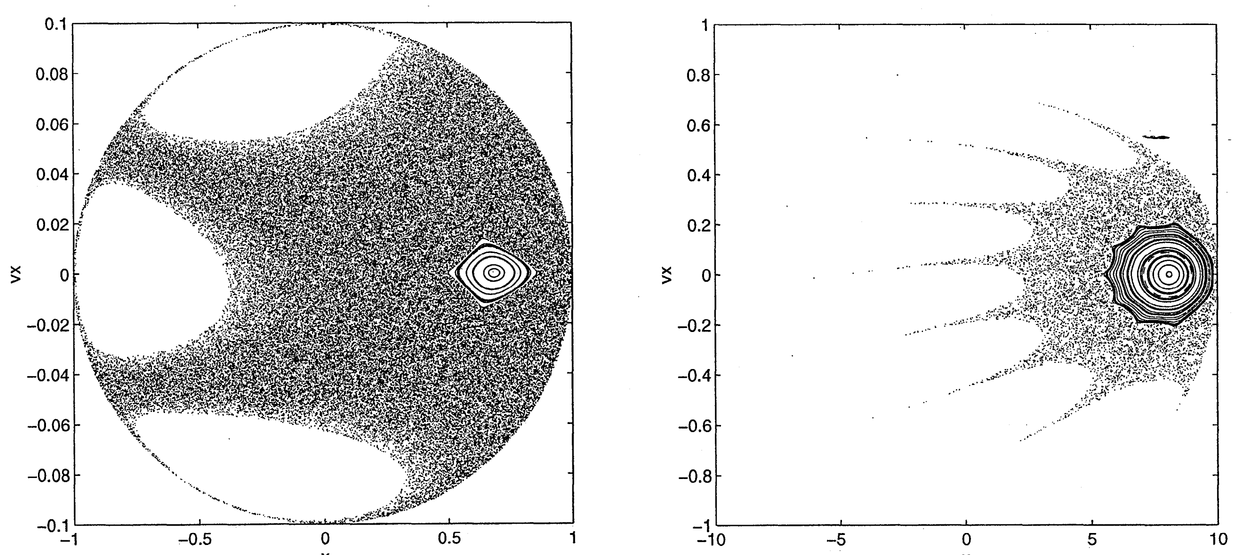

Figure 1. Poincare SOS for $b_y = 0$ and $v = 0.1$ (left), 1.0 (right)

