New physics effects in $b \to s$ transitions with complex Wilson coefficients.

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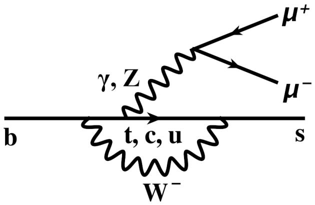
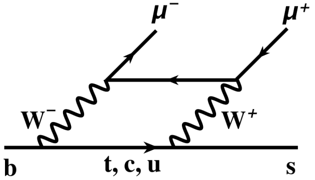
Anomalies in flavor physics

- Flavor changing neutral currents (FCNCs), potential probe of physics at higher energy scales $\rightarrow$ loop-suppressed amplitudes within the Standard Model (SM).



- Deviations in few angular observables ($P_5' \sim 3\sigma$) and theoretically clean ratios (R_K and R_{K^*}) from their respective SM predictions.



Theory

- Matching between the Standard Model and the effective theory amplitudes $\rightarrow$ Wilson coefficients at the scale $\mu_0 \sim M_W$.
- Renormalization group evolution of the Wilson coefficients down to the scale $\mu_0 \sim m_b$.

$$\mathcal{H}_{eff} = -\frac{4\,G_F}{\sqrt{2}}\left(\lambda_t \mathcal{H}_{eff}^{(t)} + \lambda_u \mathcal{H}_{eff}^{(u)}\right) \tag{1}$$

$$\begin{aligned}
\mathcal{H}_{eff}^{(t)} &= C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i=3}^{6} C_i \mathcal{O}_i + \sum_{i=7,8,9,10,P,S} (C_i \mathcal{O}_i + C_i' \mathcal{O}_i') \\
\mathcal{H}_{eff}^{(u)} &= C_1 (\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2 (\mathcal{O}_2^c - \mathcal{O}_2^u)\,.
\end{aligned}$$

- W.C's $\rightarrow$ encode short-distance physics and possible NP effects.

Operator basis

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{s}_\alpha q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A}, \\
\mathcal{O}_2 &= (\bar{s}q)_{V-A}(\bar{q}b)_{V-A}, \\
\mathcal{O}_3 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V-A}, \\
\mathcal{O}_4 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A}, \\
\mathcal{O}_5 &= (\bar{s}b)_{V-A}\sum_q(\bar{q}q)_{V+A}, \\
\mathcal{O}_6 &= (\bar{s}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A},
\end{aligned}$$

Operator basis

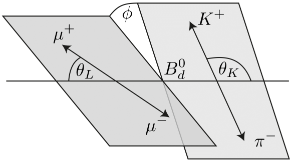
$$
\begin{aligned}
\mathcal{O}_7 &= \frac{e}{g^2} m_b(\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}, & \mathcal{O}_7^{'} &= \frac{e}{g^2} m_b(\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}, \\
\mathcal{O}_8 &= \frac{1}{g} m_b(\bar{s}\sigma_{\mu\nu} T^a P_R b) G^{\mu\nu}, & \mathcal{O}_8^{'} &= \frac{1}{g} m_b(\bar{s}\sigma_{\mu\nu} T^a P_L b) G^{\mu\nu}, \\
\mathcal{O}_9 &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l), & \mathcal{O}_9^{'} &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l), \\
\mathcal{O}_{10} &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l), & \mathcal{O}_{10}^{'} &= \frac{e^2}{g^2} (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu \gamma_5 l), \\
\mathcal{O}_S &= \frac{e^2}{16\pi^2} m_b(\bar{s} P_R b)(\bar{l} l), & \mathcal{O}_S^{'} &= \frac{e^2}{16\pi^2} m_b(\bar{s} P_L b)(\bar{l} l), \\
\mathcal{O}_P &= \frac{e^2}{16\pi^2} m_b(\bar{s} P_R b)(\bar{l}\gamma_5 l), & \mathcal{O}_P^{'} &= \frac{e^2}{16\pi^2} m_b(\bar{s} P_L b)(\bar{l}\gamma_5 l) \quad (2)
\end{aligned}
$$

