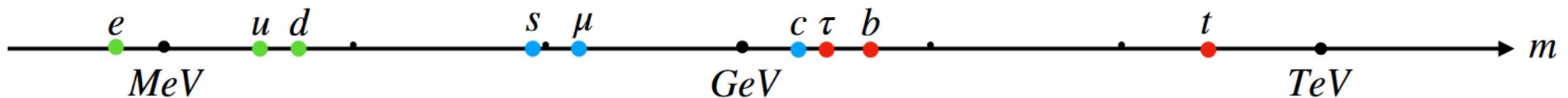


# A Unified Approach to the Flavour Puzzle

Joe Davighi, University of Zurich

Particle Physics Seminar @ University of Warwick, 10<sup>th</sup> November 2022



# Outline

1. Review of the flavour puzzle(s)
2. Flavour model building in the LHC era
  - a) Horizontal flavour symmetries
  - b) Flavour-deconstructed gauge symmetries
  - c) Unified (gauge & flavour) symmetries
3. Electroweak flavour unification via  $SU(4) \times Sp(6)_L \times Sp(6)_R$  gauge group

Davighi, Tooby-Smith, [2201.07245](#)

# The Flavour Puzzle



# The Flavour Puzzle(s)

The matter content of the Standard Model is a source of many mysteries!

**Puzzle 1:** SM fermions in 5 (6) ad hoc representations of SM gauge group:

$$\begin{array}{lll} q_L \sim (\mathbf{3}, \mathbf{2})_{1/6} & u_R \sim (\mathbf{3}, \mathbf{1})_{2/3} & d_R \sim (\mathbf{3}, \mathbf{1})_{-1/3} \\ l_L \sim (\mathbf{1}, \mathbf{2})_{-1/2} & e_R \sim (\mathbf{1}, \mathbf{1})_{-1} & \nu_R \sim (\mathbf{1}, \mathbf{1})_0 \end{array}$$

Hints at unification e.g. quark-lepton unification?  $SU(5)$ ?  $SO(10)$ ?

Traditionally, unification was done **ignoring flavour**.

(More later...)

Georgi, Glashow, [1974](#)

Georgi, [1975](#), and Fritzsche, Minkowski, [1975](#)

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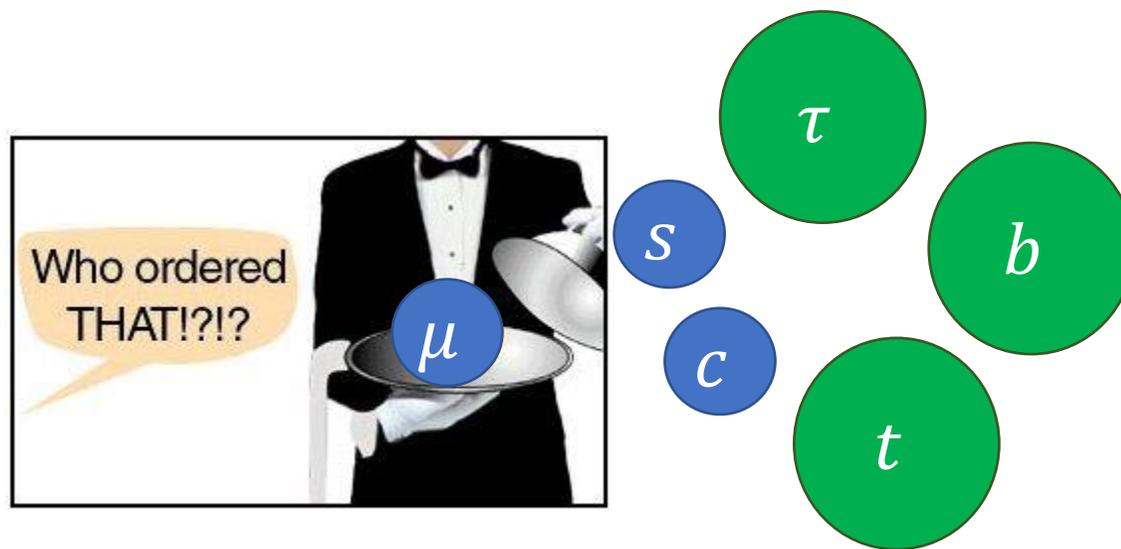


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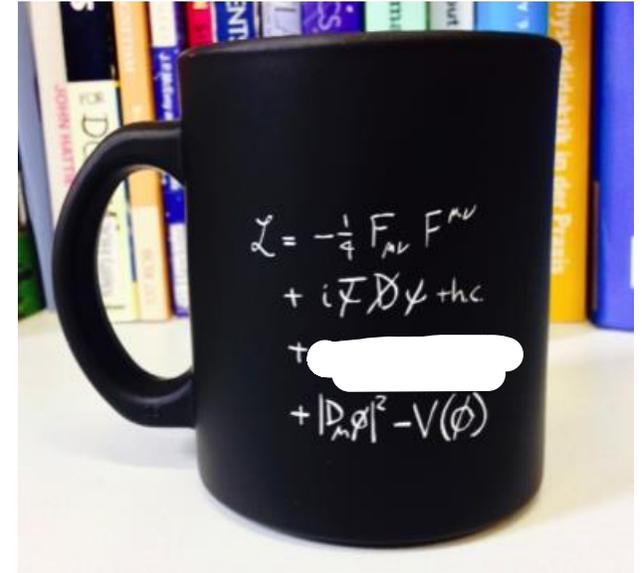
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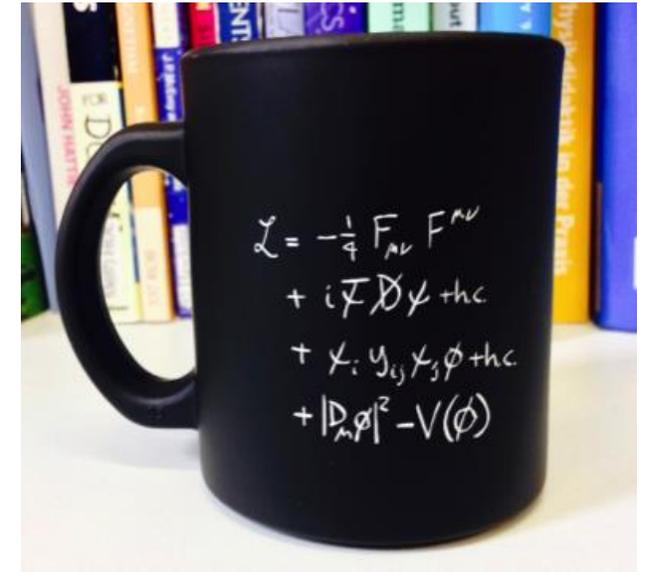
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Yukawa couplings to the Higgs ( $\mathbf{Y}$ ) break this symmetry:

$$U(3)^5 \rightarrow G_{\text{acc}} := U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$$



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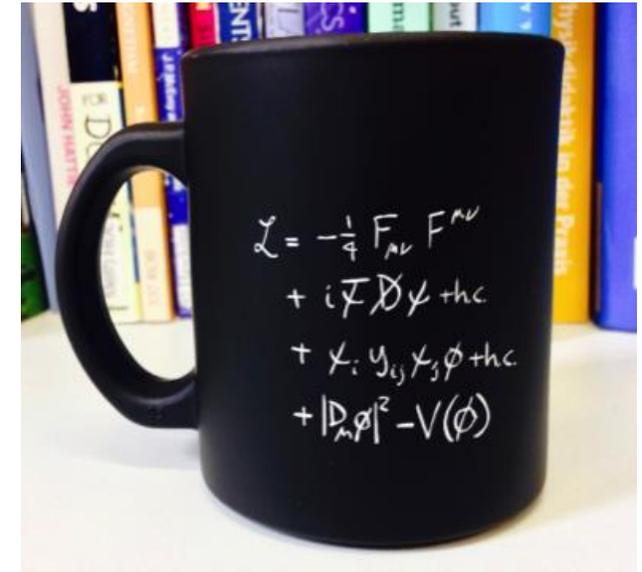
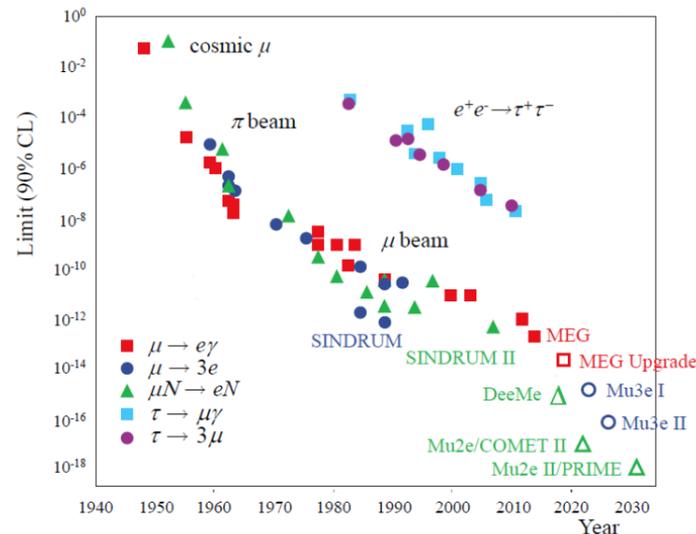
These are **very good symmetries** of Nature, probing scales  $\Lambda_{\text{acc}} \gg \text{TeV}$

Example: proton half-life

$$\tau(p \rightarrow \pi^0 e^+) \gtrsim 10^{34} \text{ yr},$$

$$\tau(p \rightarrow \bar{\nu} K^+) \gtrsim 10^{34} \text{ yr}$$

If due to dim-6 SMEFT,  $\Lambda \gtrsim 10^{13} \text{ TeV}$ .  
**Strongest bound we have on NP!**



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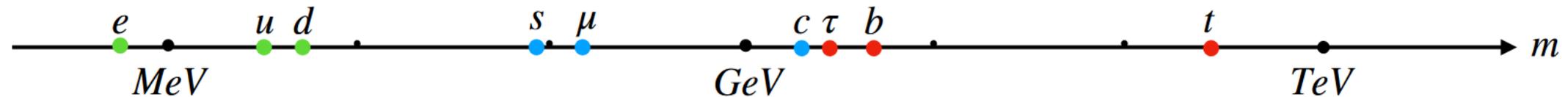
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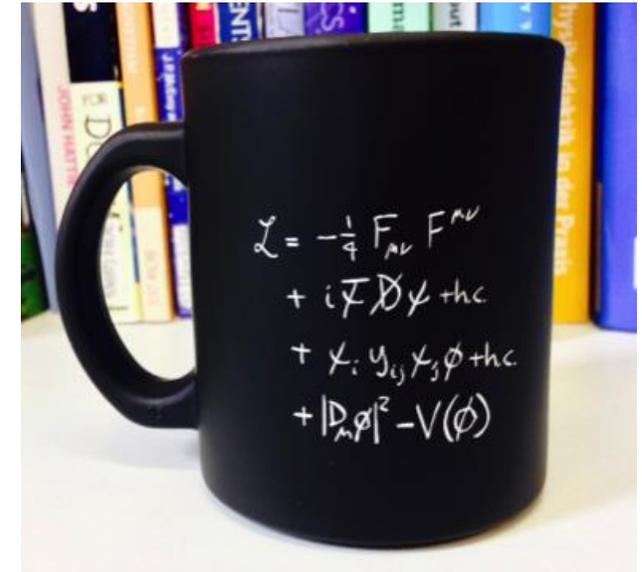
The breaking by **Y** to  $G_{\text{acc}}$  is not “random”, but very structured:

