


Formation of quasi-periodic slow magnetoacoustic wave trains by the heating/cooling misbalance

Cite as: Phys. Plasmas **26**, 082113 (2019); <https://doi.org/10.1063/1.5115224>

Submitted: 15 June 2019 . Accepted: 18 July 2019 . Published Online: 19 August 2019

D. I. Zavershinskii , D. Y. Kolotkov , V. M. Nakariakov , N. E. Molevich , and D. S. Ryashchikov 

COLLECTIONS

 This paper was selected as Featured



View Online



Export Citation



CrossMark

ARTICLES YOU MAY BE INTERESTED IN

[Influence of 3D plasmoid dynamics on the transition from collisional to kinetic reconnection](#)

Physics of Plasmas **26**, 072121 (2019); <https://doi.org/10.1063/1.5100737>

[Whistler modes excited by magnetic antennas: A review](#)

Physics of Plasmas **26**, 080501 (2019); <https://doi.org/10.1063/1.5097852>

[Three-dimensional modeling and hydrodynamic scaling of National Ignition Facility implosions](#)

Physics of Plasmas **26**, 050601 (2019); <https://doi.org/10.1063/1.5091449>







AVS Quantum Science

A high impact interdisciplinary journal for **ALL** quantum science



ACCEPTING SUBMISSIONS

Formation of quasi-periodic slow magnetoacoustic wave trains by the heating/cooling misbalance

Cite as: Phys. Plasmas **26**, 082113 (2019); doi: [10.1063/1.5115224](https://doi.org/10.1063/1.5115224)

Submitted: 15 June 2019 · Accepted: 18 July 2019 ·

Published Online: 19 August 2019



View Online



Export Citation



CrossMark

D. I. Zavershinskii,^{1,2}  D. Y. Kolotkov,^{3,a)}  V. M. Nakariakov,^{3,4}  N. E. Molevich,^{1,2}  and D. S. Ryashchikov^{1,2} 

AFFILIATIONS

¹Department of Physics, Samara National Research University, Samara 443086, Russia

²Department of Theoretical Physics, Lebedev Physical Institute of Russian Academy of Sciences, Samara Branch, Samara 443011, Russia

³Physics Department, Centre for Fusion, Space and Astrophysics, University of Warwick, Coventry CV4 7AL, United Kingdom

⁴St. Petersburg Branch, Special Astrophysical Observatory, Russian Academy of Sciences, St. Petersburg 196140, Russia

^{a)}E-mail: D.Kolotkov.1@warwick.ac.uk

ABSTRACT

Slow magnetoacoustic waves are omnipresent in both natural and laboratory plasma systems. The wave-induced misbalance between plasma cooling and heating processes causes the amplification or attenuation, and also dispersion, of slow magnetoacoustic waves. The wave dispersion could be attributed to the presence of characteristic time scales in the system, connected with the plasma heating or cooling due to the competition of the heating and cooling processes in the vicinity of thermal equilibrium. We analyzed linear slow magnetoacoustic waves in a plasma in thermal equilibrium formed by a balance of optically thin radiative losses, field-align thermal conduction, and an unspecified heating process. The dispersion is manifested by the dependence of the effective adiabatic index of the wave on the wave frequency, making the phase and group speeds frequency-dependent. The mutual effect of the wave amplification and dispersion is shown to result in the occurrence of an oscillatory pattern in an initially broadband slow wave, with the characteristic period determined by the thermal misbalance time scales, i.e., by the derivatives of the combined radiation loss and heating function with respect to the density and temperature, evaluated at the equilibrium. This effect is illustrated by estimating the characteristic period of the oscillatory pattern, appearing because of thermal misbalance in the plasma of the solar corona. It is found that by an order of magnitude, the period is about the typical periods of slow magnetoacoustic oscillations detected in the corona.

Published under license by AIP Publishing. <https://doi.org/10.1063/1.5115224>

I. INTRODUCTION

Magnetohydrodynamic (MHD) waves in natural and laboratory plasma systems are the subject of intensive recent studies.^{1,2} The growing interest in MHD waves is, in particular, connected with their potential to act as seismological probes in remote diagnostics of plasmas, which requires a detailed understanding of the effects affecting wave excitation, propagation, and damping (see, e.g., Refs. 3 and 4 for the discussion and implications of the MHD coronal seismology methods). The importance of MHD waves is also stimulated by recent case studies revealing their potential ability to locally heat the corona.⁵ However, a full picture of the role of MHD waves in the energy transport through the upper layers of the solar atmosphere is to be understood. An interesting feature of compressive MHD waves is possible overstability caused by the misbalance of the local energy losses, e.g., dissipative processes and radiation, and heating (e.g., Ref. 6 and references therein). The instability of a plasma caused by the thermal misbalance has intensively been studied in the context of star formation,⁷

solar prominence formation,⁸ and edge-localized modes in tokamaks,⁹ see also Ref. 10 for a comprehensive review. An important example of a potentially thermally unstable plasma is the corona of the Sun, in which the observed local thermal equilibrium is supported by a competition of the radiative and thermal conductive energy losses with a yet unidentified heating mechanism that could be connected, for example, with magnetic reconnection or wave dissipation.¹¹ Slow magnetoacoustic waves that are confidently detected in the corona^{12,13} have the energy clearly insufficient to heat the coronal plasma,¹⁴ but they are a promising tool for the plasma diagnostics, including its thermodynamical properties.

A perturbation of an initial thermal equilibrium by a compressive wave leads to the misbalance between the heating and cooling rates. This in turn can affect the wave via the temperature and density variations, thus establishing a feedback between the perturbed medium and the perturbing wave, which resulted in the wave overstability. The thermal misbalance is known to lead to either the damping or

amplification of compressive waves.⁶ In the latter case, the plasma acts as an active medium. A traditional description of the thermal overstability of MHD waves is the evolutionary equation method, based usually on the assumption that the nonadiabatic effects are weak. In this limit, the overstability is independent of the wavelength. In combination with short-wavelength dissipation (e.g., by finite thermal conduction, viscosity, or resistivity) and the waveguide dispersion caused by a plasma nonuniformity, it may lead to the occurrence of stationary nonlinear dissipative structures, such as autowaves and autosolitons.^{15,16}

Stronger heating/cooling misbalance violates the assumption of weak nonadiabaticity, making the effect frequency- (or wavelength-) dependent, i.e., causing linear wave dispersion.^{17–19} This dispersion is not connected with the plasma nonuniformity that is often attributed to the observed dispersive effects.²⁰ In the latter case, the geometrical dispersion is known to result in the development of quasiperiodic fast magnetoacoustic wave trains with the periodicity determined by the properties of the waveguiding nonuniformity (see, e.g., Ref. 1 for a comprehensive discussion of this topic in the context of the solar corona and Earth's magnetosphere). For slow waves, this effect has not been considered due to the relatively weak geometrical dispersion. However, the dispersion caused by a thermal misbalance may be sufficiently strong.

