Challenges and future in three-body heavy meson decays

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Motivation

- Standard Model works quite well but... some gaps!
  → baryogenesis !

- 1967, the Russian physicist Andrey Sakharov proposed three conditions for generating the observed matter/anti-matter asymmetry of the Universe:
  1) baryon number violation
  2) C and CP violation
  3) departure from thermal equilibrium

- CP-Violation on Hadronic decays
  - SM predicts CPV in B sector but ….. lot to be understood
    ↳ LHCb massive phase-space localized Asymmetry in $B^{\pm} \to h^{\pm}h^{-}h^{+}$
  - 2019 1st observation in charm $D^0(\bar{D}^0) \to \pi^+\pi^- - K^+K^-$ LHCb
    ↳ CPV on three-body?

→ can lead to new physics

- D and B three-body HADRONIC decays are dominated by low E resonances

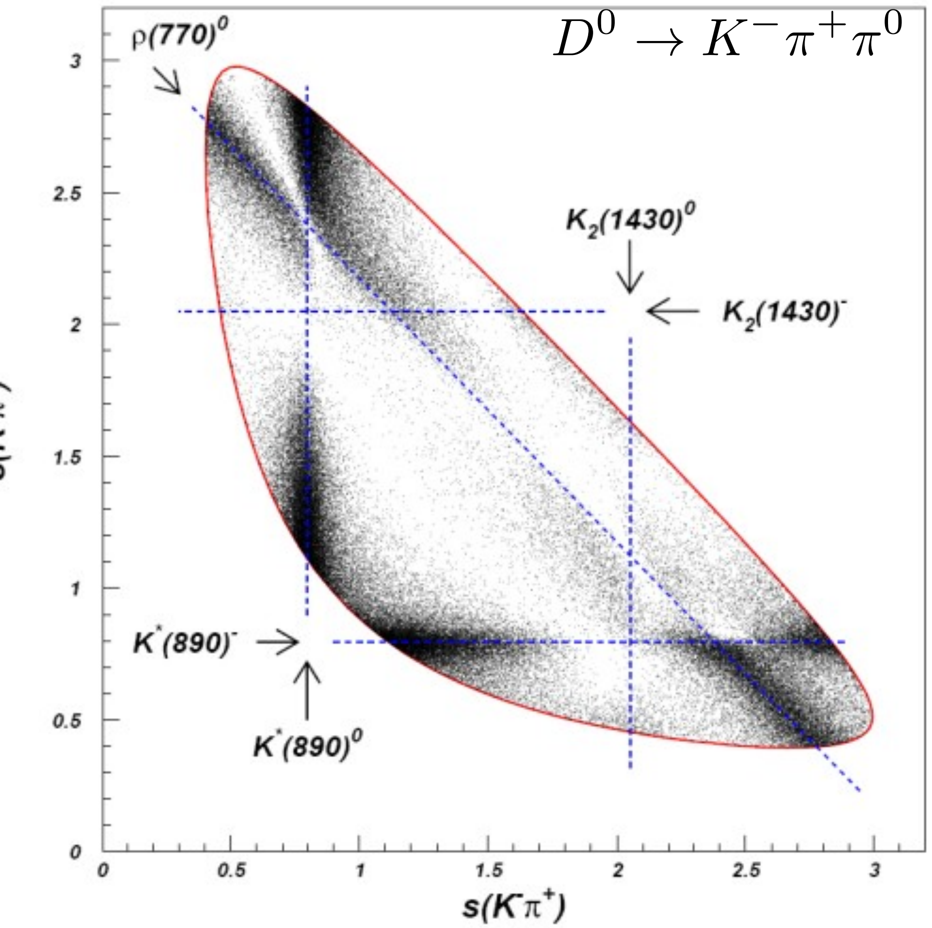


image credit:Brian Meadows

- spectroscopy: new resonances, their properties…
- information of MM interactions

1st observation of
σ [$f_0(600)$] and κ [$K_0^*(700)$]
in D decays

- build up the idea that the main dynamic in 3-body is driven by 2-body resonances

