Supplemental Material to the Review Paper "Magnetohydrodynamic Waves in the Solar Corona" by *Nakariakov & Kolotkov, Ann. Rev. Astron. Astrophys. 2019*

1. To Sec. 2 of the main paper, on "Theoretical modelling of MHD waves in plasma structures of the corona"

Figures 1 and 2 illustrate the spatial structure of lowest-m fast magnetoacoustic modes of the straight plasma cylinder of the radius r, modelled as

$$r(z, \phi) = r_0 + \tilde{r} \cos\left(k_\parallel z + m\phi\right), \tag{1}$$

where r_0 and $\tilde{r}$ are the equilibrium value and small-amplitude perturbation, respectively; z and ϕ are the parallel to the cylinder's main axis and azimuthal coordinates with the wave numbers $k_\parallel$ and m, respectively.

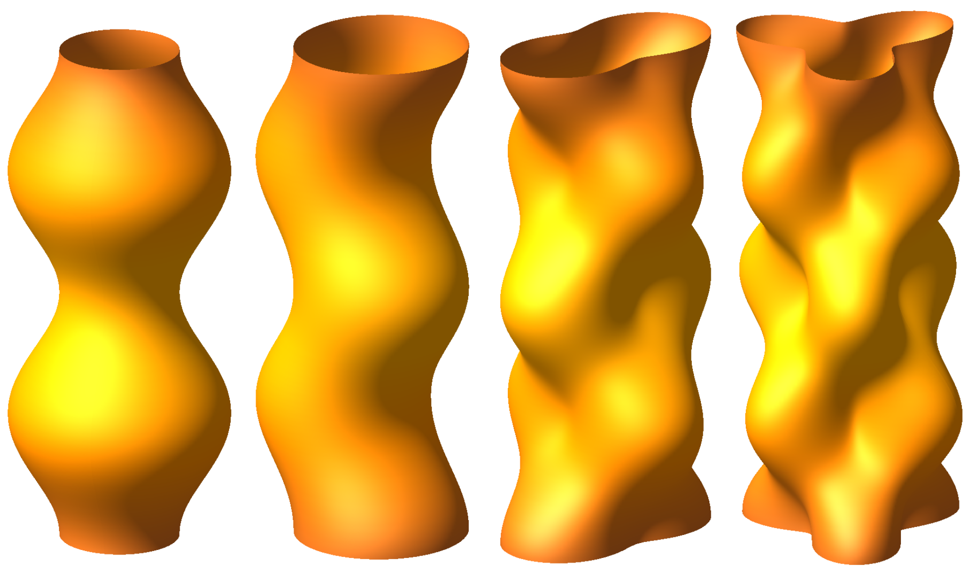

Figure 1: Illustration of the perturbation of the plasma cylinder's boundary in fast magnetoacoustic modes, determined by Eq. (1) with the azimuthal wave numbers (left-to-right): sausage, $m = 0$, linearly polarised kink, i.e., a superposition of $m = \pm 1$, and ballooning (fluting), $m = \pm 2$ and $m = \pm 3$, modes.

Figure 3 shows perturbations of the plasma density by a fast magnetoacoustic wave, guided along a zero-β plasma non-uniformity stretched along a straight and uniform magnetic field $\boldsymbol{B_0}$ directed along the z-axis and with the transverse density profile of a symmetric Epstein form,

$$\rho(x) = (\rho_0 - \rho_\infty)\mathrm{sech}^2\left(\frac{x}{d}\right) + \rho_\infty, \tag{2}$$

where ρ_0 and ρ_∞ are values of the plasma density at $x = 0$ and $x \to \infty$, and d is a characteristic width of the non-uniformity (see Cooper et al. 2003).