Experimental Inputs

- $B^{(0,+)} \to K^{(0,+)}\mu^+\mu^-$: Differential branching fractions and isospin asymmetries (LHCb and Belle), binned data on the angular observables(A_{FB} and F_H) for $B^+ \to K^+\mu^+\mu^-$ (CMS) and inputs on R_K (LHCb and Belle).
- $B \to K^*\mu^+\mu^-$: Differential branching fractions and isospin asymmetries (LHCb), angular observables in $B^0 \to K^{*0}\mu^+\mu^-$ decays (LHCb and ATLAS), P_4' and P_5' for $B^0 \to K^{*0}\mu^+\mu^-$(Belle) and R_{K^*} (LHCb and Belle).
- $B_s \to \phi\mu^+\mu^-$: Differential branching fractions and angular observables (LHCb).
- $B_s \to \mu\mu$: Branching fraction (HFLAV 2019).
- Radiative modes.

Decay distribution

- **For a vector meson in final state :**



$$\frac{1}{d(\Gamma+\bar{\Gamma})/dq^2}\frac{d^4(\Gamma+\bar{\Gamma})}{dq^2 d\Omega} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_L)sin^2\theta_K + F_L cos^2\theta_K + \frac{1}{4}(1-F_L)sin^2\theta_K cos2\theta_l - F_L cos^2\theta_K cos2\theta_l + S_3 sin^2\theta_K sin^2\theta_l cos2\phi + S_4 sin2\theta_K sin2\theta_l cos\phi + S_5 sin2\theta_K sin\theta_l cos\phi + \frac{4}{3}A_{FB} sin^2\theta_K cos\theta_l + S_7 sin2\theta_K sin\theta_l sin\phi + S_8 sin2\theta_K sin2\theta_l sin\phi + S_9 sin^2\theta_K sin^2\theta_l sin2\phi\Big] \quad (3)$$

- $P_1 = \frac{2S_3}{1-F_L}$, $P_2 = \frac{2}{3}\frac{A_{FB}}{(1-F_L)}$, $P_3 = \frac{-S_9}{1-F_L}$, $P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}$
- **Normalized angular distribution for pseudoscalar meson :**

$$\frac{1}{\Gamma_l}\frac{d\Gamma_l}{dcos\theta} = \frac{3}{4}(1-F_H^l)(1-cos^2\theta) + \frac{1}{2}F_H^l + A_{FB}^l cos\theta \quad (4)$$

Analysis

- Complex new physics WCs : $C_7^{'}$, ΔC_9, $C_9^{'}$, ΔC_{10}, $C_{10}^{'}$, C_S, $C_S^{'}$, C_P and $C_P^{'}$.
- 3 datasets: Moment(LHCb 2016), Likelihood(LHCb 2016), Likelihood(LHCb 2020) - **No asymmetric observables**.
- Statistical analysis optimizing a χ^2 statistic.

$$\chi^2(C^{NP}) = [O_{exp} - O_{th}(C^{NP})]^T [C_{exp} + C_{th}]^{-1} [O_{exp} - O_{th}(C^{NP})]$$

- In the post-process for each fit, obtain fit-quality using p-value and find outliers.
- Model selection out of all possible combinations of the WCs(1022 combinations - Real + Real and Imaginary) - Akaike's Information Criterion (AIC*c*) and cross - validation.