In this paper, we demonstrate the formation of a quasiperiodic structure in a linear slow magnetoacoustic wave, i.e., the formation of linear quasiperiodic slow magnetoacoustic wave trains, in a thermally active plasma due to the linear dispersion associated with the thermal misbalance. The discussed effect is generic and may appear in different plasma environments. In this work, we focus on the general consideration of the role of the thermal misbalance in magnetoacoustic wave dynamics and illustrate this effect in the plasma of the solar corona.

II. GOVERNING EQUATIONS

We consider slow magnetoacoustic waves in a uniform medium in the infinite field approximation, which allows us to study the wave dynamics in terms of a reduced one-dimensional hydrodynamic model (see also Refs. 21–26 where this approximation is extensively used for modeling slow magnetoacoustic waves)

$$\rho \frac{dV_z}{dt} = -\frac{\partial P}{\partial z}, \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho V_z) = 0, \quad (2)$$

$$P = \frac{k_B T \rho}{m}, \quad (3)$$

$$C_V \frac{dT}{dt} - \frac{k_B T}{m \rho} \frac{d\rho}{dt} = -Q(\rho, T) + \frac{\kappa}{\rho} \frac{\partial^2 T}{\partial z^2}, \quad (4)$$

where V_z is the velocity component along the z -axis coinciding with the magnetic field direction; ρ , T , and P are the density, temperature, and pressure, respectively; and k_B is the Boltzmann constant, m is the mean particle mass, C_V is the specific heat capacity at constant volume, κ is the field-aligned thermal conductivity, and d/dt stands for the convective derivative. The heating/cooling function

$$Q(\rho, T) = L(\rho, T) - H(\rho, T) \quad (5)$$

combines the effects of the heating $H(\rho, T)$ and radiative losses $L(\rho, T)$. Astrophysical plasmas are often observed to be approximately

isothermal along the magnetic field, see, e.g., Fig. 9 in Ref. 27 and references therein for recent detection of an isothermal plasma in active regions of the solar corona. Hence, we consider an isothermal initial equilibrium, at which $Q(\rho_0, T_0) = 0$, where the index 0 indicates the equilibrium quantities. Model (1)–(4) implies that we focus on the propagation of waves strictly along the ambient magnetic field lines. Hence, the latter is not explicitly present in the governing equations. Under this approximation, the magnetic field is assumed to be infinitely strong so that it acts as an infinitely stiff guiding background for the field-aligned motions in slow waves. Therefore, in this approximation, the waves do not perturb the field, and their speed is independent of it. In a low- β plasma, the validity of this approximation can be illustrated by the following simple estimation: for example, $\beta = 0.1$ and adiabatic index $\gamma = 5/3$; the standard sound speed c_s is found to differ from the tube speed $c_T = c_s / \sqrt{1 + (\gamma/2)\beta}$ (for obliquely propagating slow waves) by less than 4%. In the zero- β limit considered in this paper, the slow waves were shown to degenerate into pure acoustic waves [see, e.g., wave equation (8) in Ref. 28 and dispersion relation (74) in Ref. 29 for $c_T \rightarrow c_s$ and for the Alfvén speed $c_A \rightarrow \infty$]. Equations (1)–(4) thus coincide with the equations of one-dimensional acoustics.

For clarity, the optically thin radiation loss function in the solar corona can be modeled as

$$L(\rho, T) = \chi \rho T^\beta, \quad (6)$$

where the parameters χ and β depend on the temperature and are determined, for example, from the CHIANTI atomic database^{30,31} (see Fig. 1). We would like to stress that in this work, we do not aim to address any specific problem of the solar corona. The plasma of the corona is mentioned here as an illustrative example only, as the most nearest candidate among the thermally active astrophysical plasmas.

The heating function could be taken in the form

$$H(\rho, T) = h \rho^a T^b, \quad (7)$$

where the constant h is determined from the thermal equilibrium condition $Q(\rho_0, T_0) = 0$, and the indices a and b are associated with the specific heating mechanism.^{19,32} In particular, $a = 0$ and $b = 1$ correspond to the Ohmic heating that is used as an illustrative example in this paper. As the radiation and heating depend on ρ and T differently, the wave perturbations of these quantities cause the thermal misbalance that can either damp or magnify the wave. In other words, the considered waves do not contribute to the heating process but may alter its efficiency via perturbations of the physical parameters of the plasma, which affect the heating.

III. DISPERSION RELATION AND CHARACTERISTIC TIME SCALES

Consider the dynamics of a small-amplitude perturbation, governed by Eqs. (1)–(4) supplemented with expressions (6) and (7). Linearizing it around the initial equilibrium and excluding all variables except the density perturbation ρ_1 , we obtain

$$\begin{aligned} \frac{\partial^3 \rho_1}{\partial t^3} - \gamma \frac{k_B T_0}{m} \frac{\partial^3 \rho_1}{\partial t \partial z^2} &= \frac{\kappa}{\rho_0 C_V} \left(\frac{\partial^4 \rho_1}{\partial z^2 \partial t^2} - \frac{k_B T_0}{m} \frac{\partial^4 \rho_1}{\partial z^4} \right) \\ &- \frac{Q_{[\rho]T}}{C_V} \left(\frac{\partial^2 \rho_1}{\partial t^2} - \frac{Q_{[p]T} k_B T_0}{Q_{[\rho]T} m} \frac{\partial^2 \rho_1}{\partial z^2} \right), \end{aligned} \quad (8)$$