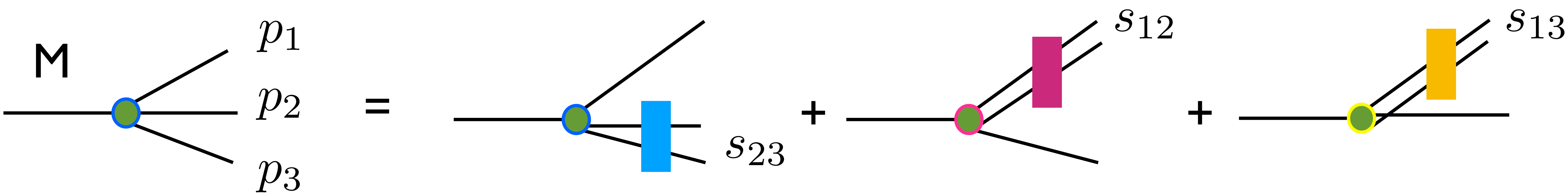
- new high data sample from LHCb ⟶ more to come from LHCb and Belle II

  ↳ simple models (only focus on two-body resonances) are not enough to explain data anymore

theoretical challenge !

Three-body kinematics : DALITZ plot

- How to describe the kinematics of three-body HADRONIC decays?



- Mandelstam variables for 3-body

$$s_{12} = (p_1 + p_2)^2 = m_{12}^2$$

$$s_{13} = (p_1 + p_3)^2 = m_{13}^2$$

$$s_{23} = (p_2 + p_3)^2 = m_{23}^2$$

→ $s_{12} + s_{13} + s_{12} = M^2 + m_1^2 + m_2^2 + m_3^2$

- In the rest frame of M (**P**=0): final particle are in the same plane
  - ↳ final particle distribution in the phase-space will depend on: - average of spin
    - Euler angles

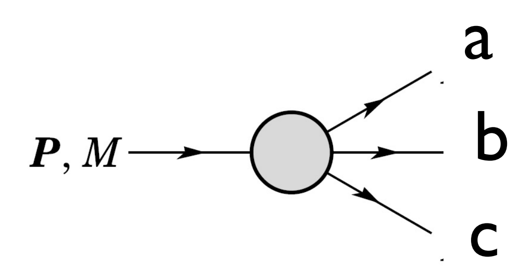
→ decay rate can be written as: $d\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32M^2}|\bar{\mathcal{M}}|^2 s_{12}s_{23}$

↳ Amplitude, dynamic!

Three-body kinematics : DALITZ plot

- The phase-space is NOT one-dimension!