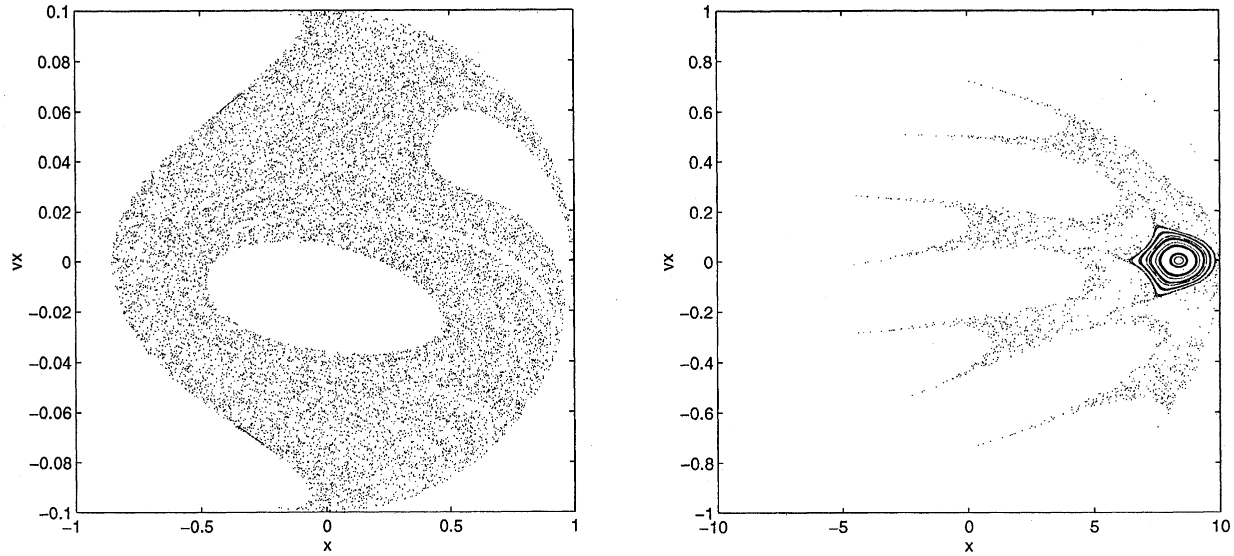

Figure 2. Poincare SOS for $b_y = 0.1$ and $v = 0.1$ (left), 1.0 (right)

chaos", that is, minimum region in phase space occupied by regular orbits at $B_y \sim B_z$ (i.e. $b_2 \sim 0.1$ here) has previously been conjectured by *Karimabadi et al.*, [1990] and demonstrated by means of surfae of section plots.

The form of (25) and (26) indicate that three parameters $\bar{h} = 2\bar{H}/mL^2\Omega_0^2$, $b_2 = B_y/B_0$ and $b_1 = B_z/B_0$ are needed to determine the fate of non current-carrying J conserving orbits in the case of general z dependence. Since $\bar{h} = \hat{H}/2$ and $b_1 = b_n$ are the parameter pair shown by *Chen*, [1992] (and references therein) to be required to completely specify the dynamics in the $B_y = 0$ system, we have identified the third parameter needed for general z dependence when $B_y \neq 0$. In the case of the parabolic field where $f(z) = z$, a single parameter such as κ is sufficient to completely specify the dynamics when $B_y = 0$ [*Buchner and Zelenyi*,1989] where $\kappa = b_1\bar{h}^{-\frac{1}{4}}$. A modified κ parameter, $\kappa_{ns} = \kappa(1 + (B_y/B_z)^2)^{\frac{3}{4}}$ specified by parameter pair κ and $\kappa_s = b_2\bar{h}^{-\frac{1}{4}}$ has been proposed for the $B_y \neq 0$ case, on the basis of SOS plots, and the minimum radius of curvature [*Karimabadi et al.*, 1990] and inspection of the equations of motion [*Buchner and Zelenyi*, 1991]. Since $\kappa = b_1\bar{h}^{-\frac{1}{4}}$ $\kappa_s = b_2\bar{h}^{-\frac{1}{4}}$ and $\kappa_{ns} = \kappa_{ns}(\kappa, \kappa_s)$ we would expect that overall trends in dynamical behavior demonstrated by surface of section plots will be ordered by either κ, κ_s as given by *Buchner and Zelenyi*, [1991] or $\bar{h}, b_1, b_2$, as given by *Karimabadi et al.*, [1990] (shown by varying b_2 for a given b_1 and $\bar{h}$) and here (shown by varying b_2 and $\bar{h}$ for a given b_1). It should be stressed, however, that (25) and (26) derived here