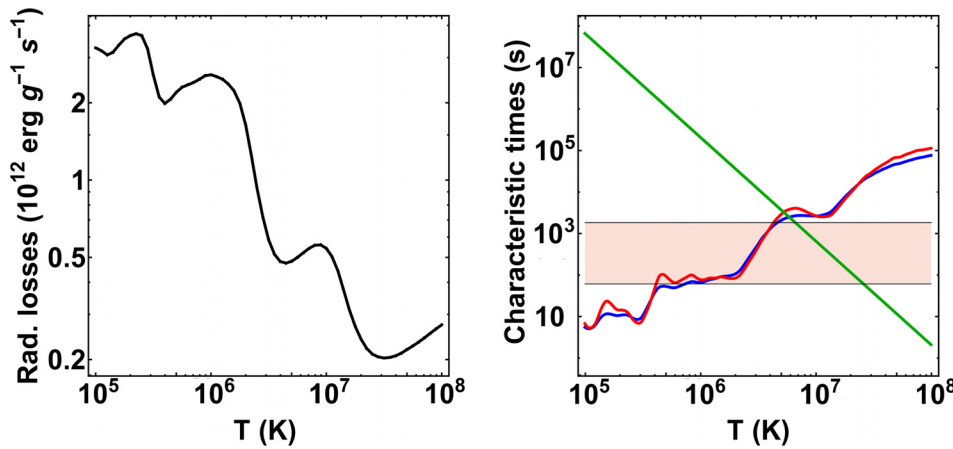


FIG. 1. Radiative losses per unit mass (black solid line) obtained from CHIANTI v. 8.0.7 atomic database for the plasma concentration $n_0 = 10^{10} \text{ cm}^{-3}$ (left) and the absolute values of the misbalance characteristic times τ_1 and τ_2 [see Eq. (11), blue and red solid lines, respectively] and the thermal conduction time τ_{cond} [see Eq. (9), green line] for $\lambda = 100 \text{ Mm}$ (right). The pink-shaded area in the right-hand panel shows typical observed periods of slow magnetoacoustic waves in the solar corona, namely, from 1 to 30 min.

where $Q_{[\rho]T} = (\partial Q / \partial T)_\rho$, $Q_{[p]T} = (\partial Q / \partial T)_\rho - (\rho_0 / T_0)(\partial Q / \partial \rho)_T = (\partial Q / \partial T)_p$. Being a third-order equation with respect to time, Eq. (8) describes three wave modes, which are two slow magnetoacoustic modes and one entropy mode (see, e.g., Ref. 33 and references therein, for the description of the physical properties of the latter, which are out of the scope of this study). Previous theoretical estimations^{14,34} show that the characteristic time scale of the thermal conduction is highly sensitive to the equilibrium temperature and density of the plasma and to the wavelength of the oscillation, λ

$$\tau_{\text{cond}} = \rho_0 C_V \lambda^2 / \kappa, \quad (9)$$

where the thermal conduction coefficient κ could be estimated as $\kappa = 10^{-11} T_0^{5/2} \text{ W m}^{-1} \text{ K}^{-1}$. On the other hand, there is a broad variety of the temperatures and densities in the astrophysical plasma structures (see, e.g., Ref. 35 for the properties of coronal loops, including those associated with the direct observations of slow oscillations in Ref. 36). Hence, in the further analysis, we address rather dense ($n_0 = 10^{10} \text{ cm}^{-3}$) and warm ($1 \text{ MK} < T_0 < 3 \text{ MK}$) plasma of the solar corona and assume the oscillation wavelength to be sufficiently long ($\lambda \sim 100 \text{ Mm}$), for which the thermal conduction time is a few orders of magnitude longer than typically observed slow magnetoacoustic oscillation periods (see Fig. 1). This allows us to neglect the effect of thermal conduction on the slow wave in the following calculations.

Dispersion relations for slow waves in the presence of the heating/cooling misbalance are obtained from Eq. (8) by assuming the harmonic dependence upon the time and spatial coordinates

$$\frac{\omega^2}{k^2} = \frac{1 - i\omega\tau_1}{1 - i\omega\tau_2} \gamma_Q \frac{k_B T_0}{m}, \quad (10)$$

where ω and k are the cyclic frequency and wavenumber, respectively. Dispersion relation (10) is a limiting case of the dispersion relations derived in Refs. 18 and 19 when neglecting the effects of thermal conduction and oblique propagation. Equation (10) includes characteristic times

$$\tau_1 = \gamma C_V / Q_{[p]T}, \quad \tau_2 = C_V / Q_{[\rho]T}, \quad (11)$$

whose absolute values determine the time scales at which the dispersive properties of the wave, caused by the thermal misbalance, are most pronounced. Figure 1 illustrates the dependence of $|\tau_{1,2}|$ on

temperature, in the case of the solar coronal plasma. The ratio of the wave period and the characteristic times τ_1 and τ_2 determines two qualitatively different limits in the slow wave evolution, i.e., the high-frequency (HF) limit, $\omega \gg 1/\min\{|\tau_1|, |\tau_2|\}$, and the low-frequency (LF) limit, $\omega \ll 1/\max\{|\tau_1|, |\tau_2|\}$. Characteristic times in form (11) allow for a direct association of a weak/strong nonadiabaticity with the high-frequency/low-frequency limits, respectively. Indeed, considering the derivatives $Q_{[p]T}$ and $Q_{[\rho]T}$ to be small, the characteristic times τ_1 and τ_2 tend to infinity, thus corresponding to the high-frequency regime. In this limit, the right-hand side of the linearized energy equation (4) can be assumed to be small (cf. Refs. 6 and 26). Likewise, the low-frequency regime corresponds to the large values of those derivatives (small values of τ_1 and τ_2), resulting in the domination of the thermal effects in the linearized energy equation (4). The new thermal misbalance time scales $\tau_{1,2}$ are not associated with the radiative cooling time of the background plasma, which determines the cooling rate in the case of steady, i.e., nonvarying, heating or in its full absence. For example, Ref. 37 considered such a constant heating term, neither contributing into the wave dynamics nor being affected by it. In contrast to this, we account for the heating and radiative cooling processes, both varied by the wave. One of the important implications is that the effects discussed below occur even in isothermal waves.²³ In this regime, the waves are not subject to damping by thermal conduction, while the cooling and heating functions, and hence their wave-induced misbalance, are affected by the perturbations of the density in the wave.