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12},s_{23})|^2$$

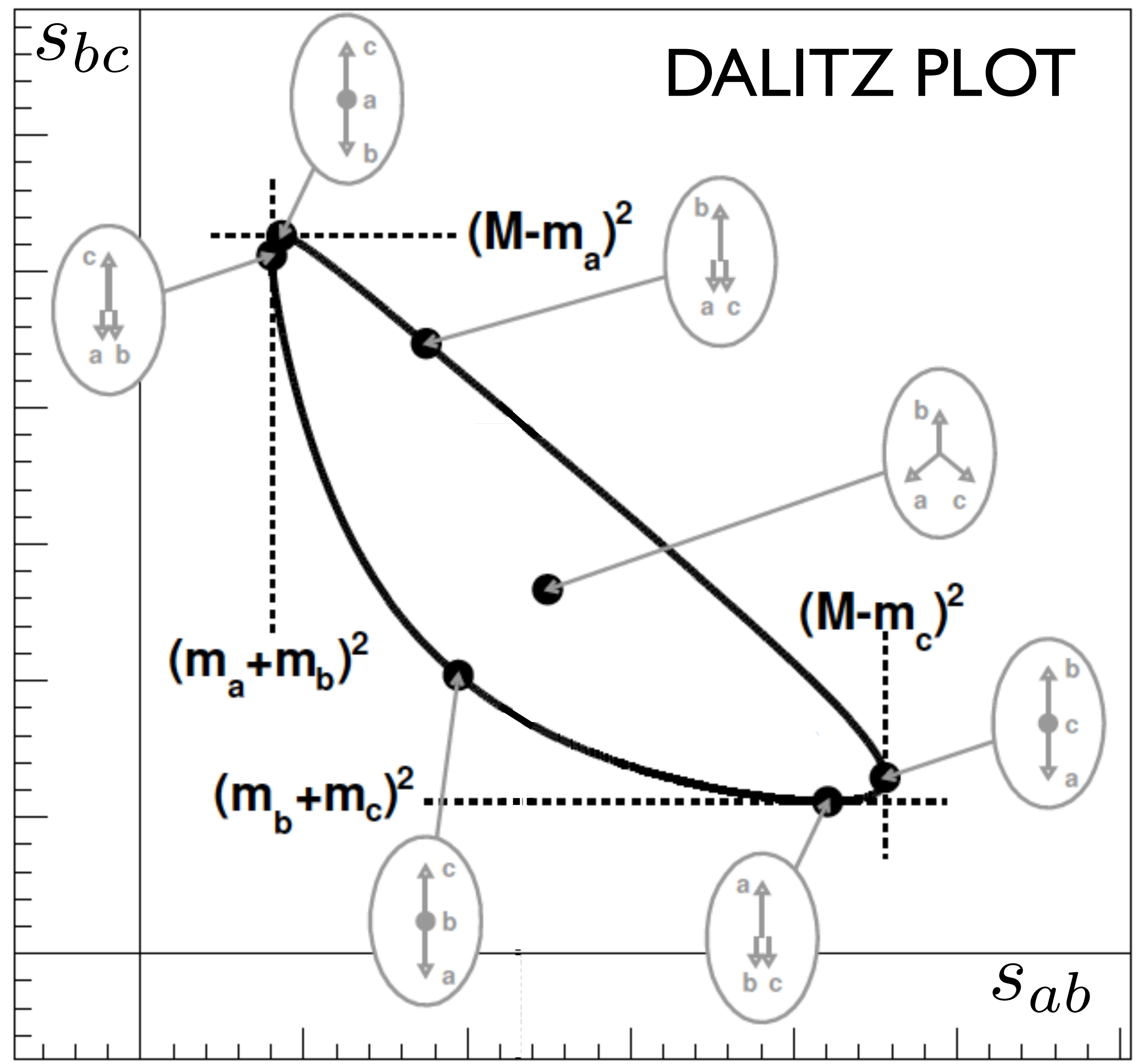


- DP proposed by Richard Dalitz (1925-2006) in 1953

- the perimeter depends on the masses
  min: $s_{ij} > (m_i + m_j)^2$
  max: in s_{ij} , $(M - m_k)^2$

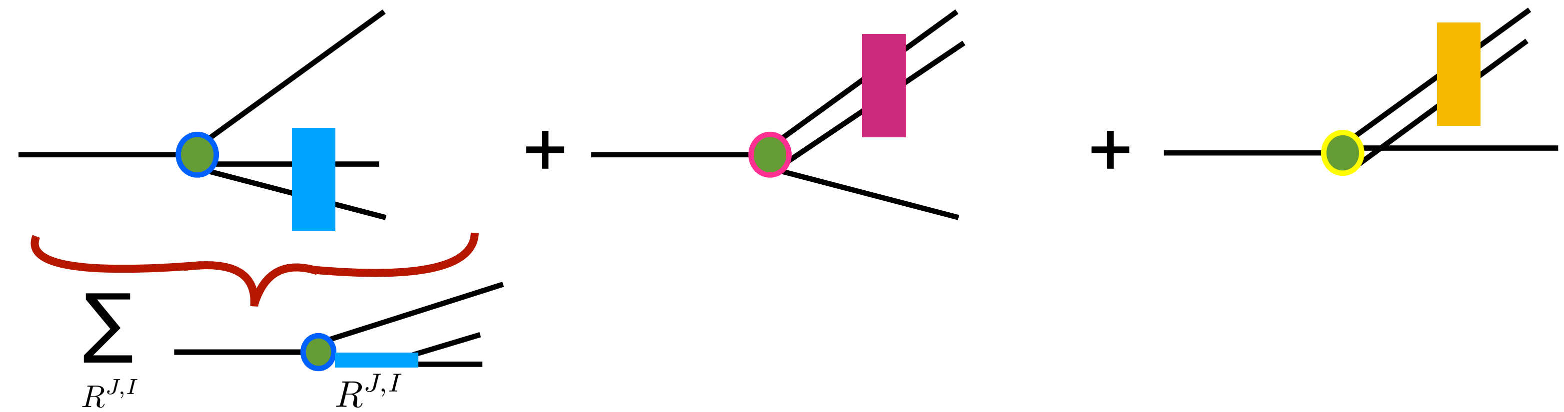
- inside this contour there are all combinations of momenta distribution