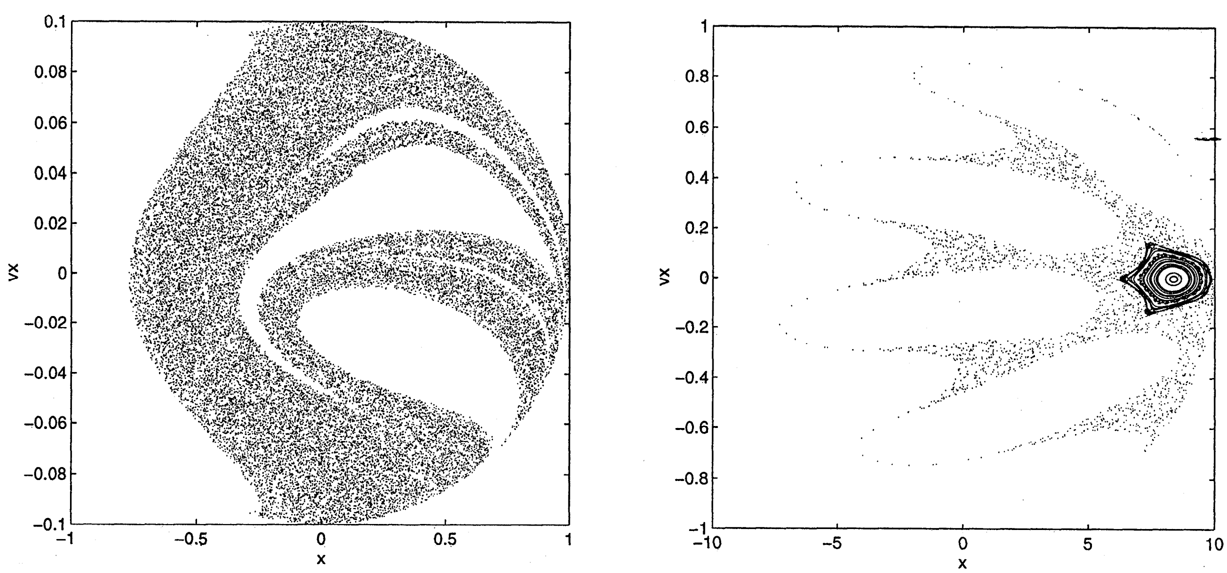

Figure 3. Poincare SOS for $b_y = 0.15$ and $v = 0.1$ (left), 1.0 (right)

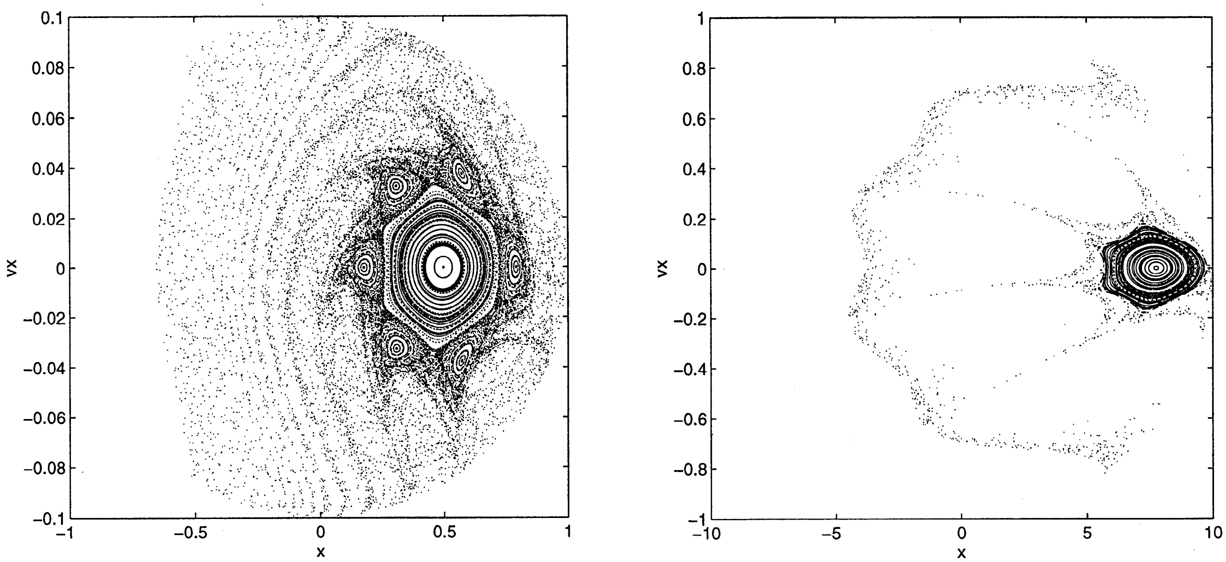

Figure 4. Poincare SOS for $b_y = 0.3$ and $v = 0.1$ (left),1.0 (right)

cannot be rewritten as functions of κ, κ_s which implies that quantitative information, such as the stability criteria and volume in phase space occupied by regular J conserving orbits will in general depend upon $\bar{h}, b_1, b_2$.