The combination of parameters on the right-hand side of Eq. (10)

$$\gamma_{\text{eff}} = \gamma_Q \frac{1 - i\omega\tau_1}{1 - i\omega\tau_2} = \begin{cases} \gamma \equiv \frac{C_p}{C_V}, & \text{HF regime} \\ \gamma_Q \equiv \frac{Q_{[p]T}}{Q_{[\rho]T}}, & \text{LF regime} \end{cases} \quad (12)$$

can be treated as a frequency-, density-, and temperature-dependent effective adiabatic index of a slow magnetoacoustic wave in a plasma with the heating/cooling misbalance, where γ_Q is the effective adiabatic index in the low-frequency limit $\omega \ll 1/\max\{|\tau_1|, |\tau_2|\}$, determined by the heating and cooling processes only. The dependence of γ_Q on the temperature is shown in Fig. 2, which is obtained using the radiative loss function calculated with CHIANTI v. 8.0.7 and assuming the

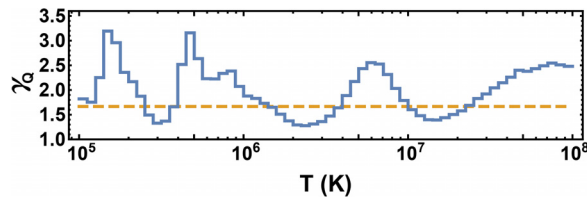


FIG. 2. Low-frequency effective adiabatic index γ_Q [see Eq. (12), blue solid line], determined for the radiative cooling by CHIANTI and heating by the Ohmic dissipation. The dashed orange line shows the standard (high-frequency) adiabatic index $\gamma = 5/3$.

Ohmic heating. The value of γ_Q can be either higher or lower than the standard adiabatic index $\gamma = 5/3$, ranging from about 1.4 to 3.2 in the considered temperature interval and for the chosen heating model.

In the limits $\omega \times \min\{|\tau_1|, |\tau_2|\} \rightarrow \infty$ and $\omega \times \max\{|\tau_1|, |\tau_2|\} \rightarrow 0$, Eq. (10) reduces to the equations describing the propagation of slow waves without dispersion and dissipation at the phase speeds

$$c_s = \sqrt{\gamma \frac{k_B T_0}{m}}, \quad c_{sQ} = \sqrt{\gamma_Q \frac{k_B T_0}{m}}, \quad (13)$$

respectively. In these limiting cases, the misbalance does not cause any dispersion and damping/amplification of slow waves. In the specific case $\tau_1 = \tau_2$ providing $\gamma = \gamma_Q$ and $c_s = c_{sQ}$, slow waves also propagate without any dispersion and damping or amplification. In contrast, for the interim frequencies including those comparable to the characteristic time scales $|\tau_{1,2}|^{-1}$, the effect of misbalance may be important. It is interesting that for the typical temperature interval, $\sim 10^6$ – 10^7 K within which slow magnetoacoustic waves are usually detected in the corona,^{12,13} the values of $\tau_{1,2}$ are found to be comparable to their typical oscillation periods (ranging from about 1 min to 30 min, see Fig. 1).

Consider the LF and HF limits of Eq. (10) keeping the first order of the small parameter $\omega \times \max\{|\tau_1|, |\tau_2|\}$ or $1/(\omega \times \min\{|\tau_1|, |\tau_2|\})$, respectively

$$\omega^2 - c_{sQ}^2 k^2 = -ik^2 \omega \tau_2 (c_s^2 - c_{sQ}^2), \quad \text{LF regime}, \quad (14)$$

$$\omega^2 - c_s^2 k^2 = -ik^2 (c_s^2 - c_{sQ}^2) / \omega \tau_2, \quad \text{HF regime}. \quad (15)$$

Unlike the zero-order approximation (that is $\omega \times \max\{|\tau_1|, |\tau_2|\} \rightarrow 0$ and $\omega \times \min\{|\tau_1|, |\tau_2|\} \rightarrow \infty$) described above, in these limits, both the wave dispersion and decay/amplification appear. Moreover, implying the assumption of a weak amplification/attenuation on a wavelength, i.e., assuming the frequency ω to be always real, while the wavenumber is complex, $k = k_R + ik_I$, with $k_R \gg k_I$, Eqs. (14) and (15) further reduce to

$$kc_{sQ} \approx \omega + i\omega^2 \tau_2 (\gamma - \gamma_Q) / 2\gamma_Q, \quad \text{LF regime}, \quad (16)$$

$$kc_s \approx \omega + i\tau_2^{-1} (\gamma - \gamma_Q) / 2\gamma, \quad \text{HF regime}. \quad (17)$$

The latter corresponds to the specific case considered in Ref. 6, where the slow wave evolves without dispersion but with the amplification or attenuation due to a nonzero imaginary part k_I . This analysis implies that the amplification/attenuation of slow waves by the thermal misbalance persists across the whole frequency spectrum, from the LF to the HF limit, while the phase speed approaches the constant values c_{sQ} and c_s in those limits.

IV. WAVE SPEED AND INCREMENT/DECREMENT

Under the assumption $k_R \gg k_I$ which is satisfied when $|(\tau_2 - \tau_1)/\tau_1| \ll 1$ (i.e., the values of the characteristic misbalance times are sufficiently close to each other, or equivalently, $|(\gamma_Q - \gamma)/\gamma| \ll 1$), the dispersion equation (10) gives us the frequency-dependent phase and group speeds

$$c_{ph}(\omega) = \frac{\omega}{k_R} \approx \sqrt{\frac{c_{sQ}^2 + \omega^2 \tau_2^2 c_s^2}{1 + \omega^2 \tau_2^2}}, \quad (18)$$

$$c_{gr}(\omega) = \left(\frac{\partial k_R}{\partial \omega} \right)^{-1} = \frac{c_{ph}^3(\omega)}{c_{ph}^2(\omega) - \Lambda(\omega)}, \quad (19)$$

where $\Lambda(\omega) = \omega^2 \tau_2^2 (c_s^2 - c_{sQ}^2) / (1 + \omega^2 \tau_2^2)^2$. In the high-frequency ($\omega \gg 1/\min\{|\tau_1|, |\tau_2|\}$) and low-frequency ($\omega \ll 1/\max\{|\tau_1|, |\tau_2|\}$) limits, both the phase and group speeds tend to the constant values c_s and c_{sQ} (13), respectively.