Mass hierarchies:  $m_3 \gg m_2 \gg m_1$

Small mixing angles:  $V_{us} \sim \lambda \sim 0.2, V_{cb} \sim \lambda^2, V_{ub} \sim \lambda^3$



*This structure is highly suggestive of a **dynamical BSM theory of flavour!***



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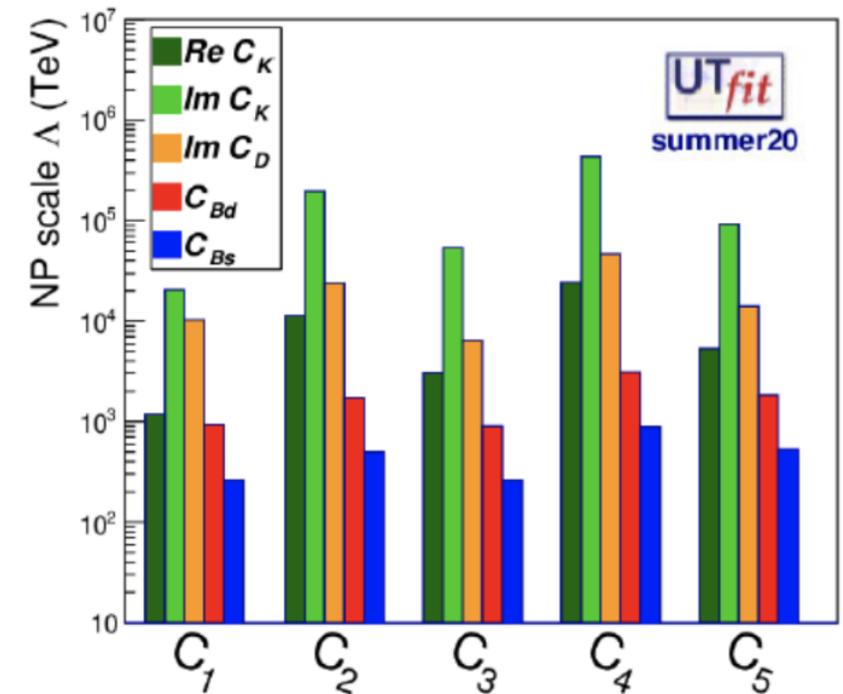
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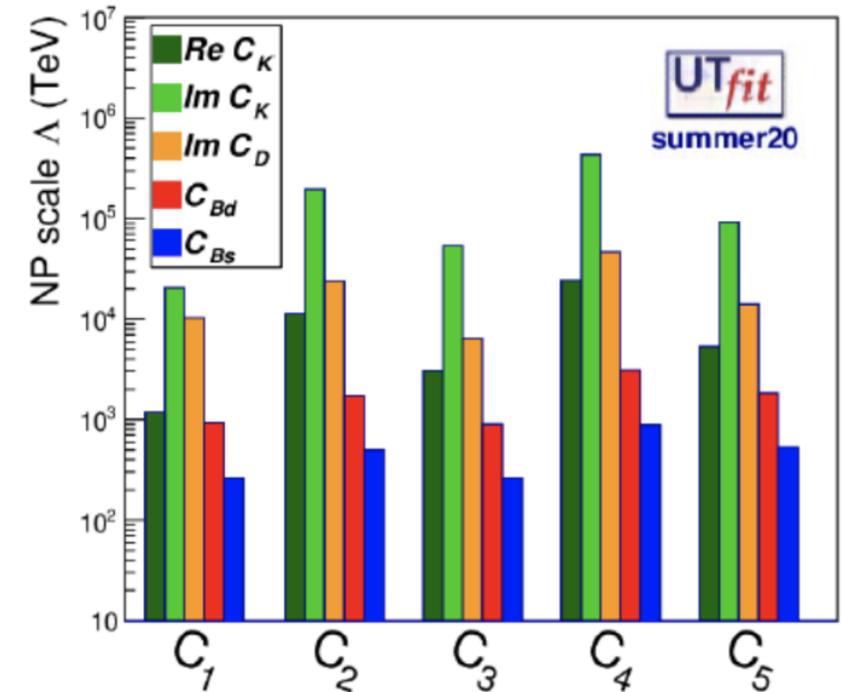
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... So, should we really have expected fireworks at the LHC? Certainly, they could not have had “generic” flavour structure. Either way, no fireworks yet...

Neutral meson mixing constraints



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# Flavour model building (in the LHC era)

# Global symmetries: a big clue for model building

Puzzle 3

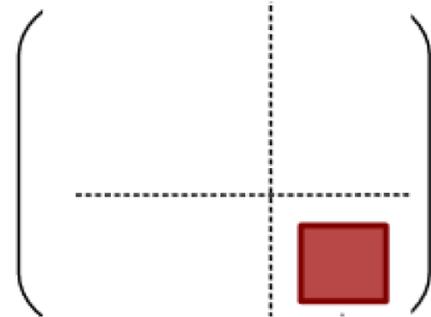
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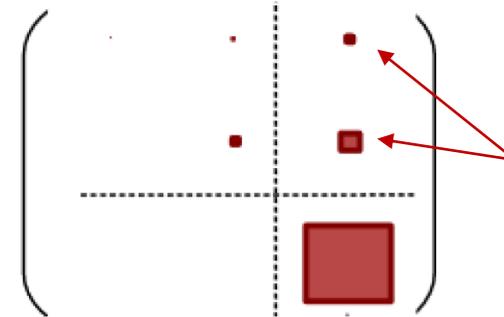
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Yukawa matrices have approximate global symmetries:  $G_{\text{approx}} = U(2)^5$  acting on light families



Exact  $U(2)$  limit

$\approx$



Observed Yukawa

$U(2)$ -breaking  
spurions

Barbieri et al, [1105.2296](#)

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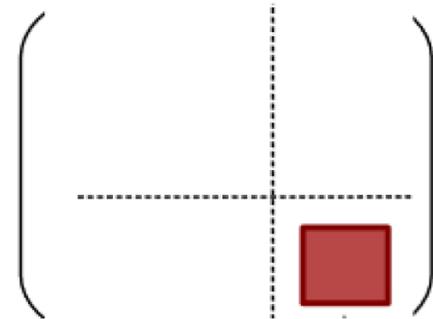
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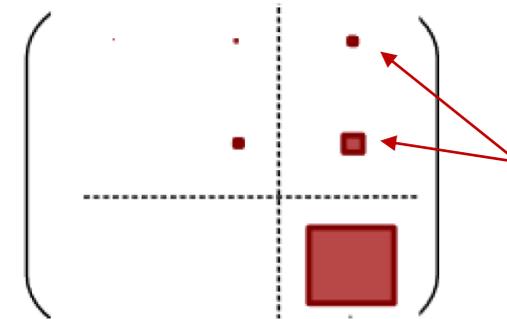
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$$\mathcal{L} \supset \bar{\psi}_3 H \psi_3 + \left(\frac{w}{\Lambda}\right)^{n_{ij}} \bar{\psi}_i H \psi_j$$

$w$  = vev of some BSM scalar

$\Lambda$  = some heavier NP scale

$n_{ij}$  = an integer

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- Perhaps as likely to see **indirect evidence of NP** in rare heavy flavoured decays e.g. at LHCb, Belle II

# Anchoring the low scale

By and large, any theory of flavour takes the form:

$$\mathcal{L} \supset \bar{\psi}_3 H \psi_3 + \left(\frac{w}{\Lambda}\right)^{n_{ij}} \bar{\psi}_i H \psi_j$$

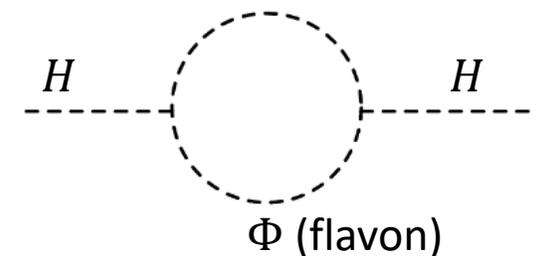
$w$  = vev of some BSM scalar  
 $\Lambda$  = some heavier NP scale  
 $n_{ij}$  = an integer

All the observed hierarchies depend only on *ratios of scales*. E.g.  $\frac{m_2}{m_3} \sim \left(\frac{w}{\Lambda}\right)^{n_{22}}$

Unfortunately, this means the scales of NP responsible for flavour *could* be very far off...  
(the old rationale for postponing the flavour puzzle)

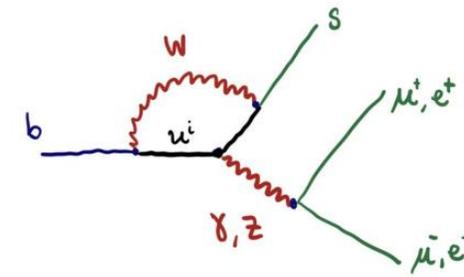
BUT, there are two good reasons for **anchoring** the low scale  $w$  not too far above TeV

1. Naturalness – a big scale separation would destabilize the Higgs
2. There are already **hints** of new flavoured physics at TeV scale, in **rare B decays**



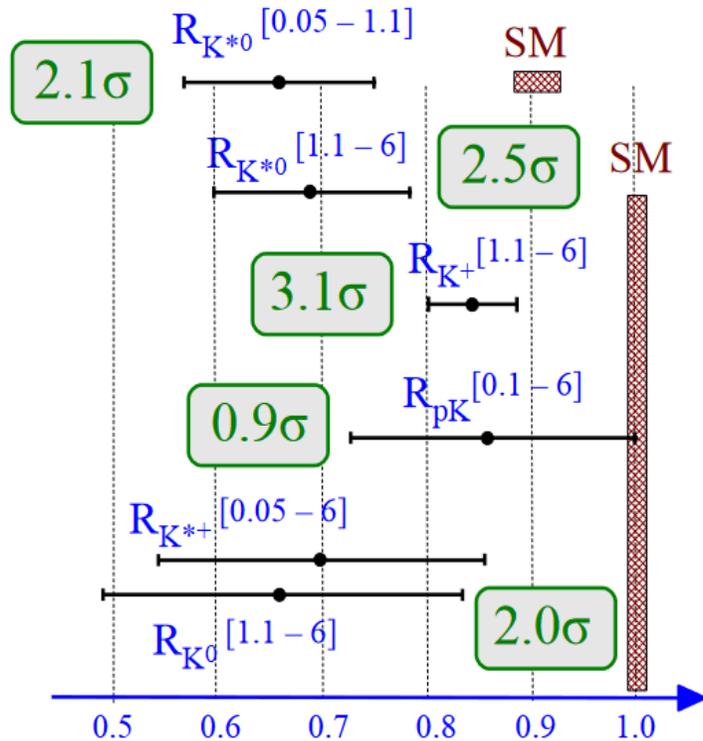
Interlude: a quick review of the flavour anomalies...