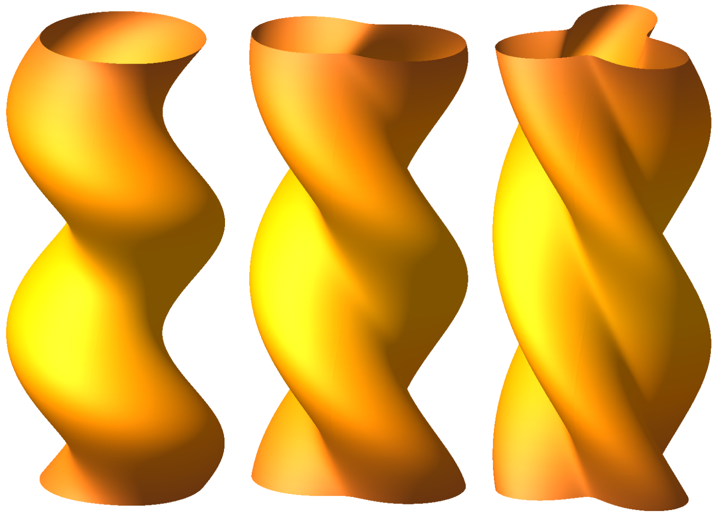

Figure 2: Circularly polarised perturbations of the plasma cylinder's boundary in fast magnetoacoustic modes, see Eq. (1): kink, $m = +1$, and ballooning, $m = +2$ and $m = +3$, modes (left-to-right).

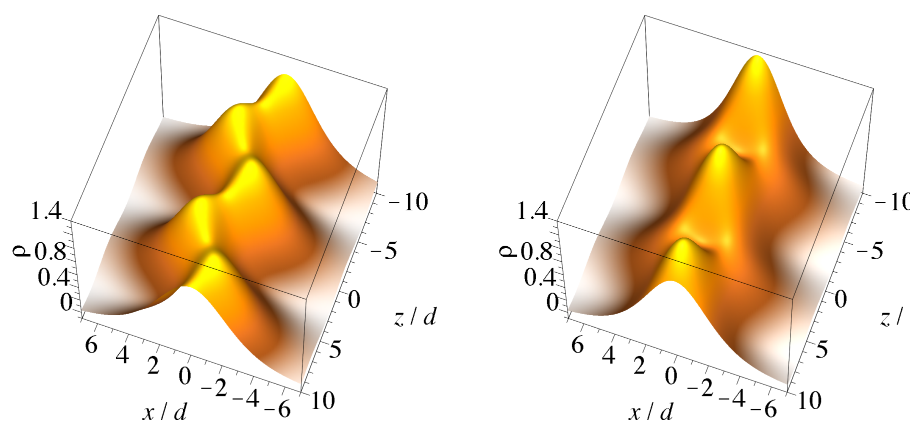

Figure 3: Structure of the density variation in the fast kink ($|m| = 1$, left) and sausage ($m = 0$, right) modes of a plasma non-uniformity with the symmetric Epstein transverse profile, see Eq. (2). The density is normalised as $(\rho - \rho_\infty) / (\rho_0 - \rho_\infty)$.

2. To Sec. 7 of the main paper, on "Quasi-periodic rapidly propagating wave trains"

Figure 4 illustrates a dispersively evolving propagating fast magnetoacoustic wave train and its wavelet spectrum, formed in a zero-β plasma non-uniformity with a transverse profile $\rho(x)$ given by Eq. (2). The initial equilibrium is perturbed by an axisymmetric ($m = 0$) velocity pulse

$$V_0(x, z) = A_0 x \exp\left[-\frac{x^2}{\lambda_x^2}\right] \exp\left[-\frac{(z - z_0)^2}{\lambda_z^2}\right], \tag{3}$$

where A_0 is an arbitrary amplitude, $\lambda_x = d$, $\lambda_z = 0.5d$, and $z_0 = 2d$. The evolution of such an initial pulse guided by the plasma non-uniformity is modelled as $V(x,z,t) \propto V_x(x) \exp[i(k_\parallel z - \omega t)]$. The transverse structure of the plasma velocity $V_x(x)$ oscillating in a trapped regime is obtained from the following ordinary differential equation,