In contrast to c_s which is a standard value of the sound speed in an ideal medium, c_{sQ} is defined by the heating and cooling processes. In particular, the low-frequency slow wave that is highly influenced by the thermal misbalance can propagate at the phase speed which is substantially different from that of the high-frequency wave. The effect of this dispersion is most pronounced when the wave period is about the characteristic times $|\tau_{1,2}|$ and reaches its maximum near the frequency

$$\omega_M \approx (\tau_1 \tau_2)^{-1/2} = \sqrt{\frac{Q_{[p]T} Q_{[p]T}}{C_V C_p}}, \quad (20)$$

which is determined as the frequency at which $dc_{ph}/d\omega$ is the highest.

The discussed scenario is illustrated in Fig. 3 (left-hand panels) which shows how c_{ph} and c_{gr} vary with the wave period and plasma temperature. The departure of c_{gr} from c_s is quantified via the introduction of a normalized difference $(c_{gr} - c_s)/c_s$ whose absolute value grows with the increase in dispersion, thus delineating the parametric region where the discussed effect is the most pronounced. As seen in Fig. 3, c_{gr} could be either greater or lower than c_s depending upon a specific combination of the wave period and plasma temperature. We need to mention here that according to Fig. 3, the highest deviation of c_{gr} from c_s is detected to be about 10% which is consistent with the above-made assumption of a relatively weak dispersion and amplification/attenuation of the wave on the wavelength, $k_R \gg k_I$.

The slow wave damping/amplification due to the wave-induced thermal misbalance is determined by the wave increment/decrement k_I obtained from dispersion relation (10)

$$k_I \approx \frac{\omega^2 \xi}{2c_{ph}^3(\omega) \rho_0}, \quad \xi = \frac{\rho_0 \tau_2 (c_s^2 - c_{sQ}^2)}{1 + \omega^2 \tau_2^2}, \quad (21)$$

where ξ is an effective thermal misbalance-caused bulk viscosity coefficient.^{17,38} Similar to the effective phase and group speeds (18) and (19), the increment/decrement k_I is frequency-dependent, indicating that different wave harmonics are amplified/attenuated differently. In the low-frequency limit, it reduces to the quadratic dependence upon the wave frequency ω , see Eq. (16). In the high-frequency limit, it reaches a constant maximum value $k_{I\infty} = (\gamma - \gamma_Q) / 2\gamma c_s \tau_2$ (cf. Ref. 6).

Figure 3 (right-hand panels) illustrates the dependence of the wave increment k_I and its value normalized to the wavelength $\lambda = 2\pi/k_R$

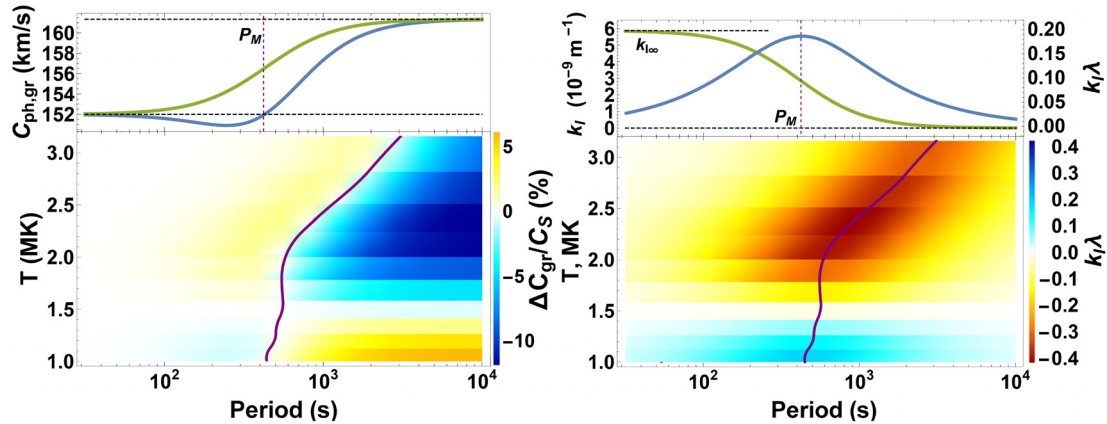


FIG. 3. Left top: dependence of the phase c_{ph} (green solid line) and group c_{gr} (blue solid line) speeds on the wave period, for the temperature of 1.06 MK. Left bottom: normalized difference between the group and standard sound speed $(c_{gr} - c_s)/c_s$ as a function of the wave period and temperature T . The orange/blue color scheme indicates the regions where c_{gr} is greater/lower than c_s , respectively. Top right: dependence of the spatial increment k_l (green solid line) and the increment normalized to the wavelength λ (blue solid line), for the temperature of 1.06 MK. For higher temperatures (≥ 1.6 MK), k_l changes its sign to negative, while its qualitative behavior with the wave period remains similar. Bottom right: variation of the normalized increment $k_l \lambda$ with the wave period and plasma temperature. The orange/blue colors indicate the regions of the wave amplification and damping, respectively. The purple lines in all panels show the oscillation period P_M .

(which is effectively equivalent to an inverse spatial quality factor of the wave) upon the wave period and plasma temperature. It allows for a clear localization of the discussed effect in the parametric space, revealing the regions of the wave damping and amplification. The most efficient amplification/attenuation coincides with the maximum of the dispersion effect and occurs at the frequency ω_M (20). Likewise, similar to the effect of the dispersion, $k_l \lambda$ tends to zero in the low- and high-frequency limits, indicating a low-efficiency damping/amplification of slow waves in those limits.

Equation (21) also implies that the sign of k_l is fully determined by the sign of the effective viscosity coefficient ζ , caused by the thermal misbalance. Thus, the slow wave is amplified in the case of a negative ζ and damped in the opposite case. Therefore, the condition of the wave amplification is

$$\tau_2(\gamma - \gamma_Q) < 0, \quad (22)$$

which is identical to the isentropic instability condition obtained in Ref. 39.

V. THERMAL OVERSTABILITY

Figure 4 shows the results of the numerical solution of equation (8), illustrating the dispersive and damping/amplification effects on the evolution of a broadband pulse. The initial shape of the pulse is Gaussian, $\rho_1 = A_0 \exp(-z^2/w^2)$, where A_0 and w are the initial amplitude and width, respectively. The derivatives $Q_{[p]T}$ and $Q_{[p]T}$ of the heating/cooling function are taken to be positive, which allows us to exclude effects of the isobaric and isochoric instabilities, see Ref. 39 for details, and thus focus on the effects associated with the wave evolution.