# Neutral current $b \rightarrow sll$



LFUV ratios (clean) @ LHCb

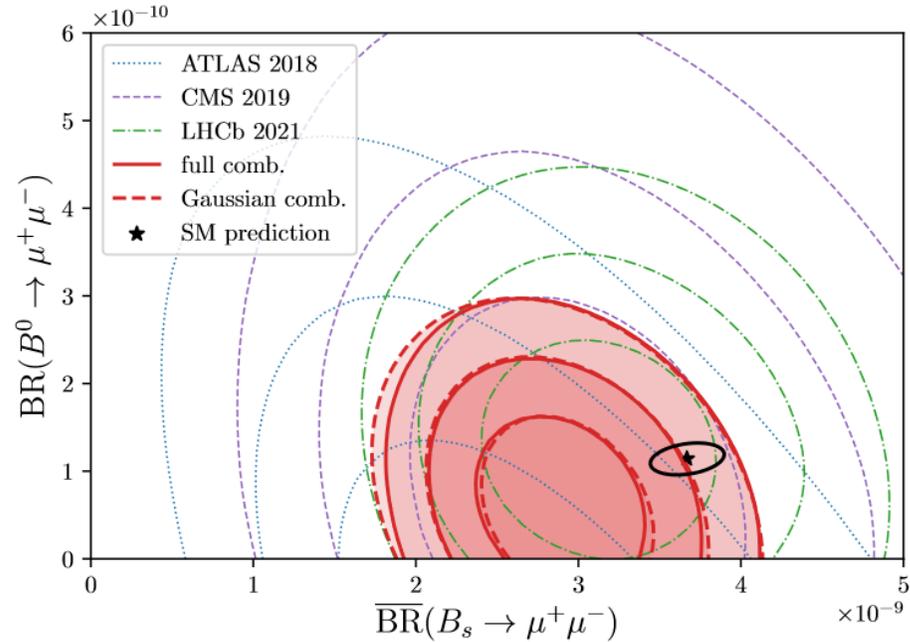
$$R_H = \frac{\text{BR}(B \rightarrow H\mu\mu)}{\text{BR}(B \rightarrow H\mu\mu)}$$



Summary from G. Isidori @ Planck 22

$\text{BR}(B_s \rightarrow \mu\mu)$  (clean)

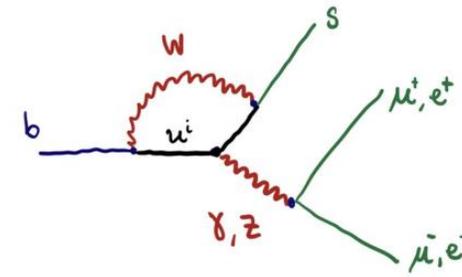
In 2021



Pull =  $2.3\sigma$

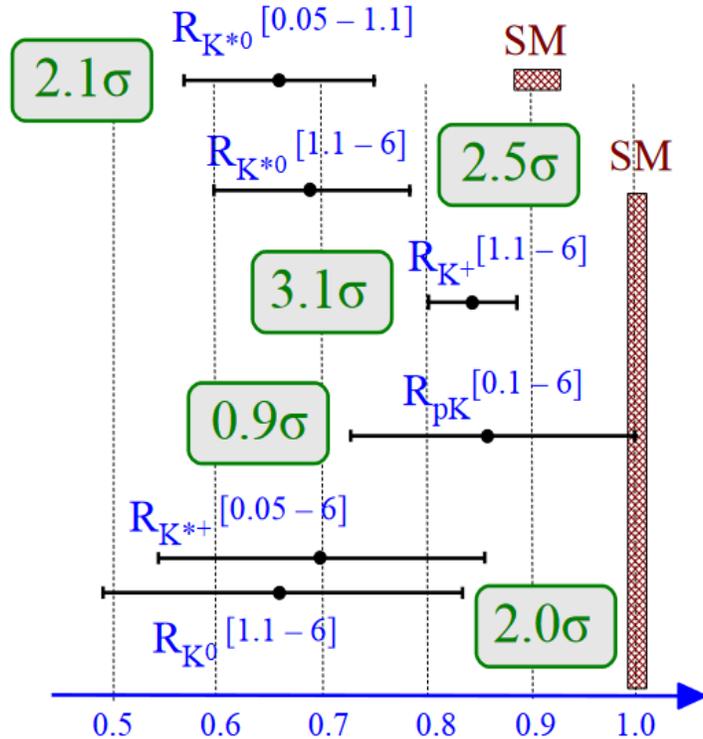
Altmannshofer, Stangl [2103.13370](https://arxiv.org/abs/2103.13370)

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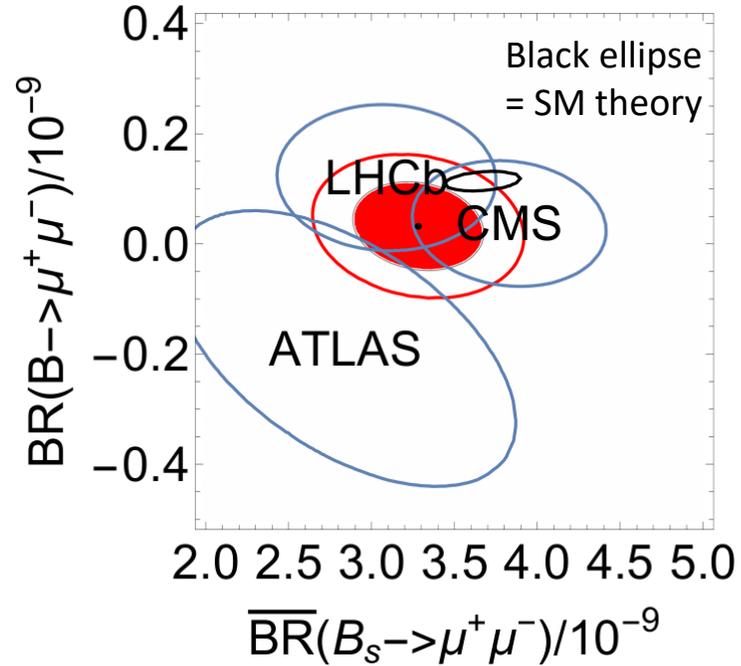
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In 2022



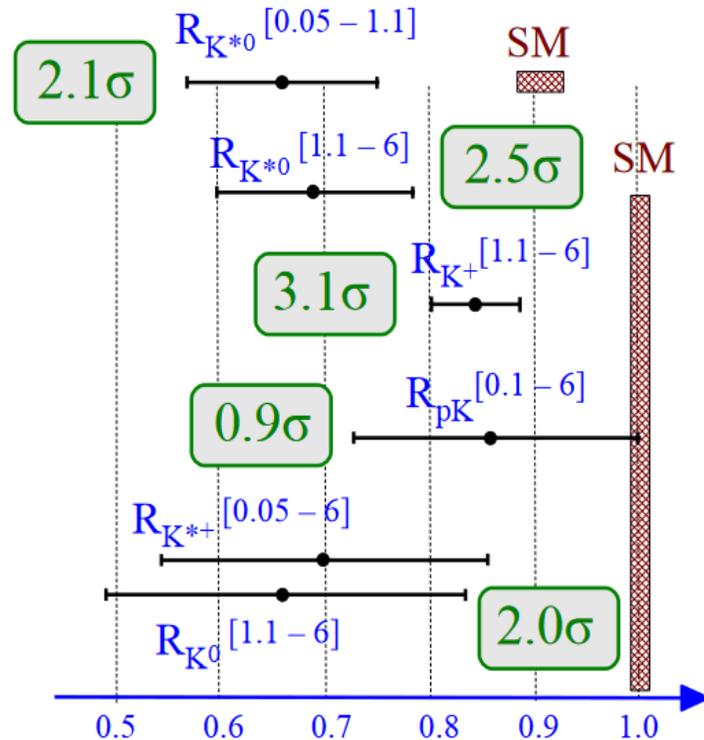
**Pull = 1.6  $\sigma$**  (agrees with SM now)

Allanach, Davighi (in progress)

# Neutral current $b \rightarrow sll$

LFUV ratios (clean) @ LHCb

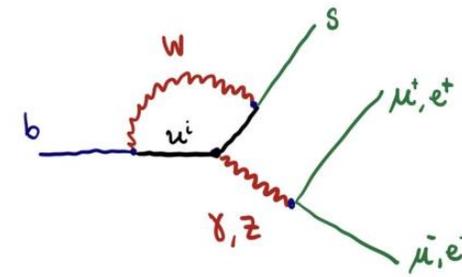
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Also tensions in muon-only observables where hadronic uncertainties in TH predictions could be unexpectedly big. E.g. angular analyses for

1.  $B \rightarrow K^* \mu\mu$
2.  $B_s \rightarrow \phi \mu\mu$



A more or less **coherent** set of deviations in  $bsll$  that can be explained by NP contribution to e.g.

$$(\overline{s_L} \gamma_\rho b_L) (\overline{\mu_L} \gamma^\rho \mu_L)$$

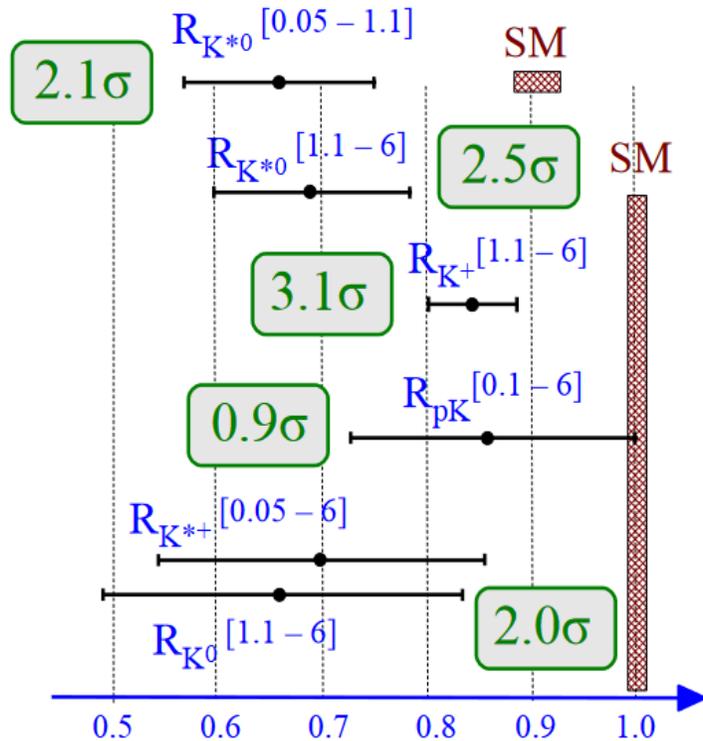
Conservative estimate in 2021: global significance  $\sim 4.3 \sigma$   
Expect similar conclusion despite  $\text{BR}(B_s \rightarrow \mu\mu)$  update