One operator scenarios

Dataset	χ^2/DOF	p-val(%)	Value	χ^2/DOF	p-val(%)	Value
			C_7'			ΔC_9
Likelihood 2020	279.5/210	9.4×10^{-2}	$Re(C_7') \to$ -0.039±0.013 $Im(C_7') \to$ -0.026±0.101	202.89/210	62.5	$Re(\Delta C_9) \to$ -1.10±0.11 $Im(\Delta C_9) \to$ 1.27±0.37
Likelihood 2016	306.45/249	0.7	$Re(C_7') \to$ -0.03±0.01 $Im(C_7') \to$ -0.002±0.025	235.79/249	71.6	$Re(\Delta C_9) \to$ -1.21±0.14 $Im(\Delta C_9) \to$ -1.25±0.44
Moments 2016	291.85/271	18.4	$Re(C_7') \to$ -0.031±0.016 $Im(C_7') \to$ -0.0057±0.0300	241.7/271	89.9	$Re(\Delta C_9) \to$ -1.24±0.18 $Im(\Delta C_9) \to$ 1.19±0.48
			C_9'			ΔC_{10}
Likelihood 2020	287.9/210	2.9×10^{-2}	$Re(C_9') \to$ -0.077±0.149 $Im(C_9') \to$ -0.70±0.54	276.17/210	0.15	$Re(\Delta C_{10}) \to$ 0.64±0.18 $Im(\Delta C_{10}) \to$ 1.79±0.29
Likelihood 2016	310.72/249	0.47	$Re(C_9') \to$ -0.13±0.15 $Im(C_9') \to$ -0.15±0.71	303.22/249	1.1	$Re(\Delta C_{10}) \to$ 0.39±0.15 $Im(\Delta C_{10}) \to$ 0.45±0.50
Moments 2016	295.4/271	14.8	$Re(C_9') \to$ -0.060±0.148 $Im(C_9') \to$ -0.084±0.423	281.7/271	31.5	$Re(\Delta C_{10}) \to$ 0.51±0.14 $Im(\Delta C_{10}) \to$ -0.11±0.68
			C_{10}'			C_S
Likelihood 2020	278.1/210	0.1	$Re(C_{10}') \to$ 0.33±0.11 $Im(C_{10}') \to$ -0.21±0.81	288.55/210	2.6×10^{-2}	$Re(C_S) \to$ -0.029±0.483 $Im(C_S) \to$ -0.032±0.440
Likelihood 2016	303.0/249	1.1	$Re(C_{10}') \to$ 0.33±0.11 $Im(C_{10}') \to$ 0.02±0.28	311.19/249	0.4	$Re(C_S) \to$ -0.04±0.04 $Im(C_S) \to$ 0.0017±0.3043
Moments 2016	290.2/271	20.2	$Re(C_{10}') \to$ 0.28±0.12 $Im(C_{10}') \to$ -0.0030±0.3175	295.4/271	14.8	$Re(C_S) \to$ -0.027±0.279 $Im(C_S) \to$ 0.030±0.251
			C_P			C_S'
Likelihood 2020	288.52/210	2.6×10^{-2}	$Re(C_P) \to$ -0.0075±0.0135 $Im(C_P) \to$ 0.003±0.241	288.51/210	2.6×10^{-2}	$Re(C_S') \to$ -0.044±0.053 $Im(C_S') \to$ 0.0055±0.3001
Likelihood 2016	311.22/249	0.4	$Re(C_P) \to$ -0.0047±0.1564 $Im(C_P) \to$ 0.02±0.85	311.22/249	0.4	$Re(C_S') \to$ -0.04±0.17 $Im(C_S') \to$ -0.01±0.62
Moments 2016	295.2/271	15	$Re(C_P) \to$ 0.26±0.12 $Im(C_P) \to$ -0.019±0.847	295.4/271	14.8	$Re(C_S') \to$ -0.035±0.157 $Im(C_S') \to$ -0.020±0.263
			C_P'			
Likelihood 2020	288.49/210	2.6×10^{-2}	$Re(C_P') \to$ 0.0078±0.0125 $Im(C_P') \to$ -0.002±0.182			
Likelihood 2016	311.2/249	0.4	$Re(C_P') \to$ 0.007±0.013 $Im(C_P') \to$ -0.0027±0.2648			
Moments 2016	295.4/271	14.8	$Re(C_P') \to$ 0.0061±0.0135 $Im(C_P') \to$ 0.0021±0.3384			