$$\frac{d^2 V_x}{dx^2} + \left[\frac{\omega^2}{C_\mathrm{A}^2(x)} - k_\parallel^2\right] V_x = 0, \tag{4}$$

where $C_\mathrm{A} = B_0/\sqrt{4\pi\rho(x)}$ is the transverse profile of the Alfvén speed coinciding with that of the fast magnetoacoustic speed in a zero-β limit. The phase speed $V_\mathrm{ph} = \omega/k_\parallel$ of the considered axisymmetric perturbation (3), travelling along the plasma non-uniformity (2), as a function of the parallel wave number $k_\parallel$ for the fundamental transverse mode was obtained from the following algebraic equation,

$$\frac{|k_\parallel| d}{C_\mathrm{A0}^2}(V_\mathrm{ph}^2 - C_\mathrm{A0}^2) - \frac{2}{|k_\parallel| d} = \frac{3}{C_{\mathrm{A}\infty}} \sqrt{C_{\mathrm{A}\infty}^2 - V_\mathrm{ph}^2}, \tag{5}$$

where C_A0 and $C_{\mathrm{A}\infty}$ are the Alfvén speeds at $x = 0$ and $x \to \infty$, respectively (see Cooper et al. 2003).

LITERATURE CITED

Cooper FC, Nakariakov VM, Williams DR. 2003. *Astron. Astrophys.* 409:325–330

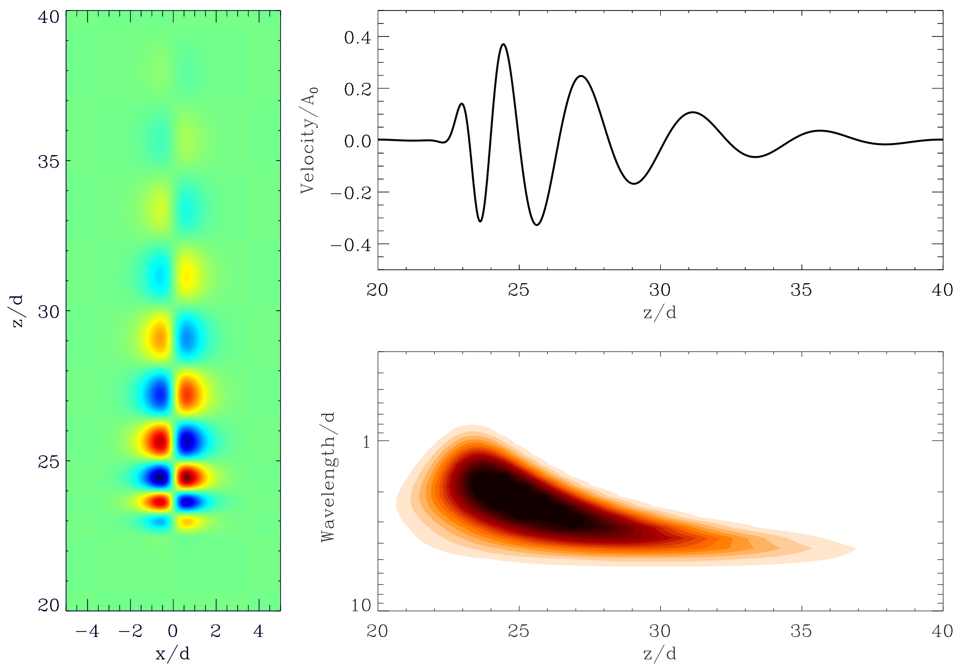

Figure 4: Left: Snapshot of the plasma velocity perturbed by a fast magnetoacoustic wave train developed from a broadband pulse guided by a zero-β plasma slab with a transverse density profile (2), at a certain instance of the computational time. Top right: Fast magnetoacoustic wave train shown in the left-hand panel, at a fixed transverse coordinate $x = 0.5d$. Bottom right: Morlet wavelet spectrum of a fast magnetoacoustic wave train shown in the top panel, manifesting typical *"crazy tadpole"* signatures, i.e. a narrowband "tail" travelling ahead and a broadband trailing "head".