Isidori, Lancierini, Owen, Serra, 2104.05631

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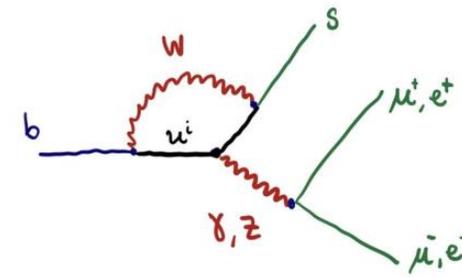
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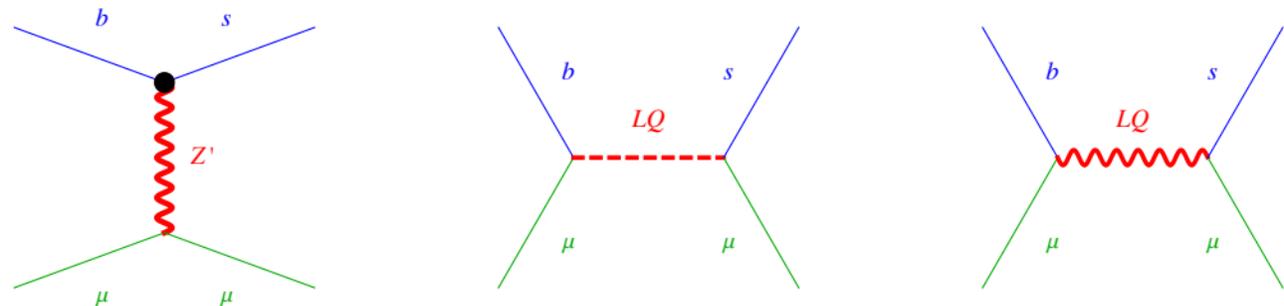
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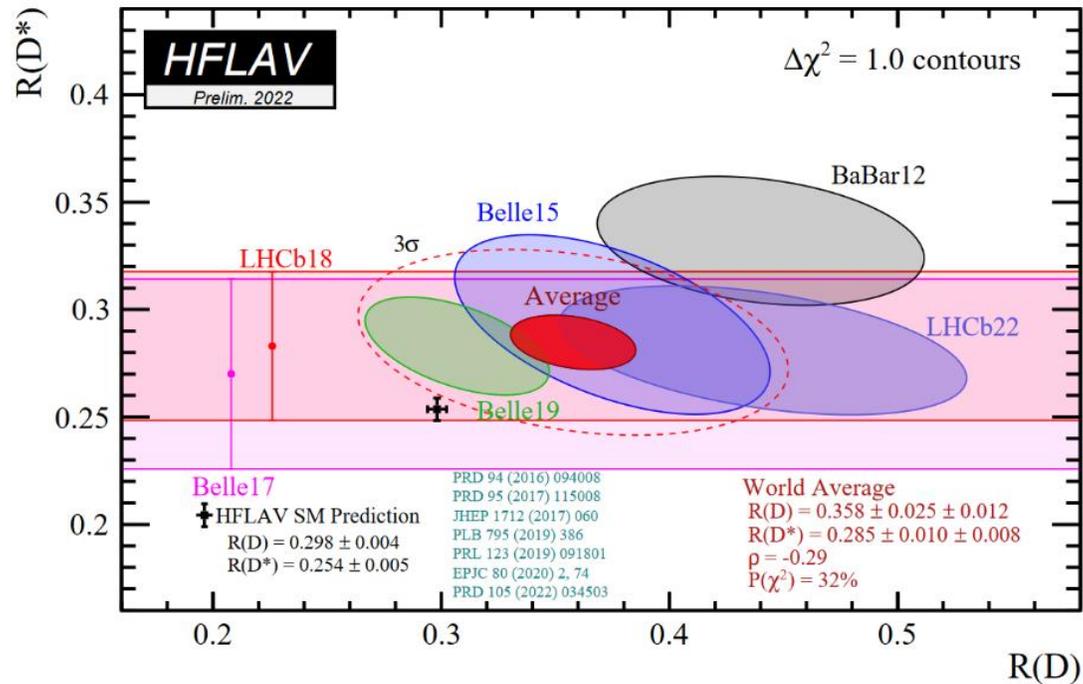
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New particle explanations. Mass/g  $\sim 3 \text{ TeV}/0.1$  (weak effect)



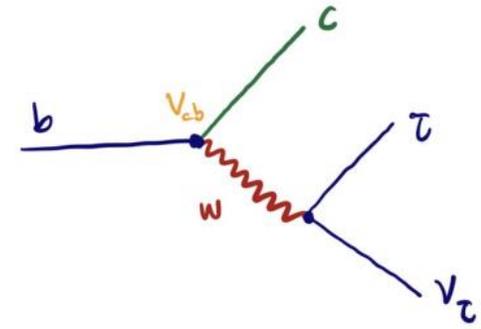
# Charged current $b \rightarrow cl\nu$



- Measurements continue to show good agreement
- Significance remains just above  $3\sigma$  after LHCb 2022

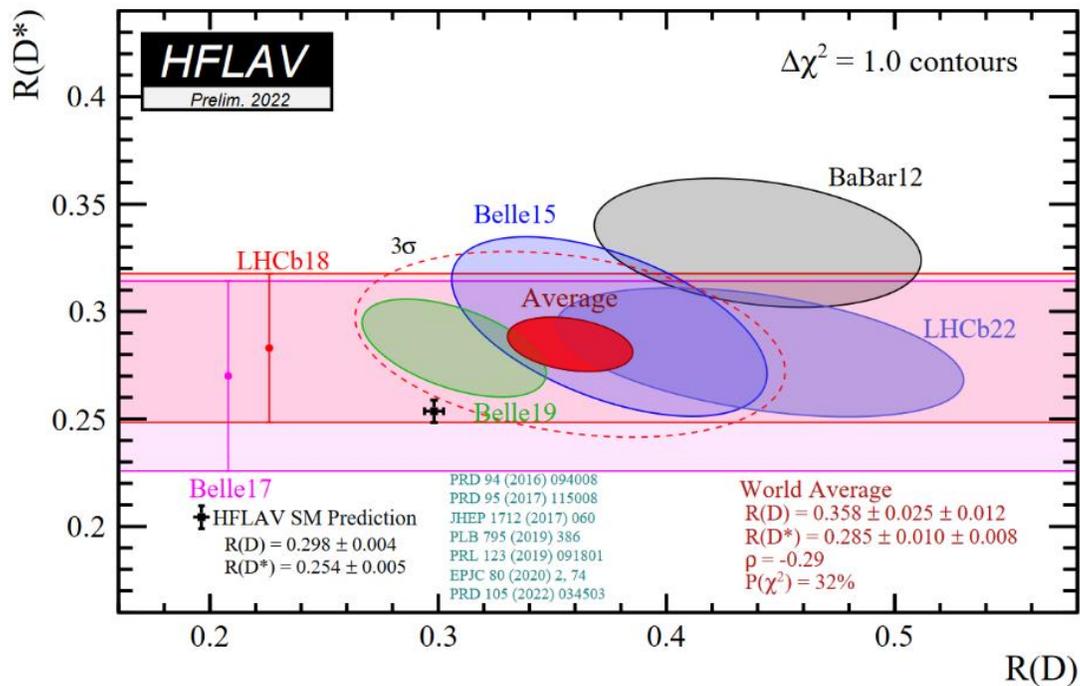
Here, LFUV ratios are

$$R_H = \frac{\text{BR}(B \rightarrow H\tau\nu)}{\text{BR}(B \rightarrow H\mu\nu)}$$

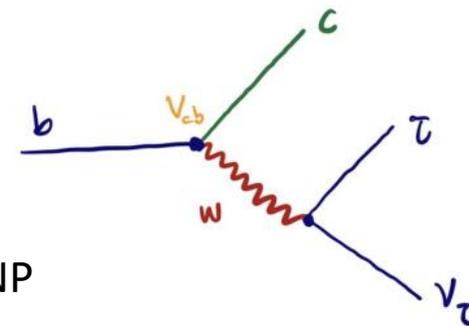


2022 update from LHCb:  
First measurement of  $R_D$  at a collider!

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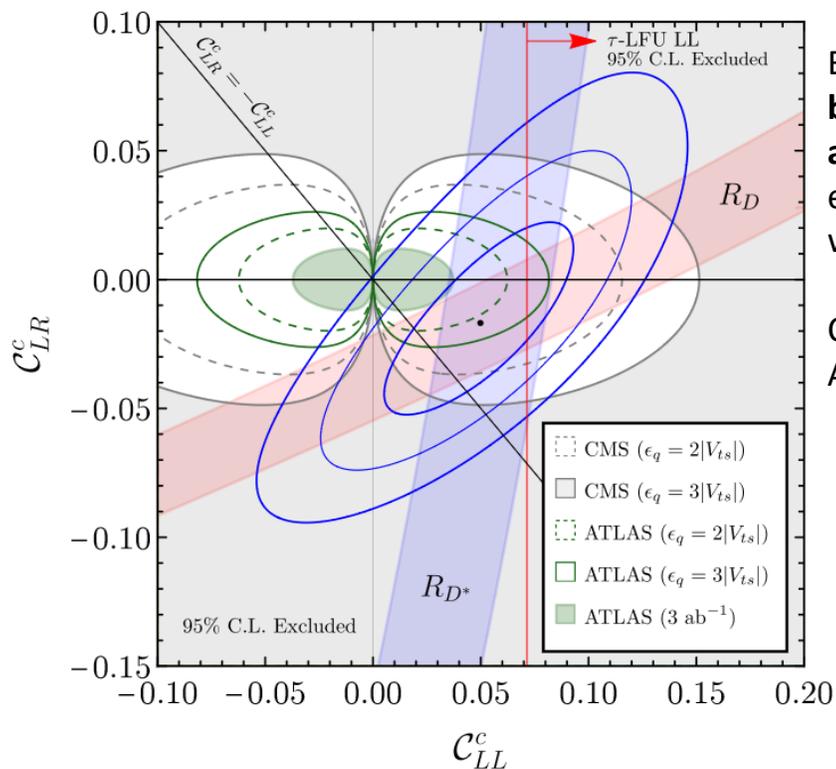


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New physics explanations:

- SM process here is tree-level, so the NP effect is BIG! **Mass/g  $\sim 3$  TeV/1**
- The  $W'$ /charged Higgs explanations ruled out by LHC high  $p_T$  &  $B_c$  lifetime
- **Leptoquark** is best explanation



Best-fit region is **already being squeezed by ATLAS and CMS**. Here are 95% exclusion from  $pp \rightarrow \tau\tau$  with b-tag

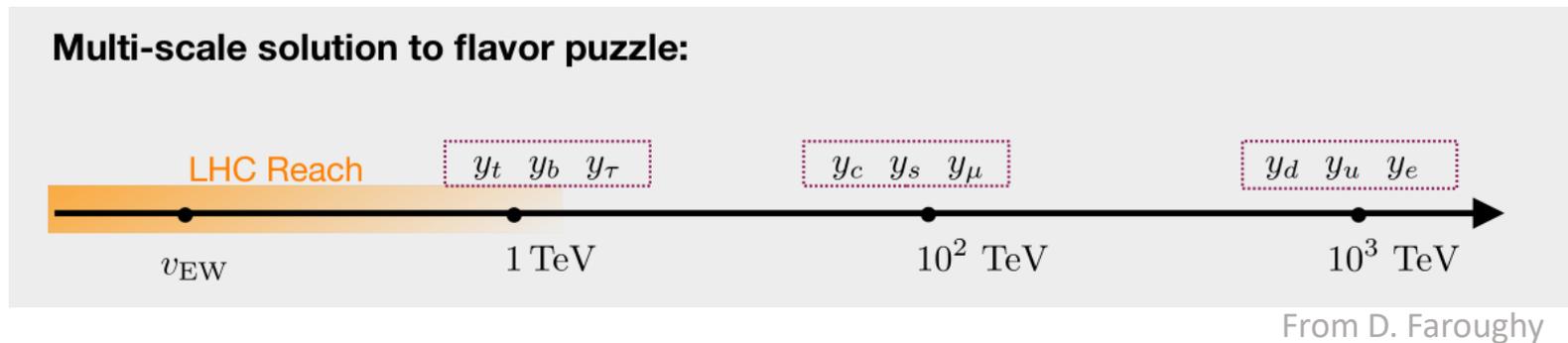
CMS: [2208.02717](#)

ATLAS: [2002.12223](#)

These anomalies are, for now, only hints of BSM.