The top panel of Fig. 4 shows the case of the wave amplification and negative dispersion, i.e., with $\gamma_Q > \gamma$. The discussed regime implies that longer-wavelength harmonics travel faster [see Eqs. (18) and (19) and the left-hand panels of Fig. 3], and the most efficient gain of the energy from the medium occurs in the vicinity of ω_M [see Eqs. (20)

and (21) and the right-hand panels of Fig. 3]. At the initial stage of the wave evolution, this leads to the development of a quasiperiodic wave train, in which longer-wavelength spectral components travel faster and hence overtake the shorter wavelengths. The combination of this effect with amplification results in the occurrence of a quasimonochromatic amplitude-modulated signal with the dominant period $P_M = 2\pi\omega_M^{-1}$. Its amplitude is substantially higher than that of the initial perturbation, which could be negligibly small. In the example

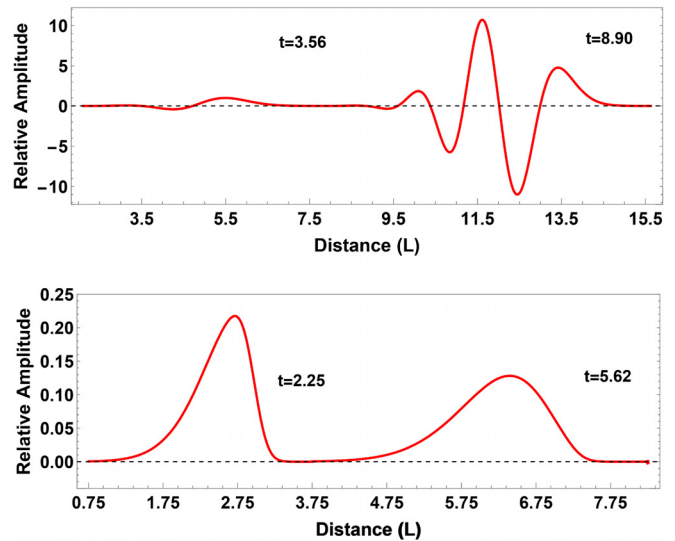


FIG. 4. Evolution of the plasma density in a slow magnetoacoustic pulse of an initial Gaussian shape at two different elapsed times after the excitation; for $w=0.2$, $\tau_1/P_M = 0.188$, and $\tau_2/P_M = 0.135$ (providing $\gamma_Q \approx 1.2$, bottom); and $w=1$, $\tau_1/P_M = 0.119$, and $\tau_2/P_M = 0.214$ (providing $\gamma_Q \approx 3$, top). The amplitude is measured in units of the initial amplitude. The time is normalized to $P_M = 2\pi\omega_M^{-1}$. The horizontal axis shows the distance from the wave excitation point and is normalized to $L = P_M \sqrt{k_B T_0/m}$.

shown in Fig. 4, the apparent dominant periodicity of the wave train is about $1.24P_M$ or 469 s for $P_M \approx 378$ s at the temperature of 1.0 MK, for which the wavelength and the phase speed are about 84 Mm and 179 km s^{-1} [corresponding to the effective adiabatic index of about 2.3, determined as the real part of Eq. (12)], respectively. The effective adiabatic index γ_{eff} becomes frequency and temperature dependent too. It may explain its observational estimations recently made in Ref. 40 (see also Ref. 41).

The bottom panel of Fig. 4 shows an alternative scenario, with the damping and positive dispersion, corresponding to $\gamma_Q < \gamma$. The initially Gaussian pulse becomes asymmetric, broadens, and decreases in its amplitude with time. In this analysis, the asymmetric shape of the perturbation is a purely linear effect caused by both the dispersion of the wave speed, due to which the higher harmonics propagate faster, and by the fact that the initial perturbation also excites the entropy mode (in addition to two slow magnetoacoustic ones), which breaks the symmetry in the distribution of the initial energy across harmonics. On the other hand, those faster propagating higher-frequency components decay with a higher decrement which results in an additional apparent broadening of the pulse and an overall decrease in its amplitude. In this case, the wave decays faster than the oscillatory wake forms.

VI. CONCLUSIONS

We demonstrated that the presence of characteristic times determined by the thermal misbalance leads to the occurrence of quasiperiodic slow magnetoacoustic wave patterns in a uniform plasma. The thermal misbalance is associated with different dependences of the radiative cooling and an unspecified heating function on the quantities perturbed by the wave in the vicinity of the equilibrium. Due to the effective slow wave magnification, the amplitude of the initial perturbation rapidly grows, implying that any low-amplitude fluctuation of thermodynamical parameters would be sufficient for the development of those quasiperiodic structures. The periodicity is created by the competition of the wave dispersion and amplification, both caused by the thermal misbalance. The characteristic period is determined by the dependence of the heating/cooling function on the plasma parameters and is not connected with other characteristic times, e.g., the transverse travel time across a waveguiding plasma nonuniformity.

In this study, we considered the linear perturbations of a dense and warm plasma, with the wavelengths long enough to make the wavelength-dependent nonadiabatic effects, such as thermal conduction and viscosity, negligible. This allowed us to isolate and investigate the role of the effect of the thermal misbalance in the dynamics of slow magnetoacoustic waves. This approach allowed us to identify a new mechanism for the formation of quasiperiodicity in a slow magnetoacoustic wave excited by a broadband, impulsive driver. A further development of the presented theory would require addressing specific physical problems in specific plasma environments and accounting for appropriate additional nonadiabatic effects and also nonlinearity. This would bring additional time scales, e.g., connected with the thermal conduction and viscosity times,²³ which would lead to a more effective decay of the shorter-wavelength spectral components and could allow for a stabilization of the wave amplitude. In this case, the characteristic period of the slow wave train could be used as a promising seismological tool for the diagnostics of the parameters of the plasma heating function, hence stimulating the search for this effect in observations. For example, the detected quasiperiodic behavior may be responsible for the

quasiperiodic pulsations observed in impulsive energy releases and often associated with the evolution of a slow magnetoacoustic mode.⁴² It is also worth noting here that the apparent variation of the instantaneous frequency in the detected quasiperiodic wave train, occurring as an effect of the discussed dispersion, can readily cause the observed nonstationarity of those quasiperiodic pulsations (see Ref. 43 for the most recent comprehensive review of this topic). Another interesting development of the proposed theory could be accounting for the effect of the plasma inhomogeneity on the discussed slow magnetoacoustic wave trains. In particular, slow waves were shown to be a subject to an effective phase mixing due to the transverse nonuniformity of the plasma temperature.⁴⁴ Likewise, the parallel inhomogeneity with the spatial scale comparable to the wavelength may result in additional wave amplification or damping (see, e.g., Refs. 45 and 46 where a similar dispersion relation was derived for the inhomogeneous flows of nonequilibrium gas).