Results (Likelihood 2020 dataset)

- With CP-asymmetric observables only in $B_s \to \phi\mu\mu$ -

χ^2_{Min}/DOF	p-value (%)	Scenario
206.517/211	57.4	$Re(\Delta C_9) \to -1.05 \pm 0.11$
202.889/210	62.5	$Re(\Delta C_9) \to -1.10 \pm 0.11$ $Im(\Delta C_9) \to 1.27^{+0.33}_{-0.43}$

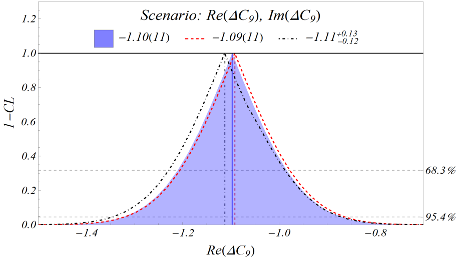
- Without any CP-asymmetric observables -

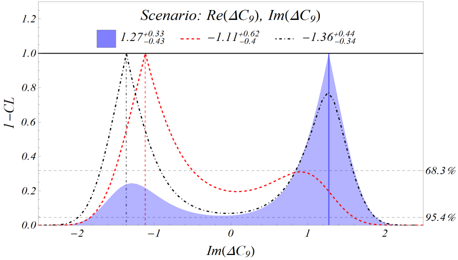
χ^2_{Min}/DOF	p-value (%)	Scenario
198.226/199	50.2	$Re(\Delta C_9) \to -1.05 \pm 0.11$
194.926/198	54.8	$Re(\Delta C_9) \to -1.11 \pm 0.12$ $Im(\Delta C_9) \to -1.36^{+0.44}_{-0.34} \cup [0.84,\ 1.59]$

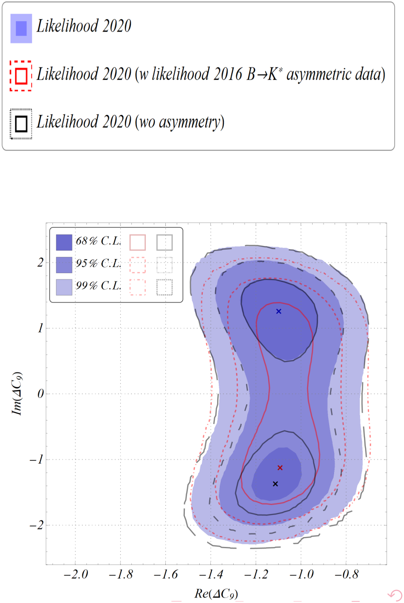
- With CP-asymmetric observables in both $B_s \to \phi\mu\mu$ and $B \to K^*\mu\mu$ (From Likelihood 2016 dataset) -

χ^2_{Min}/DOF	p-value (%)	Scenario
239.768/246	60	$Re(\Delta C_9) \to -1.06 \pm 0.11$
238.105/245	61.2	$Re(\Delta C_9) \to -1.09 \pm 0.11$ $Im(\Delta C_9) \to -1.11^{+0.62}_{-0.4}$