But if they are real, they are the kind of thing we would expect at the LHC, if taking flavour puzzle seriously. Moreover, the anomalies **anchor the lowest scale** in a BSM theory of flavour  $\sim$ TeV:

Gripaios, [0910.1789](#)



This is one reason why the B anomalies are so interesting to theorists – they could be a **window onto the flavour puzzle**

With this in mind, let's return to model building for the flavour puzzle.

## Three routes for solving the flavour puzzle

1. Horizontal flavour symmetries
2. Deconstructed gauge symmetries
3. Unified (gauge & flavour) symmetries

# 1: Horizontal flavour symmetries

A horizontal symmetry *commutes with the SM gauge symmetry*

$$G = G_{\text{SM}} \times G_F$$

Flavour symmetry  $G_F$  can be continuous or discrete, abelian or non-abelian.

Gauging  $G_F$  enforces selection rules on Yukawa couplings e.g. only allowing  $Y_{33}$ .

Froggatt, Nielsen, [1979](#)  
+many more  
Grinstein et al, [1009.2049](#)

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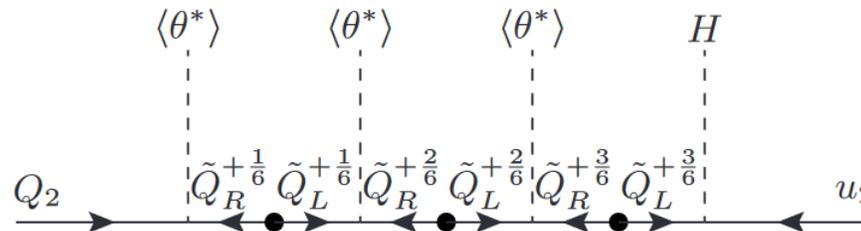
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*Example: Froggatt—Nielsen mechanism*, where  $G_F = U(1)_F$  with appropriate non-universal charges



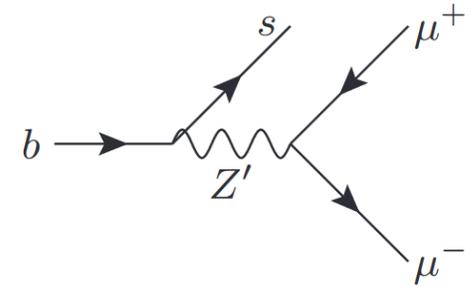
Allanach, Davighi, [1905.10327](#)

Here, the **hierarchies** come from **operator dimensions** in the low energy EFT

The heavy gauge bosons from breaking  $G_F$  will all be SM singlets  $\rightarrow$  heavy  $Z$ 's

# 1: Horizontal flavour symmetries

$$G = G_{\text{SM}} \times G_F$$



*Example 2:* Simple  $Z'$  models connecting flavour puzzle with the  $b \rightarrow sll$  anomalies

- Gauge  $G_F = U(1)_X$ ;  $X = Y_3$ . Breaking  $U(1)_X \rightarrow$  the  $Z'$

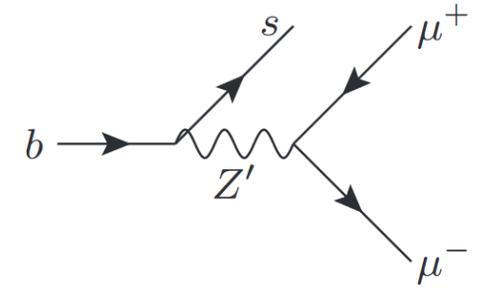
Allanach, Davighi, [1809.01158](#); [2103.12056](#); [2205.12252](#)

Only  $Y_{33}$  renormalizable:

$$\left( \begin{array}{c|c} & \\ \hline & \\ \hline & \blacksquare \end{array} \right) \approx \left( \begin{array}{c|c} \cdot & \cdot \\ \hline \cdot & \blacksquare \\ \hline & \cdot \end{array} \right)$$

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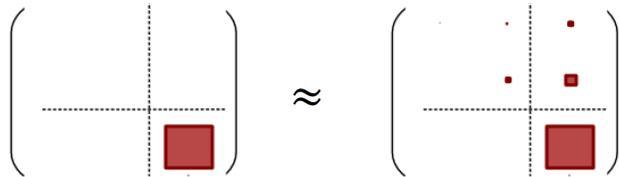


Example 2: Simple  $Z'$  models connecting flavour puzzle with the  $b \rightarrow sll$  anomalies

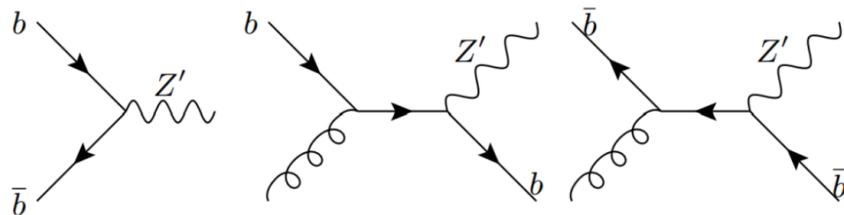
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LHC  $Z'$  production:

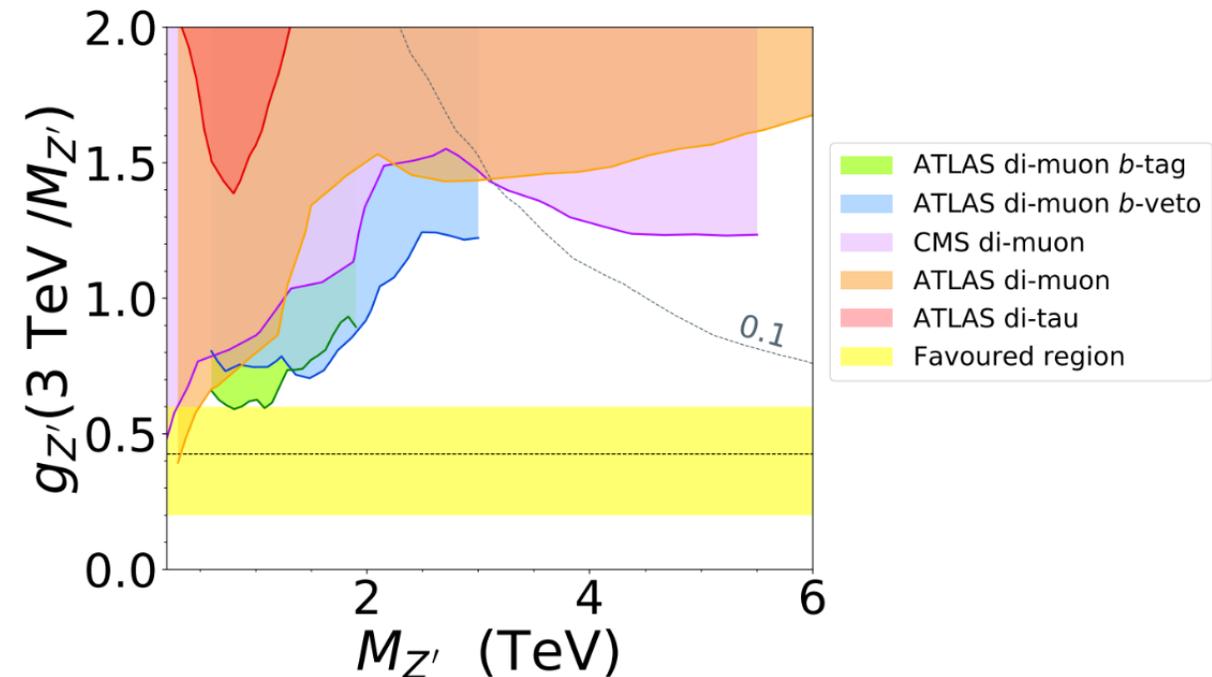


$Z'$  decay modes:

| Mode         | BR   | Mode           | BR   | Mode            | BR                          |
|--------------|------|----------------|------|-----------------|-----------------------------|
| $t\bar{t}$   | 0.42 | $b\bar{b}$     | 0.12 | $\nu\bar{\nu}'$ | 0.08                        |
| $\mu^+\mu^-$ | 0.08 | $\tau^+\tau^-$ | 0.30 | other $f_i f_j$ | $\sim \mathcal{O}(10^{-4})$ |

\*Mostly heavy flavours!

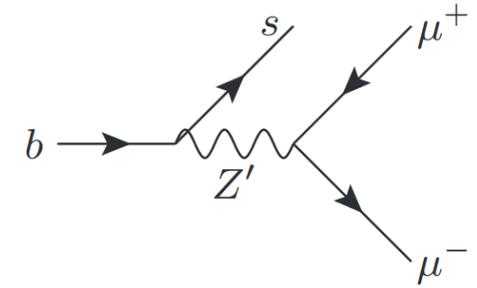
LHC  $Z'$  searches:



Allanach, Banks, [2111.06691](#)

# 1: Horizontal flavour symmetries

$$G = G_{\text{SM}} \times G_F$$



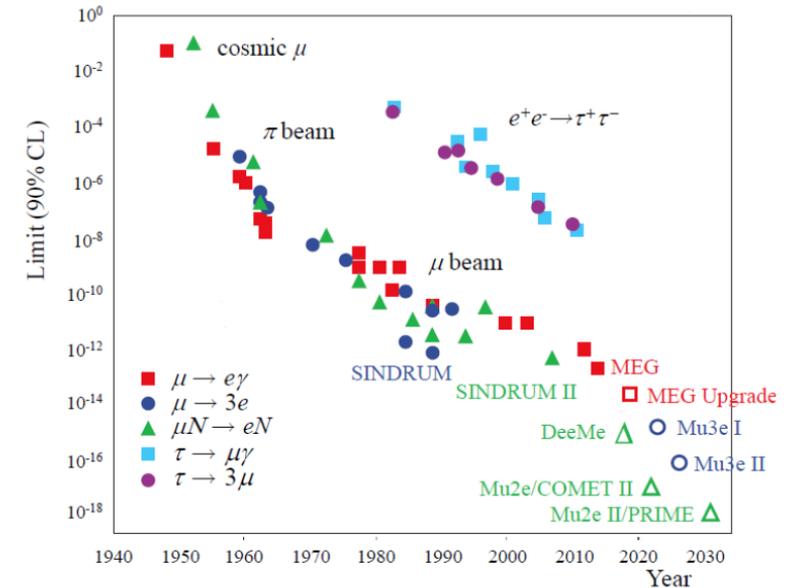
*Example 3:* Simple  $Z'$  models connecting flavour puzzle with the  $b \rightarrow sll$  anomalies + absence of LFV!