The sensitivity of transient orbits to the presence of shear field B_y has also been investigated by *Kaufmann et al.,*[1994], and ensemble average pitch angle change presented as functions of energy and shear field, and as all other parameters are fixed, this is equivalent to ordering with respect to $\bar{h}, b_1, b_2$. Consistent with the above discussion, ordering in the results with respect to κ, κ_s can also be sought as in *Kaufmann et al.,*[1994]. Trapped orbits were not investigated in *Kaufmann et al.,*[1994] however so that the contribution to the nett current that may result from their destabilization cannot be estimated from their results.

Conclusions

The dynamics for a reversal with constant linking field B_z, constant shear field B_y, and general z dependent reversing field $B_x(z)$ has been investigated. The results hold in the limit of weak linking field, that is, $B_z \ll B_x$ but are valid for any shear field magnitude B_y, and for any $\mathbf{E}.\mathbf{B} = 0$ electric field. Although this study is aimed primarily at the geotail, the formalism also applies to asymmetric geometries such as at the magnetopause.

An adiabatic invariant of the z motion, J, of particles trapped in the reversal has been shown to exist and to be conserved. From the form of the integral for J we then conjecture that the particles detrap for certain shear field strength B_y but for certain energies only:

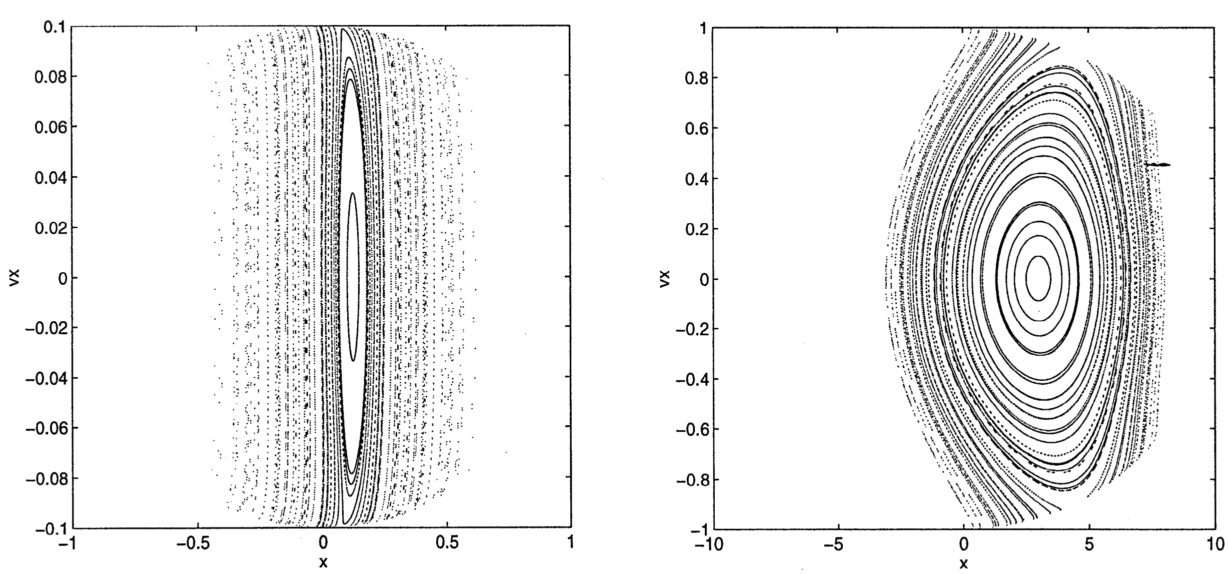

Figure 5. Poincare SOS for $b_y = 1.0$ and $v = 0.1$ (left), 1.0 (right)

this process is illustrated with numerically integrated trajectories in a simple field model with entry and exit regions. The implication is that regions in phase space usually occupied by trapped particles will, for certain energies and shear field, become accessible to particles that transit the sheet. Since trapped particles carry zero net cross-tail or cross-sheet y-directed current averaged over an orbit, whereas transiting particles carry finite net current, this may have implications for the structure and evolution of the current sheet under conditions of nonzero, and slowly evolving, shear field. Three parameters are required, in general, to indicate detrapping of particles from the sheet, or "maximal chaos" under conditions of nonzero shear field. For typical geotail parameters, in an average thickness ($\sim 1R_E$) sheet, both 1 keV protons and electrons will be "detrapped" in the presence of moderate shear field, that is, for a range of $B_y \sim B_z$ and for $B_z/B_x \sim 0.1$, J will not be conserved over all phase space. In a thin ($\sim$ 100 km) reversal, electrons will still be "detrapped", whereas protons will not; the region in proton phase space containing non-net current-carrying, trapped orbits is stable against the introduction of a shear field.