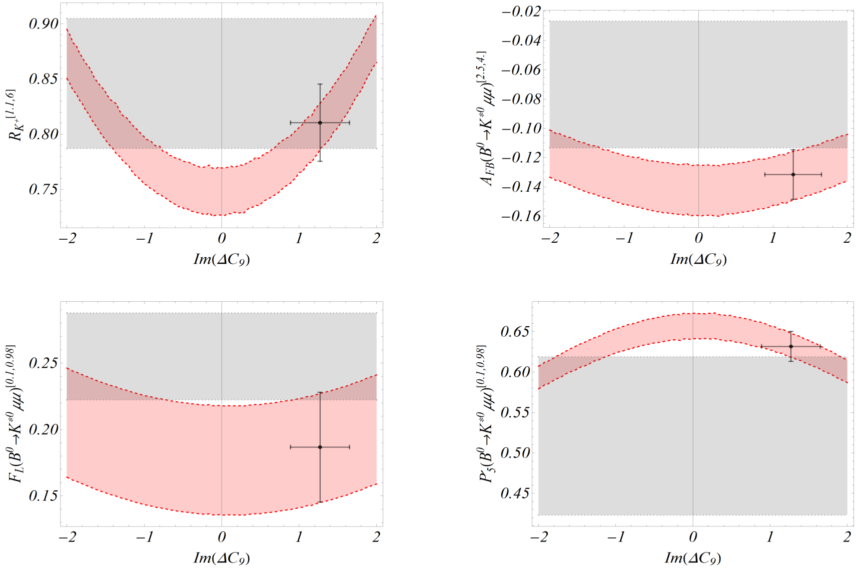
Parameter spaces



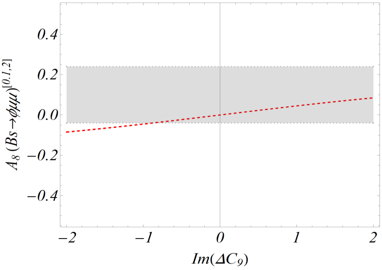


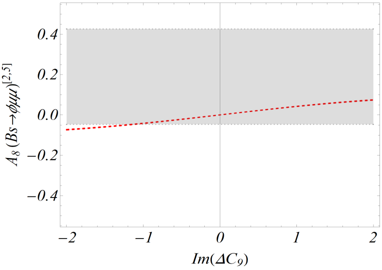


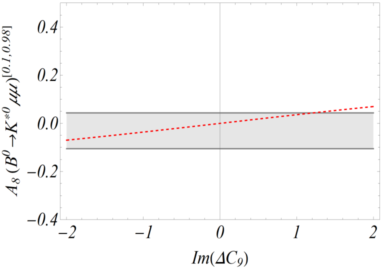
Observables sensitive to $Im(\Delta C_9)$



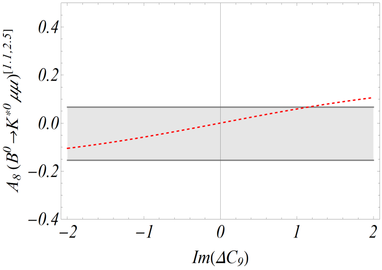
Observables responsible for sign change in $Im(\Delta C_9)$