- Gauge  $G_F = U(1)_X$ ;  $X = Y_3 + a(L_2 - L_3)$ . Breaking  $U(1)_X \rightarrow$  the  $Z'$

Davighi, [2105.06918](#)

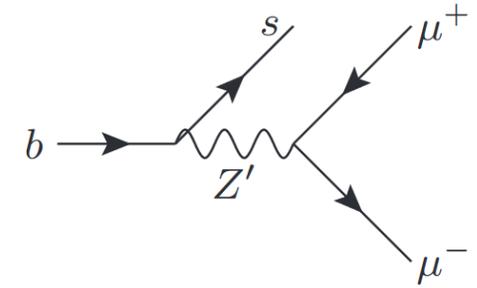
$$Y_u = y_t \begin{pmatrix} \frac{\Delta_u^{ab} \Phi}{\Lambda^2} & \frac{V_q^a}{\Lambda} \\ 0 & 1 \end{pmatrix} \quad Y_e = \begin{pmatrix} c_e \frac{e_\Phi^3}{\Lambda^3} & 0 & 0 \\ 0 & c_\mu \frac{e_\Phi^3}{\Lambda^3} & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

$$Y_d = y_b \begin{pmatrix} \frac{\Delta_d^{ab} \Phi}{\Lambda^2} & \frac{V_q^a}{\Lambda} \\ 0 & 1 \end{pmatrix} \quad \text{Quark masses + } b \rightarrow sll \\ \Lambda \sim 100 \text{ TeV}$$



# 1: Horizontal flavour symmetries

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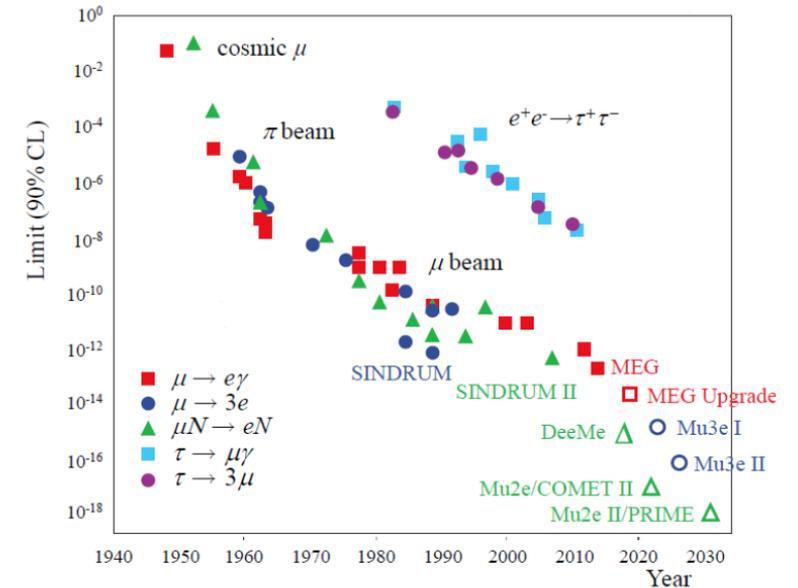
**Example 3:** Simple \$Z'\$ models connecting **flavour puzzle** with the  $b \to sll$  anomalies + **absence of LFV!**

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Davighi, [2105.06918](#)

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By gauging a combination of lepton numbers, we have **excellent protection against LFV**, despite LFUV!

$\mu \to e\gamma$ , due to dim >12 operators. Need

$$\frac{\Lambda}{\sqrt{c}} \epsilon_\Phi^{-\frac{a-3}{2}} \gtrsim 58\,000 \text{ TeV} \quad (\text{Satisfied for order-1 WCs})$$

$l_j \to 3l_i$ , due to dim >15 operators.

$$\Delta BR(\mu \to 3e) \sim \frac{m_\mu^5}{768\pi^3\Gamma_\mu} \frac{1}{\Lambda^4} \epsilon^{2a} \lesssim 10^{-29} \quad \text{etc tiny!!}$$

## 2: Deconstructed gauge symmetries

The SM gauge symmetry, which is flavour-universal, could be *deconstructed* in the UV:

$$G = G_1 \times G_2 \times G_3$$

SM Fermions:

$$\psi_1 \sim (\mathbf{R}, 1, 1)$$

$$\psi_2 \sim (1, \mathbf{R}, 1)$$

$$\psi_3 \sim (1, 1, \mathbf{R})$$

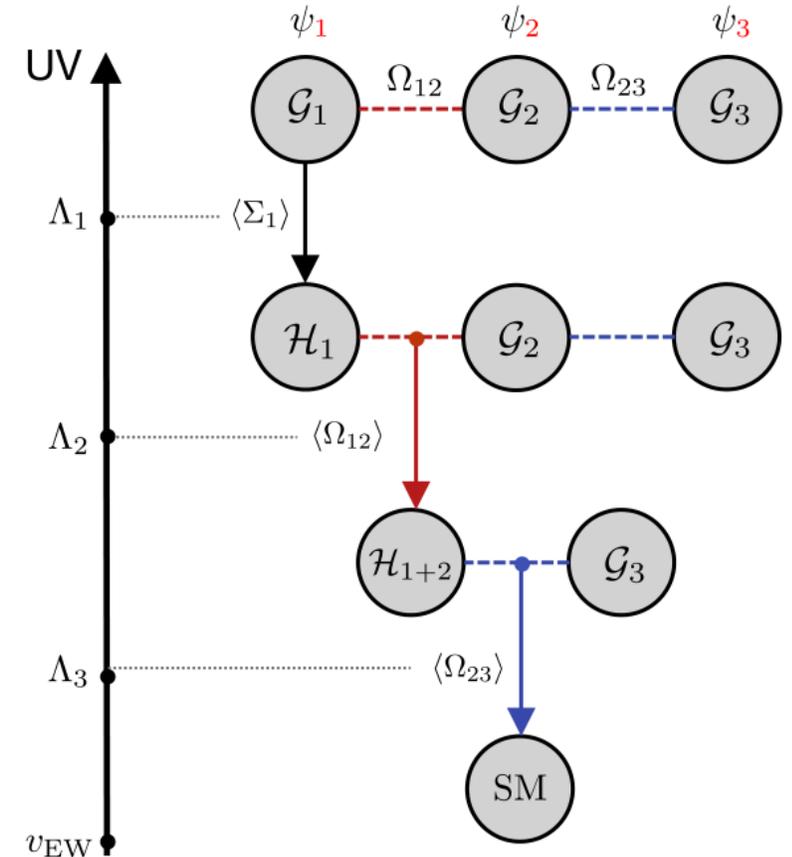
Multi-scale symmetry breaking pattern generates structure:

The **hierarchies** can come from different operator dimensions, and/or from **different scales associated with each family**

$$\Lambda_1 > \Lambda_2 > \Lambda_3$$

The “ladder of scales” does not destabilize Higgs mass!

Allwicher, Isidori, Thomsen, [2011.01946](https://arxiv.org/abs/2011.01946)



## 2: Deconstructed gauge symmetries

$$G = G_1 \times G_2 \times G_3$$

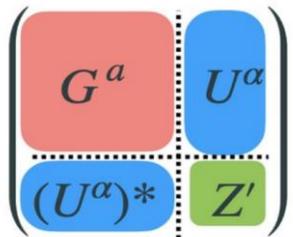
Example: 'Pati—Salam cubed' model: [Bordone et al. 1712.01368](http://1712.01368), [1805.09328](http://1805.09328)

$$G_i = [SU(4) \times SU(2)_L \times SU(2)_R]_i$$

The Pati-Salam  $SU(4)$  also **unifies quarks and leptons (puzzle 1!)**

[Pati, Salam, 1974](http://Pati, Salam, 1974)

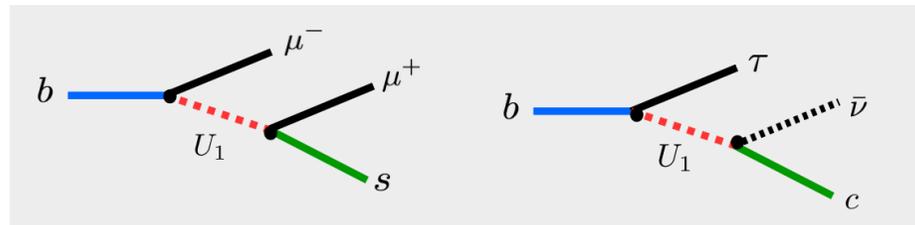
Symmetry breaking via the '4-3-2-1 model':



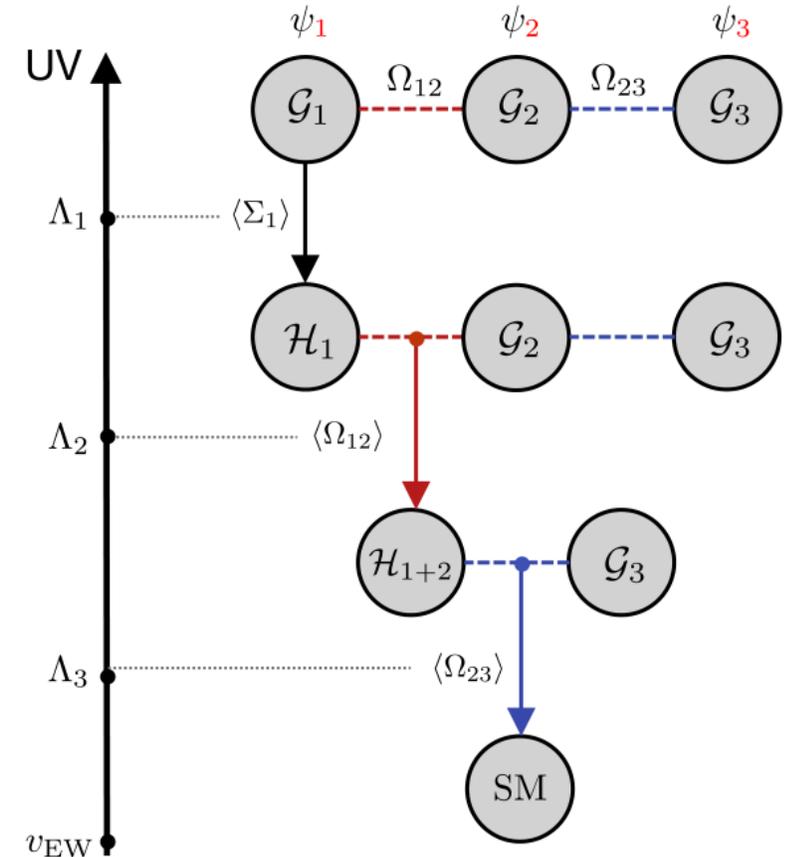
$$G \rightarrow SU(4)_3 \times SU(3)_{1+2} \times SU(2)_L \times U(1)_{Y'} \rightarrow G_{\text{SM}}$$

gives  $U_1$  leptoquark coupled mostly to 3<sup>rd</sup> family

→ only single mediator for both  $b \rightarrow sll$  and  $b \rightarrow c\tau\nu$  anomalies!