- The probability of each point inside is given by the dynamic amplitude A



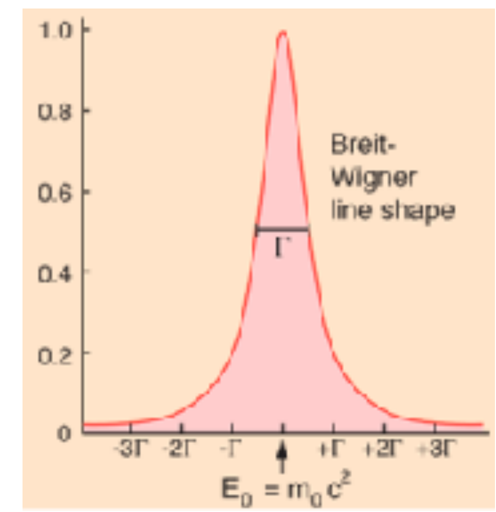
→ tool for analyse data

$\mathcal{A}(s_{12}, s_{23}) =$



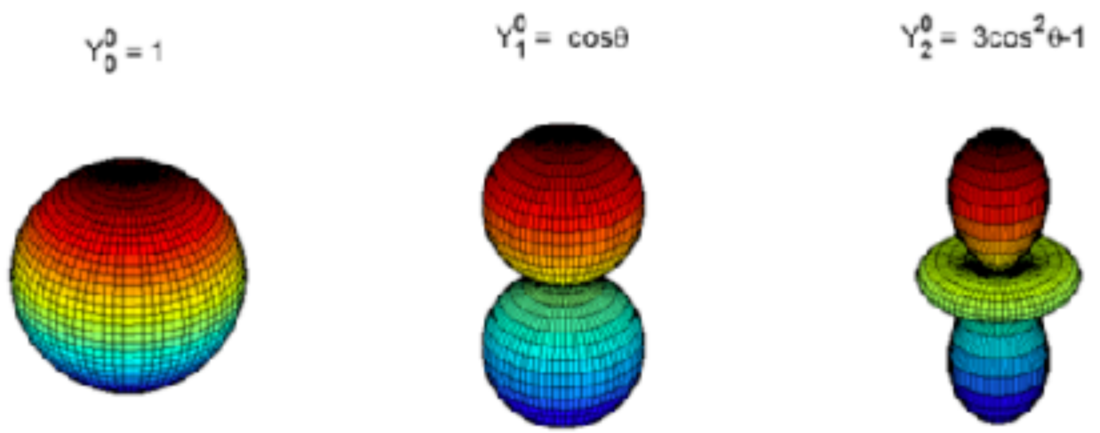
- 2-body resonances have spin and isospin well defined: $R^{J,I}$
  - typically amplitudes are bumps (like the Breit-Wigner)
  - contribute to a specific partial wave

$$\mathcal{M}_{ba}(s,t) = \sum_{j=0}^{\infty} (2j+1) \mathcal{M}^{j}_{ba}(s) P_j(\cos(\theta))$$



credit:hyperphysics.phy

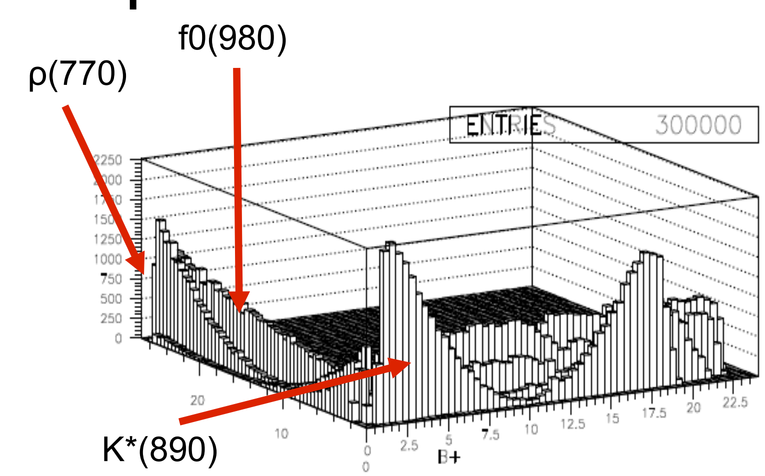
↳ besides the amplitude bump (intensity/probability) the resonance will have a spin signature in DP: $P_j(\cos(\theta))$ (same as spherical harmonics)…