ACKNOWLEDGMENTS

CHIANTI is a collaborative project involving George Mason University, the University of Michigan (USA), and the University of Cambridge (UK). V.M.N. and D.Y.K. acknowledge the support from an STFC consolidated Grant (No. ST/P000320/1). V.M.N. was supported by the Russian Foundation for Basic Research Grant No. 18-29-21016. Calculations presented in the reported study were funded by RFBR according to a research project (No. 18-32-00344). This study was supported in part by the Ministry of Education and Science of Russia under the public contract with educational and research institutions within Project No. 3.1158.2017/4.6.

REFERENCES

- ¹V. M. Nakariakov, V. Pilipenko, B. Heilig, P. Jelínek, M. Karlický, D. Y. Klimushkin, D. Y. Kolotkov, D.-H. Lee, G. Nisticò, T. Van Doorsselaere, G. Verth, and I. V. Zimovets, "Magnetohydrodynamic oscillations in the solar corona and earth's magnetosphere: Towards consolidated understanding," *Space Sci. Rev.* **200**, 75–203 (2016).
- ²S. E. Sharapov, H. J. C. Oliver, B. N. Breizman, M. Fitzgerald, and L. Garzotti, and JET Contributors, "MHD spectroscopy of JET plasmas with pellets via Alfvén eigenmodes," *Nucl. Fusion* **58**, 082008 (2018).
- ³D. J. Pascoe, "Numerical simulations for MHD coronal seismology," *Res. Astron. Astrophys.* **14**, 805–830 (2014).
- ⁴D. B. Jess, V. E. Reznikova, R. S. I. Ryans, D. J. Christian, P. H. Keys, M. Mathioudakis, D. H. Mackay, S. Krishna Prasad, D. Banerjee, S. D. T. Grant, S. Yau, and C. Diamond, "Solar coronal magnetic fields derived using seismology techniques applied to omnipresent sunspot waves," *Nat. Phys.* **12**, 179–185 (2016); e-print [arXiv:1605.06112](https://arxiv.org/abs/1605.06112) [astro-ph.SR].
- ⁵A. K. Srivastava, K. Murawski, B. Kuźma, D. P. Wójcik, T. V. Zaqarashvili, M. Stangalini, Z. E. Musielak, J. G. Doyle, P. Kayshap, and B. N. Dwivedi, "Confined pseudo-shocks as an energy source for the active solar corona," *Nat. Astron.* **2**, 951–956 (2018).
- ⁶V. M. Nakariakov, A. N. Afanasyev, S. Kumar, and Y.-J. Moon, "Effect of local thermal equilibrium misbalance on long-wavelength slow magnetoacoustic waves," *Astrophys. J.* **849**, 62 (2017).
- ⁷M.-M. Mac Low and R. S. Klessen, "Control of star formation by supersonic turbulence," *Rev. Mod. Phys.* **76**, 125–194 (2004). e-print [arXiv:astro-ph/0301093](https://arxiv.org/abs/astro-ph/0301093).
- ⁸T. Kaneko and T. Yokoyama, "Reconnection-condensation model for solar prominence formation," *Astrophys. J.* **845**, 12 (2017); e-print [arXiv:1706.10008](https://arxiv.org/abs/1706.10008) [astro-ph.SR].
- ⁹A. DePloey, R. A. M. Van der Linden, G. T. A. Huysmans, M. Goossens, W. Kerner, and J. P. Goedbloed, "Marfes: A magnetohydrodynamic stability study of two-dimensional tokamak equilibria," *Plasma Phys. Controlled Fusion* **39**, 423–438 (1997).