[Butazzo et al, 1706.07808](http://Butazzo et al, 1706.07808)  
[Angelescu et al, 1808.08179](http://Angelescu et al, 1808.08179)

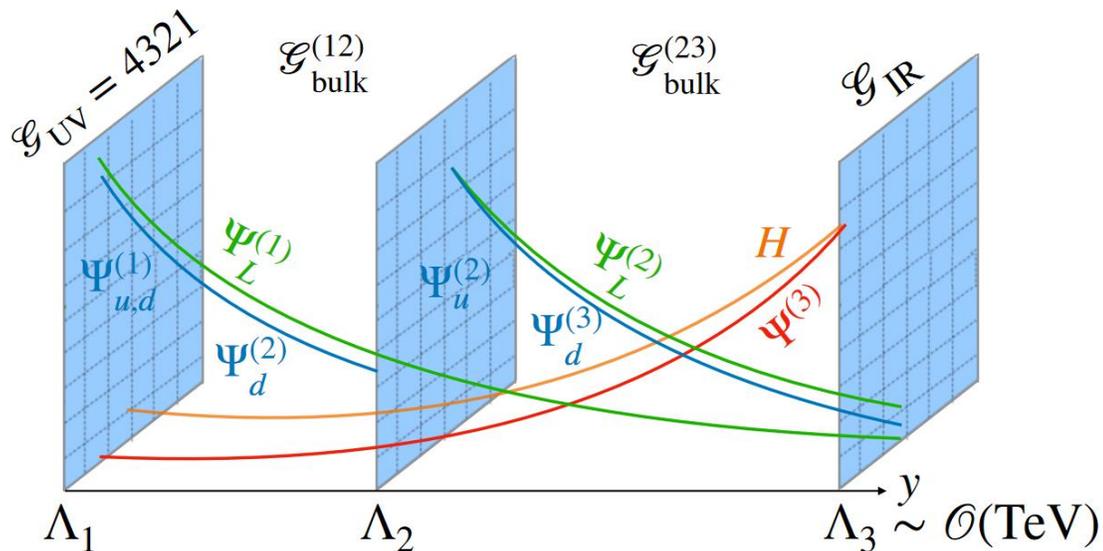


## 2: Deconstructed gauge symmetries

$$G = G_1 \times G_2 \times G_3$$

The origin of deconstruction? Flavour as a **5<sup>th</sup> dimension**, with a 4D brane for each family

Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, [2203.01952](#)

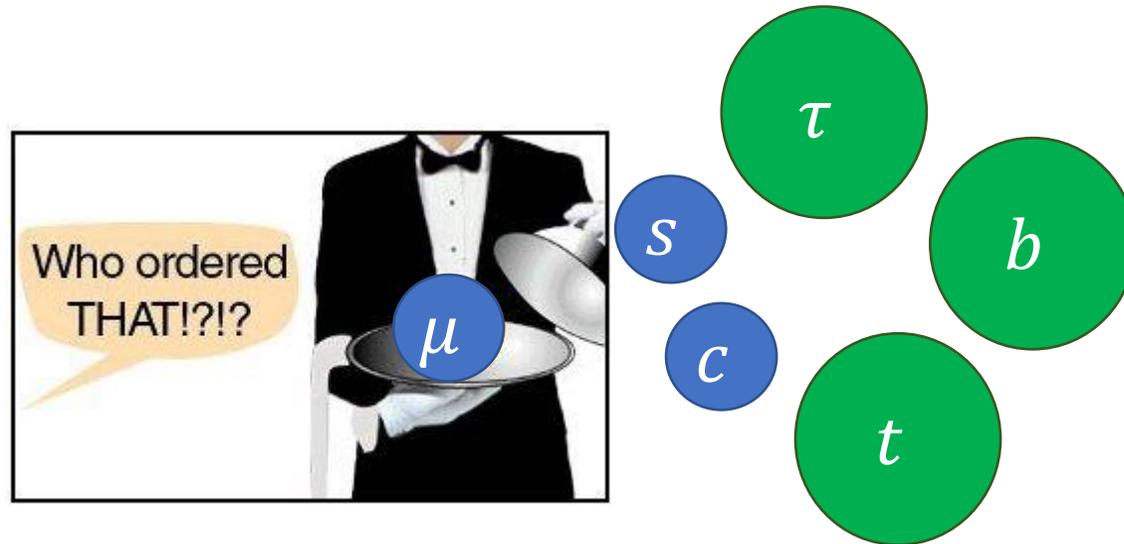


Higgs embedded as a composite pNGB!

→ Flavour puzzle & hierarchy problem solved together at TeV scale, while unifying quarks and leptons, consistent with all LHC constraints: **Puzzles 1, 3 and 4** ✓

### 3: Unifying gauge and flavour symmetries

None of the previous approaches address **Puzzle 2**: why 3 families in the first place?  
(In the 5d model, “why 3 families?” becomes “why 3 branes?”)



### 3: Unifying gauge and flavour symmetries

None of the previous approaches address **Puzzle 2**: why 3 families in the first place?

#### Gauge-flavour unification

- **3 generations** of SM fermions start life as **one particle** in the UV
- This UV fermion is acted on by a **large gauge symmetry**  $G_{123}$
- So in the UV, different generations are indistinguishable – **opposite to deconstruction** approach!
- At intermediate energies,  $G_{123}$  spontaneously broken: **flavour emerges** as a low-energy remnant



# Electroweak Flavour Unification

Davighi, Tooby-Smith, [2201.07245](#)

Davighi, [2206.04482](#)

# Gauge Flavour Unification

First challenge: embed the SM in an **anomaly-free gauge theory** that unifies the generations

$$G_{123} = ??$$

# Gauge Flavour Unification

First challenge: embed the SM in an **anomaly-free gauge theory** that unifies the generations

A comprehensive analysis of Lie algebras reveals it is not possible to unify either  $SU(5)$  or  $SO(10)$  GUT with flavour.

Allanach, Gripaios, Tooby-Smith, [2104.14555](#)

To unify gauge and flavour symmetries, we should start from **Pati-Salam** gauge group:

$$SU(4) \times SU(2)_L \times SU(2)_R$$
$$\Psi_L \sim (\mathbf{4}, \mathbf{2}, \mathbf{1})^{\oplus 3}, \quad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{2})^{\oplus 3}$$

Pati, Salam, [1974](#)

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Pati, Salam, [1974](#)

Two options:

1. Unify colour and flavour:  $SU(4) \rightarrow SU(12)$ :  $\Psi_L \sim (\mathbf{12}, \mathbf{2}, \mathbf{1}), \Psi_R \sim (\mathbf{12}, \mathbf{1}, \mathbf{2})$
2. Unify electroweak and flavour:  $SU(2)_L \times SU(2)_R \rightarrow Sp(6)_L \times Sp(6)_R$ :  $\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}), \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6})$

# Gauge Flavour Unification

*Reminder:*

The Lie group  $Sp(6)$  is a subgroup of  $SU(6)$ :

$$Sp(6) = \{U \in SU(6) | U^T \Omega U = \Omega\}, \text{ where } \Omega = \begin{pmatrix} 0 & I_3 \\ -I_3 & 0 \end{pmatrix}$$

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# Electroweak Flavour Unification (EWFU)

- Gauge group:  $G_{123} = SU(4) \times Sp(6)_L \times Sp(6)_R$
- SM fermions:

$$\Psi_L \sim (\mathbf{4}, \mathbf{6}, \mathbf{1}) \sim \begin{pmatrix} u_1^r & u_2^r & u_3^r & d_1^r & d_2^r & d_3^r \\ u_1^g & u_2^g & u_3^g & d_1^g & d_2^g & d_3^g \\ u_1^b & u_2^b & u_3^b & d_1^b & d_2^b & d_3^b \\ \nu_1 & \nu_2 & \nu_3 & e_1 & e_2 & e_3 \end{pmatrix}, \quad \Psi_R \sim (\mathbf{4}, \mathbf{1}, \mathbf{6}) \sim \text{similar}$$

# Electroweak Flavour Unification (EWFU)

By unifying all matter, such a gauge theory explains **puzzles 1** and **2** “out of the box”.

But with so much unification, can we also explain **puzzle 3**?

Puzzle 3

Mass hierarchies:

$$m_3 \gg m_2 \gg m_1$$

Small mixing angles:

$$V_{us} \sim \lambda \sim 0.2, \quad V_{cb} \sim \lambda^2, \quad V_{ub} \sim \lambda^3$$

# EWFU: generating the Yukawa structure

Two steps:

1. Flavour **deconstruction** of  $G_{123}$  at very high scale

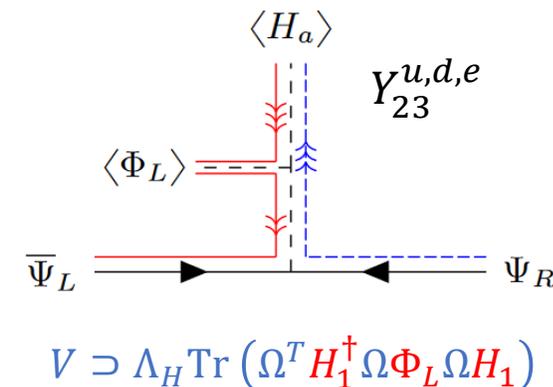
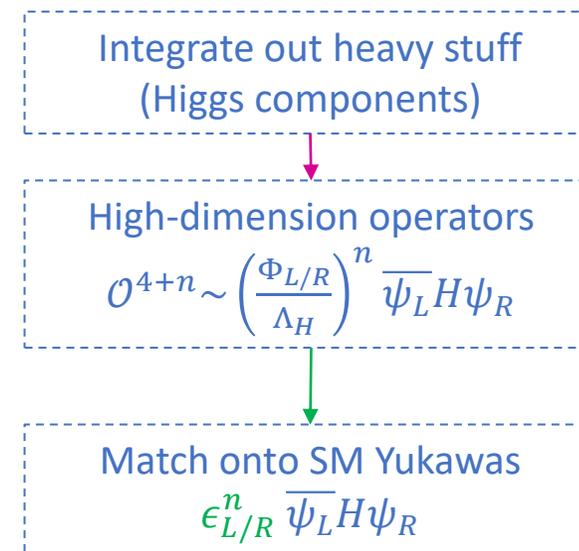
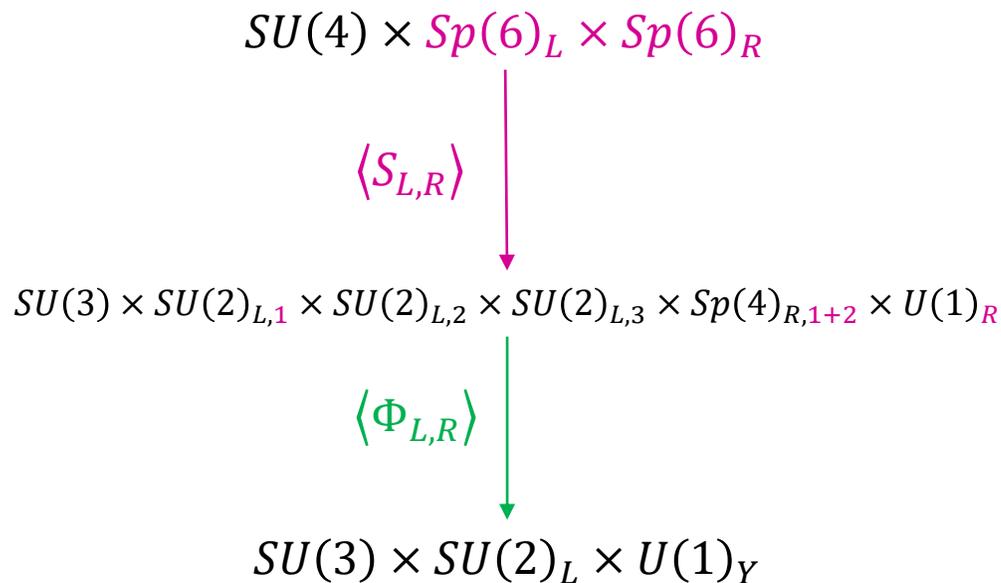
2. Break the flavour non-universal intermediate symmetry to  $G_{SM}$

$\langle \Phi_{L,R} \rangle$  have components that *link* families together. 4 independent vevs:

$$\epsilon_L^{12}, \epsilon_L^{23}, \epsilon_R^{12}, \epsilon_R^{23}$$

The scalar sector is almost minimal:

$$S_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1}), S_R \sim (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{6}), \Phi_L \sim (\mathbf{1}, \mathbf{14}, \mathbf{1}), \Phi_R \sim (\mathbf{1}, \mathbf{1}, \mathbf{14})$$