Appendix: Conservation of J

We can examine the variation in J by differentiating (25) with respect to the slow timescale τ_s. Given

$$J_0 = 2\int_{z_1}^{z_2}\left[\bar{h}_0 - (F(z_0)-\phi_0)^2 - \left(\frac{d\phi_0}{d\tau_s} + b_2 < z_0 > - z_0 b_2\right)^2\right]^{\frac{1}{2}} dz_0 \qquad \text{(A1)}$$

then since z_0 is just the variable to integration, differentiation with respect to τ_s involves only changes in ϕ_0 and $< z_0 >$. This gives

$$\frac{\partial J_0}{\partial \tau_s} = 2\int_{z_1}^{z_2}\left[\bar{h}_0 - (F(z_0)-\phi_0)^2 - \left(\frac{d\phi_0}{d\tau_s} + b_2 < z_0 > - z_0 b_2\right)^2\right]^{-\frac{1}{2}} G(z_0) dz_0 = \int_0^T G(z_0) d\tau_f \qquad \text{(A2)}$$

where

$$G = -2(F(z_0)-\phi_0)\left(\frac{d\phi_0}{d\tau_s}\right) - 2\left(\frac{d\phi_0}{d\tau_s} + b_2 < z_0 > - b_2 z_0\right) \cdot \left(\frac{d^2\phi_0}{d\tau_s^2} + b_2 \frac{d < z_0 >}{d\tau_s}\right) \qquad \text{(A3)}$$

recalling that $\phi_0 = \phi_0(\tau_s)$ and $< z_0 > = < z_0 > (\tau_s)$ only.

Integration over τ_f then gives

$$\frac{\partial J_0}{\partial \tau_s} = T\frac{d\phi_0}{d\tau_s} < F(z_0) - \phi_0 > - \left(\frac{d^2\phi_0}{d\tau_s^2} + b_2\frac{d < z_0 >}{d\tau_s}\right)\frac{d\phi_0}{d\tau_s}T \qquad \text{(A4)}$$

To order b_1^2 one obtains the equation

$$\frac{d^2\phi_2}{d\tau_f^2} + \frac{d^2\phi_0}{d\tau_s^2} + 2\frac{d^2\phi_1}{d\tau_f d\tau_s} = F(z_0) - \phi_0 - b_2\left(\frac{dz_0}{d\tau_s} + \frac{dz_1}{d\tau_f}\right) \qquad \text{(A5)}$$

For ϕ_2 to remain periodic on the τ_f timescale the following consistency condition must be satisfied:

$$\frac{d^2\phi_0}{d\tau_s^2} = < F(z_0) - \phi_0 > - b_2\frac{d < z_0 >}{d\tau_s} + \frac{b_2}{T}\frac{dT}{d\tau_s}(< z_0 > - z(\tau = 0, T, ..)) \qquad \text{(A6)}$$

This used in conjunction with (30) gives

$$\frac{1}{T}\frac{\partial J_0}{\partial \tau_s} = -\frac{b_2}{T}\frac{dT}{d\tau_s}(< z_0 > - z_0(\tau = 0, T, ..))\frac{d\phi_0}{d\tau_s} \qquad \text{(A7)}$$

Now the limits of integration can always be chosen such that the term $< z_0 > - z_0(\tau = 0, T, ..)$ will vanish to order b_1 and hence

$$\frac{\partial J_0}{\partial \tau_s} = O(b_1) \qquad \text{(A8)}$$

so J_0 is a zero-order constant of the motion.

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