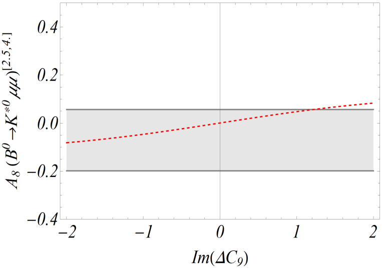


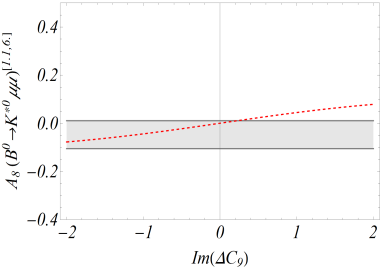




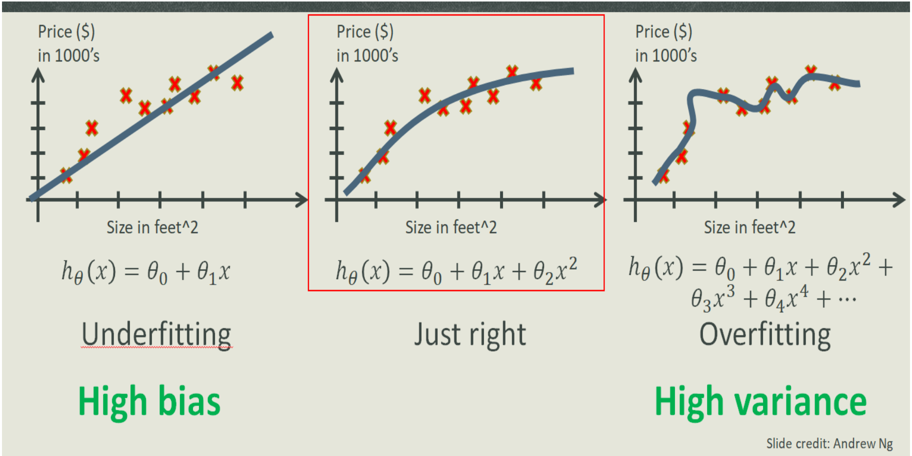
Observables responsible for sign change in $Im(\Delta C_9)$



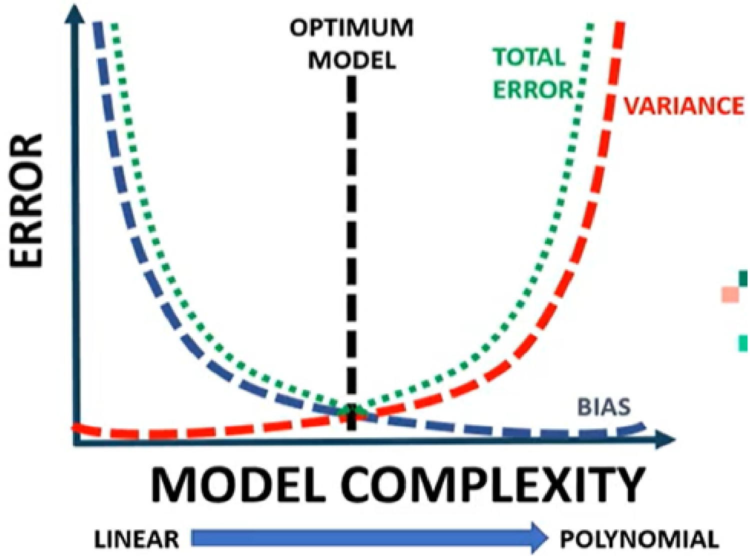




Model Selection



Model Selection



Model Selection

- **Akaike's Information Criteria -**

$$\text{AICc} = \chi^2_{min} + 2K + \frac{2K(K+1)}{n-K-1} \,,$$

n = sample size and K = no. of parameters

- **Selected models :** $\Delta AIC_c^i = AIC_c^i$ - AIC_c^{min}

Model Selection

- **Akaike's Information Criteria -**

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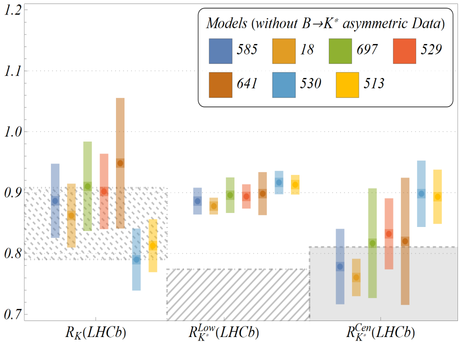
- **Selected models :** $\Delta AIC_c^i = AIC_c^i$ - AIC_c^{min}

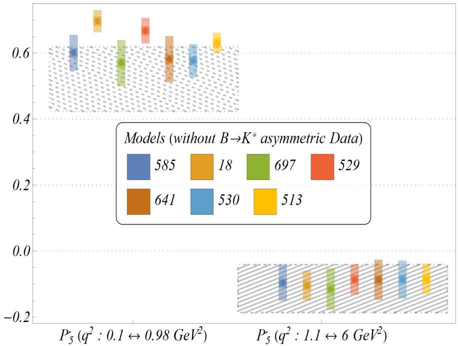
- **Cross-Validation (LOOCV) -**
  - One of the data points left out and the rest of the sample ("training set") optimized for a particular model.
  - Result used to find the predicted squared error (SE) for the left out data point.
  - Repeated for all data points and calculate MSE for the model.