# EWFU: quark masses and mixings

Puzzle 3

Mass hierarchies:  $m_3 \gg m_2 \gg m_1$   
 Small mixing angles:  $V_{us} \sim \lambda \sim 0.2, V_{cb} \sim \lambda^2, V_{ub} \sim \lambda^3$

EWFU model generates  
 Yukawa structures:

$$\frac{M^f}{v} \sim \begin{pmatrix} \epsilon_L^{12} \epsilon_L^{23} \epsilon_R^{12} \epsilon_R^{23} & \epsilon_L^{12} \epsilon_L^{23} \epsilon_R^{23} & \epsilon_L^{12} \epsilon_L^{23} \\ \epsilon_L^{23} \epsilon_R^{12} \epsilon_R^{23} & \epsilon_L^{23} \epsilon_R^{23} & \epsilon_L^{23} \\ \epsilon_R^{12} \epsilon_R^{23} & \epsilon_R^{23} & 1 \end{pmatrix}$$

Extract observables using perturbation theory:

$$\begin{aligned} m_1 &\sim \epsilon_L^{12} \epsilon_R^{12} \epsilon_L^{23} \epsilon_R^{23}, & V_{ub} &\sim \epsilon_L^{12} \epsilon_L^{23} \\ m_2 &\sim \epsilon_L^{23} \epsilon_R^{23}, & V_{cb} &\sim \epsilon_L^{23} \\ m_3 &\sim 1, & V_{us} &\sim \epsilon_L^{12} \end{aligned}$$

# EWFU: quark masses and mixings

Puzzle 3

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$$\begin{aligned} V_{ub} &\sim \epsilon_L^{12} \epsilon_L^{23} \\ V_{cb} &\sim \epsilon_L^{23} \\ V_{us} &\sim \epsilon_L^{12} \end{aligned}$$

Mixing angles  $\rightarrow \epsilon_L^{12} \sim \lambda, \epsilon_L^{23} \sim \lambda^2$

Mass hierarchies  $\rightarrow \epsilon_R^{12} \sim \lambda^2, \epsilon_R^{23} \sim \lambda$

Corresponds to a ladder of symmetry breaking scales separated by steps of  $\frac{1}{\lambda} \sim 5 \sim \mathcal{O}(1)$

... And there is **enough freedom** in the EFT coefficients to fit all the data



Work in progress

# Phenomenology of EWFU

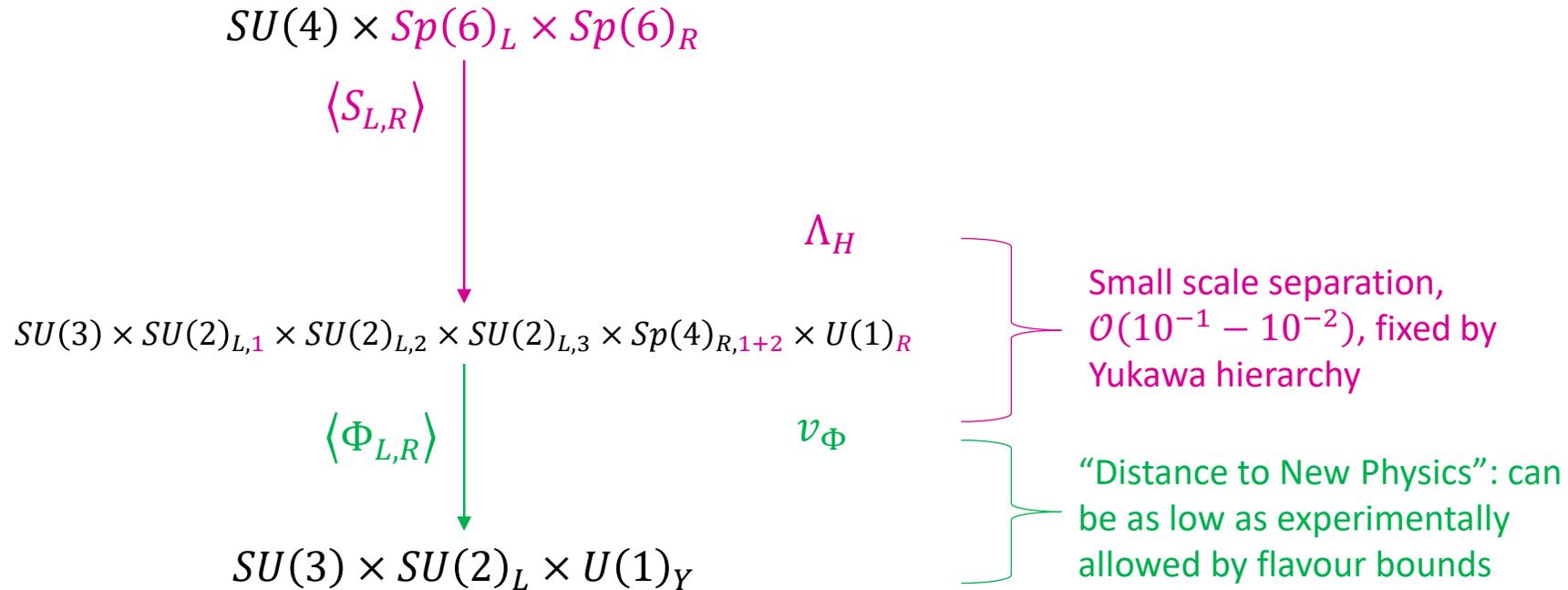
EWFU offers a new solution to

- **Puzzle 1:** why the peculiar set of 5+1 SM fermion reps?
- **Puzzle 2:** why three copies of each?
- **Puzzle 3:** why is the flavour symmetry broken in such a special way? Mass and mixing angle hierarchies

But what about **puzzle 4**? TeV scale new physics?

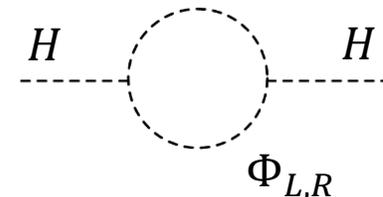
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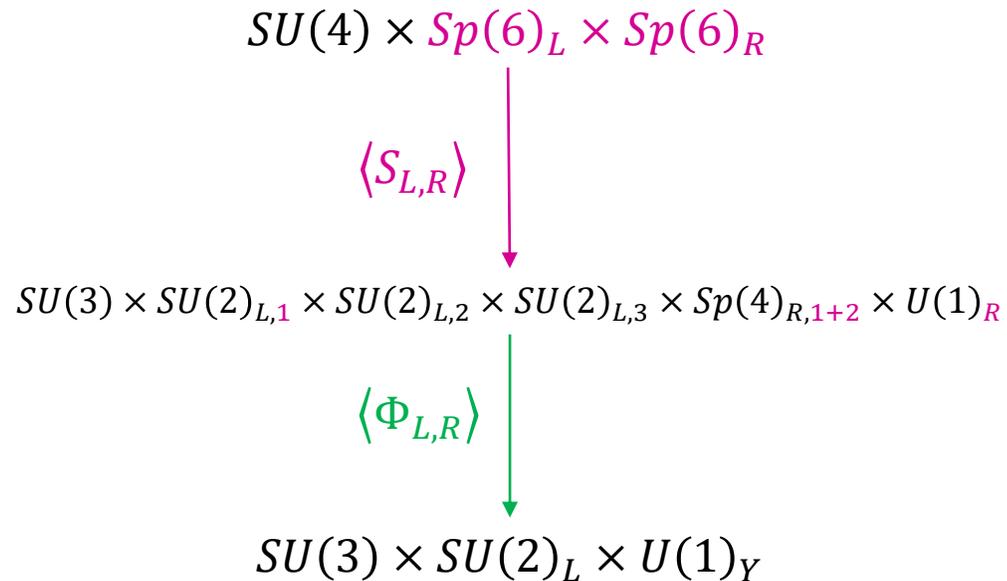


Recall there are two good reasons for  $v_\Phi$  to be low ( $\sim \text{TeV}$ ):

1. Naturalness
2. Persistent anomalies in low-energy data



# Phenomenology of EWFU: the 45 gauge bosons



## Gauge bosons (VERY HEAVY):

- $\langle S_L \rangle$ :  $(W', Z')$  triplets x3  
 $Z' \sim (\mathbf{1}, \mathbf{1})_0 \times 3$
- $\langle S_R \rangle$ :  $U_1 \sim (\mathbf{3}, \mathbf{1})_{2/3}$  leptoquark (flavour universal)  
 $Z^\pm \sim (\mathbf{1}, \mathbf{1})_1 \times 3$   
 $Z' \sim (\mathbf{1}, \mathbf{1})_0 \times 5$

## Gauge bosons (LIGHTER):

- $\langle \Phi_L \rangle$ :  $(W', Z')$  triplets x2
- $\langle \Phi_R \rangle$ :  $Z^\pm \sim (\mathbf{1}, \mathbf{1})_1 \times 3$   
 $Z' \sim (\mathbf{1}, \mathbf{1})_0 \times 4$

# Phenomenology of EWFU: the 45 gauge bosons

## Gauge bosons (LIGHTER):

$\langle \Phi_L \rangle$ :  $(W', Z')$  triplets x2

$\langle \Phi_R \rangle$ :  $Z^\pm \sim (\mathbf{1}, \mathbf{1})_1$  x3

$Z' \sim (\mathbf{1}, \mathbf{1})_0$  x4

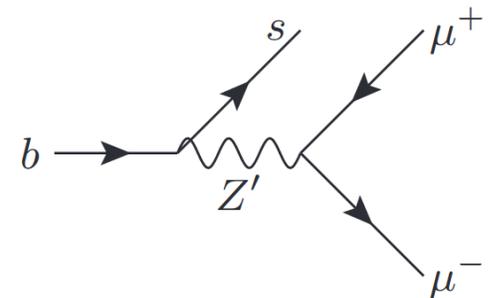
The lightest gauge bosons are **flavoured versions of EW gauge bosons, LH and RH**. For example, LH:

- one triplet coupled to 1<sup>st</sup> and 2<sup>nd</sup> families with opposite sign, mass  $m_{12} \sim g_L \epsilon_L^{12} \Lambda_H$
- one triplet coupled to 2<sup>nd</sup> and 3<sup>rd</sup> families with opposite sign, mass  $m_{23} \sim g_L \epsilon_L^{23} \Lambda_H$

New sources of quark flavour violation and LF(U)V!

*What's next for me?*

Characterise the **flavour** + **high  $p_T$**  pheno of these flavoured EW gauge bosons.



# Conclusions

- The existence of **3 generations** and the **rich Yukawa structure** are fascinating puzzles that beg for a BSM explanation
- Points to **new physics coupled predominantly to heavy generations**. Reasons for a TeV scale “anchor”:  
(a) **Naturalness** of EW sector, (b) hints of new physics in the **B anomalies**
- 3 approaches to flavour puzzle, all with TeV scale flavoured new physics:
  1. Horizontal flavour symmetry → **flavoured  $Z'$ s**
  2. Deconstructed Pati—Salam model → **flavoured  $U_1$  LQ**
  3. Electroweak flavour unification → **flavoured EW gauge bosons**

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  3. Electroweak flavour unification → **flavoured EW gauge bosons**

## A key message:

Continued **high  $p_T$  searches** with **heavy flavour** final states + continued **precision measurements of rare decays** will probe all these solutions to the flavour puzzle!

Thank you!