- ¹⁰B. Meerson, "Nonlinear dynamics of radiative condensations in optically thin plasmas," *Rev. Mod. Phys.* **68**, 215–257 (1996).
- ¹¹C. E. Parnell and I. De Moortel, "A contemporary view of coronal heating," *Philos. Trans. R. Soc. London, Ser. A* **370**, 3217–3240 (2012); e-print [arXiv:1206.6097](#) [astro-ph.SR].
- ¹²T. Wang, "Standing slow-mode waves in hot coronal loops: Observations, modeling, and coronal seismology," *Space Sci. Rev.* **158**, 397–419 (2011); e-print [arXiv:1011.2483](#) [astro-ph.SR].
- ¹³I. De Moortel and V. M. Nakariakov, "Magnetohydrodynamic waves and coronal seismology: An overview of recent results," *Philos. Trans. R. Soc. London, Ser. A* **370**, 3193–3216 (2012); e-print [arXiv:1202.1944](#) [astro-ph.SR].
- ¹⁴I. de Moortel, "Longitudinal waves in coronal loops," *Space Sci. Rev.* **149**, 65–81 (2009).
- ¹⁵V. M. Nakariakov and B. Roberts, "Solitary autowaves in magnetic flux tubes," *Phys. Lett. A* **254**, 314–318 (1999).
- ¹⁶R. Chin, E. Verwichte, G. Rowlands, and V. M. Nakariakov, "Self-organization of magnetoacoustic waves in a thermally unstable environment," *Phys. Plasmas* **17**, 032107 (2010).
- ¹⁷N. E. Molevich and A. N. Oraevskii, "Sound viscosity in media in thermodynamic disequilibrium," *J. Exp. Theor. Phys.* **67**, 504–506 (1988).
- ¹⁸M. H. Ibanez and N. M. Sanchez, "Propagation of sound and thermal waves in a plasma with solar abundances," *Astrophys. J.* **396**, 717–724 (1992).
- ¹⁹M. H. Ibanez and O. B. Escalona, "Propagation of hydrodynamic waves in optically thin plasmas," *Astrophys. J.* **415**, 335–341 (1993).
- ²⁰B. Roberts, P. M. Edwin, and A. O. Benz, "Fast pulsations in the solar corona," *Nature* **305**, 688–690 (1983).
- ²¹V. M. Nakariakov, E. Verwichte, D. Berghmans, and E. Robbrecht, "Slow magnetoacoustic waves in coronal loops," *Astron. Astrophys.* **362**, 1151–1157 (2000).
- ²²L. Ofman and T. Wang, "Hot coronal loop oscillations observed by SUMER: Slow magnetosonic wave damping by thermal conduction," *Astrophys. J. Lett.* **580**, L85–L88 (2002).
- ²³I. De Moortel and A. W. Hood, "The damping of slow MHD waves in solar coronal magnetic fields," *Astron. Astrophys.* **408**, 755–765 (2003).
- ²⁴E. Verwichte, M. Haynes, T. D. Arber, and C. S. Brady, "Damping of slow MHD coronal loop oscillations by shocks," *Astrophys. J.* **685**, 1286–1290 (2008).
- ²⁵M. S. Ruderman, "Nonlinear damped standing slow waves in hot coronal magnetic loops," *Astron. Astrophys.* **553**, A23 (2013).
- ²⁶S. Kumar, V. M. Nakariakov, and Y.-J. Moon, "Effect of a radiation cooling and heating function on standing longitudinal oscillations in coronal loops," *Astrophys. J.* **824**, 8 (2016); e-print [arXiv:1603.08335](#) [astro-ph.SR].
- ²⁷G. R. Gupta, G. Del Zanna, and H. E. Mason, "Exploring the damping of Alfvén waves along a long off-limb coronal loop, up to 1.4 R_{sun} ," *Astron. Astrophys.* **627**, A62 (2019).
- ²⁸A. N. Afanasyev and V. M. Nakariakov, "Nonlinear slow magnetoacoustic waves in coronal plasma structures," *Astron. Astrophys.* **573**, A32 (2015).
- ²⁹Y. D. Zhugzhda, "Force-free thin flux tubes: Basic equations and stability," *Phys. Plasmas* **3**, 10–21 (1996).
- ³⁰K. P. Dere, E. Landi, H. E. Mason, B. C. Monsignori Fossi, and P. R. Young, "CHIANTI—An atomic database for emission lines," *Astron. Astrophys., Suppl. Ser.* **125**, 149–173 (1997).
- ³¹G. Del Zanna, K. P. Dere, P. R. Young, E. Landi, and H. E. Mason, "CHIANTI—An atomic database for emission lines. Version 8," *Astron. Astrophys.* **582**, A56 (2015); e-print [arXiv:1508.07631](#) [astro-ph.SR].
- ³²R. B. Dahlburg and J. T. Mariska, "Influence of heating rate on the condensational instability," *Sol. Phys.* **117**, 51–56 (1988).
- ³³K. Murawski, T. V. Zaqarashvili, and V. M. Nakariakov, "Entropy mode at a magnetic null point as a possible tool for indirect observation of nanoflares in the solar corona," *Astron. Astrophys.* **533**, A18 (2011).
- ³⁴E. Priest, *Magnetohydrodynamics of the Sun* (Cambridge University Press, 2014).
- ³⁵F. Reale, "Coronal loops: Observations and modeling of confined plasma," *Living Rev. Sol. Phys.* **11**, 4 (2014).
- ³⁶G. Nisticò, V. Polito, V. M. Nakariakov, and G. Del Zanna, "Multi-instrument observations of a failed flare eruption associated with MHD waves in a loop bundle," *Astron. Astrophys.* **600**, A37 (2017); e-print [arXiv:1612.02077](#) [astro-ph.SR].
- ³⁷I. De Moortel and A. W. Hood, "The damping of slow MHD waves in solar coronal magnetic fields. II. The effect of gravitational stratification and field line divergence," *Astron. Astrophys.* **415**, 705–715 (2004).
- ³⁸V. G. Makaryan and N. E. Molevich, "Stationary shock waves in nonequilibrium media," *Plasma Sources Sci. Technol.* **16**, 124–131 (2007).
- ³⁹G. B. Field, "Thermal instability," *Astrophys. J.* **142**, 531 (1965).
- ⁴⁰S. K. Prasad, J. O. Raes, T. Van Doorselaere, N. Magyar, and D. B. Jess, "The polytropic index of solar coronal plasma in sunspot fan loops and its temperature dependence," *Astrophys. J.* **868**, 149 (2018); e-print [arXiv:1810.08449](#) [astro-ph.SR].
- ⁴¹T. Van Doorselaere, N. Wardle, G. Del Zanna, K. Jansari, E. Verwichte, and V. M. Nakariakov, "The first measurement of the adiabatic index in the solar corona using time-dependent spectroscopy of Hinode/EIS observations," *Astrophys. J. Lett.* **727**, L32 (2011).
- ⁴²T. Van Doorselaere, E. G. Kupriyanova, and D. Yuan, "Quasi-periodic pulsations in solar and stellar flares: An overview of recent results (invited review)," *Sol. Phys.* **291**, 3143–3164 (2016); e-print [arXiv:1609.02689](#) [astro-ph.SR].
- ⁴³V. M. Nakariakov, D. Y. Kolotkov, E. G. Kupriyanova, T. Mehta, C. E. Pugh, and A.-M. Broomhall, "Non-stationary quasi-periodic pulsations in solar and stellar flares," *Plasma Phys. Controlled Fusion* **61**, 014024 (2019).
- ⁴⁴Y. Voitenko, J. Andries, P. D. Copil, and M. Goossens, "Damping of phase-mixed slow magneto-acoustic waves: Real or apparent?," *Astron. Astrophys.* **437**, L47–L50 (2005).
- ⁴⁵N. E. Molevich, "Sound amplification in inhomogeneous flows of nonequilibrium gas," *Acoust. Phys.* **47**, 102–105 (2001).
- ⁴⁶R. N. Galimov, N. E. Molevich, and N. V. Troshkin, "Acoustical instability of inhomogeneous gas flows with distributed heat release," *Acta Acust. Acust.* **98**, 372–377 (2012).