Selected models (ΔAIC$c \leq 6$, $MSE_{\text{X-val}} < 1.5$)

Model	ΔAICc	$\text{MSE}_{X-\text{Val}}$	χ^2_{Min}/DOF	p-val (%)	Pull_{SM}	Result
585	0.	0.989	189.05/206	79.6	9.0	$\text{Re}(\Delta C_9) \to$ -1.36±0.24, $\text{Im}(\Delta C_9) \to$ 2.05±0.36 $\text{Re}(C_9') \to$ 0.57±0.23, $\text{Im}(C_9') \to$ 0.14±0.25 $\text{Re}(\Delta C_{10}) \to$ 0.51±0.22, $\text{Im}(\Delta C_{10}) \to$ -0.53±0.46
18	2.015	0.942	199.41/210	68.9	9.2	$\text{Re}(\Delta C_9) \to$ -1.08±0.099, $\text{Re}(C_9') \to$ 0.50±0.18
697	3.506	0.971	188.25/204	77.9	8.7	$\text{Re}(\Delta C_9) \to$ -1.4±0.24, $\text{Im}(\Delta C_9) \to$ 1.93±0.49 $\text{Re}(C_9') \to$ 0.56±0.23, $\text{Im}(C_9') \to$ 0.31±0.5 $\text{Re}(\Delta C_{10}) \to$ 0.52±0.22, $\text{Im}(\Delta C_{10}) \to$ -0.51±0.41 $\text{Re}(C_{10}') \to$ -0.032±0.177, $\text{Im}(C_{10}') \to$ 0.75±0.82
529	3.791	0.977	197.05/208	69.6	8.9	$\text{Re}(\Delta C_9) \to$ -1.11±0.11, $\text{Im}(\Delta C_9) \to$ -0.12±0.46 $\text{Re}(C_9') \to$ 0.42±0.23, $\text{Im}(C_9') \to$ -1.21±0.41
641	4.946	1.011	189.69/204	75.6	8.6	$\text{Re}(C_7') \to$ -0.0075±0.0136, $\text{Im}(C_7') \to$ -0.015±0.037 $\text{Re}(\Delta C_9) \to$ -1.07±0.13, $\text{Im}(\Delta C_9) \to$ -0.061±0.296 $\text{Re}(C_9') \to$ 0.61±0.25, $\text{Im}(C_9') \to$ -1.98±0.4 $\text{Re}(\Delta C_{10}) \to$ 0.59±0.21, $\text{Im}(\Delta C_{10}) \to$ 0.051±1.254
530	5.479	0.993	198.74/208	66.6	8.8	$\text{Re}(\Delta C_9) \to$ -1.34±0.26, $\text{Im}(\Delta C_9) \to$ 1.95±0.44 $\text{Re}(\Delta C_{10}) \to$ 0.32±0.23, $\text{Im}(\Delta C_{10}) \to$ -0.56±0.57
513	5.492	0.98	202.89/210	62.5	9.0	$\text{Re}(\Delta C_9) \to$ -1.1±0.11, $\text{Im}(\Delta C_9) \to$ 1.27±0.37

Predictions of R_{K^*} and P_5'





Conclusions

- $\mathcal{O}_9$ is the only one operator scenario with both real and complex W.Cs that is capable of explaining the present data.
- In all other one operator scenarios, the quality of fits are very poor, with the respective p-values ~ 0.
- $\mathcal{O}_9$ with complex WC, though not the best model, is the only one-operator scenario passing all the selection criteria. Some two, three and four-operator scenarios are selected as well, and all of these contain $\mathcal{O}_9$(with real or complex WC) as one of the operators.


Thank you!