scalar vector tensor

credit: knowino.org

⟶ this pattern in Dalitz Plot

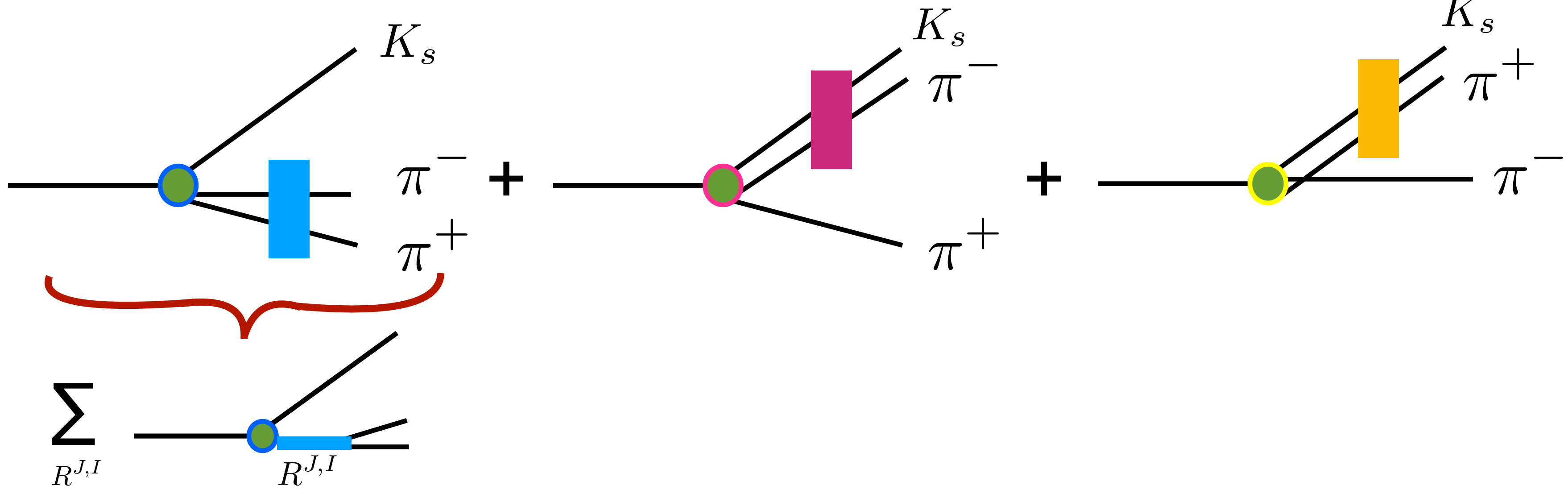


Two-body resonances signature in DP

- common cartoon to described 3-body decay

$D^0 \to K_s \pi^- \pi^+$

$\mathcal{A}(s_{12}, s_{23}) =$



- one expect to see all 3 channels res:

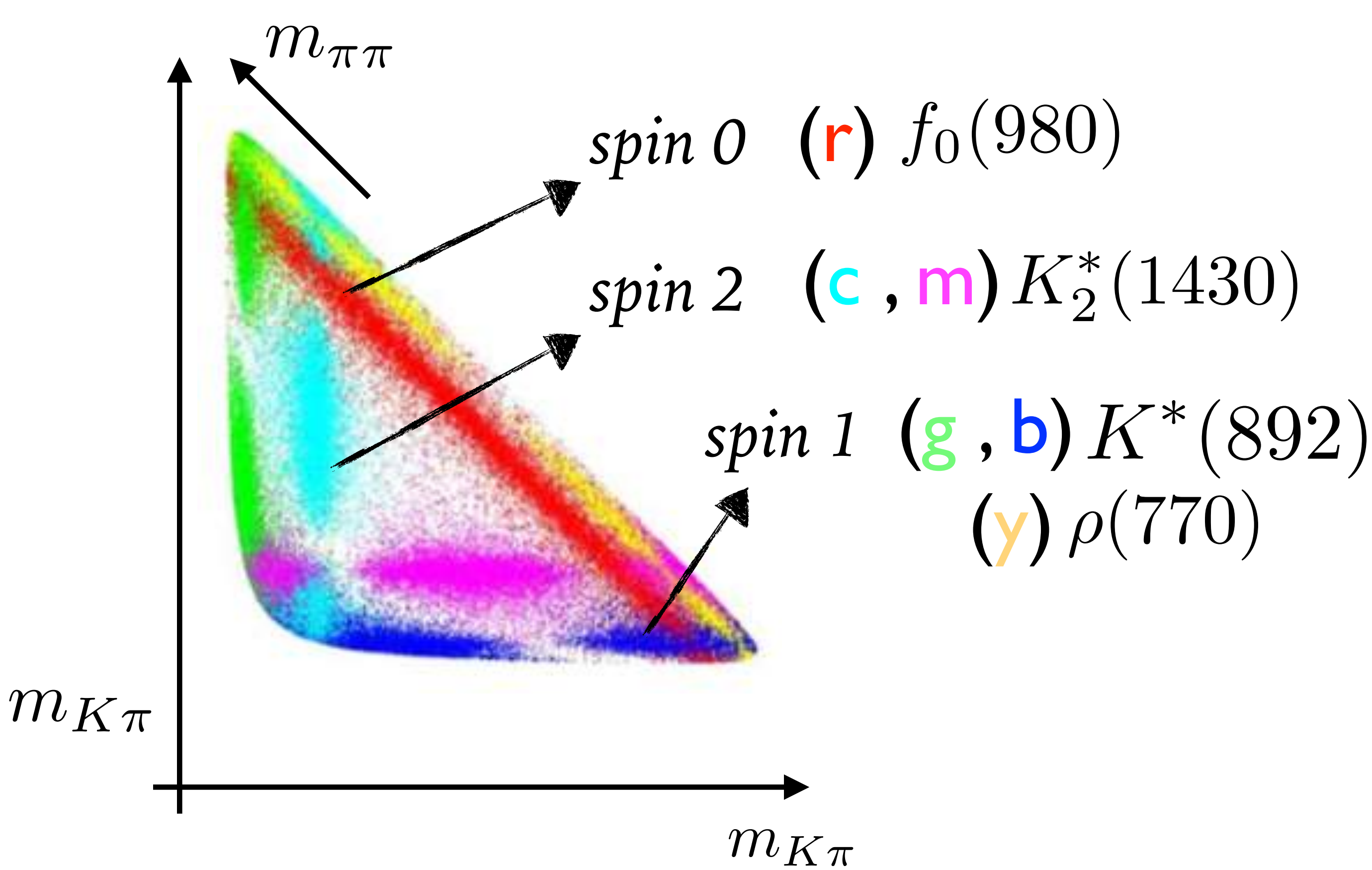
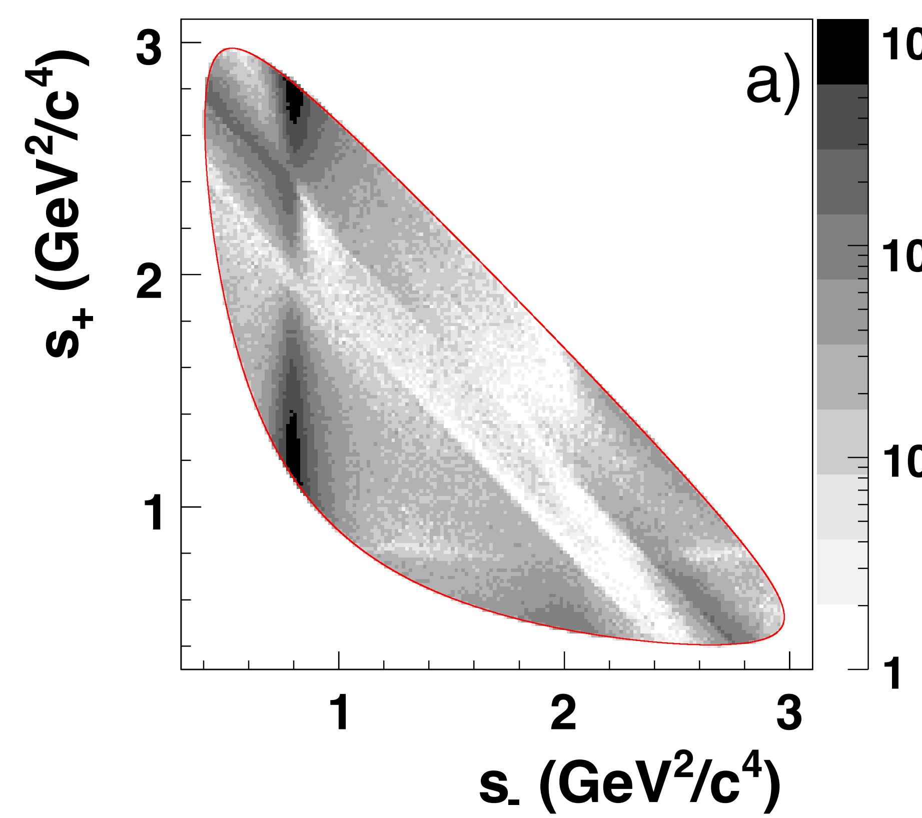


image credit:Tom Latham

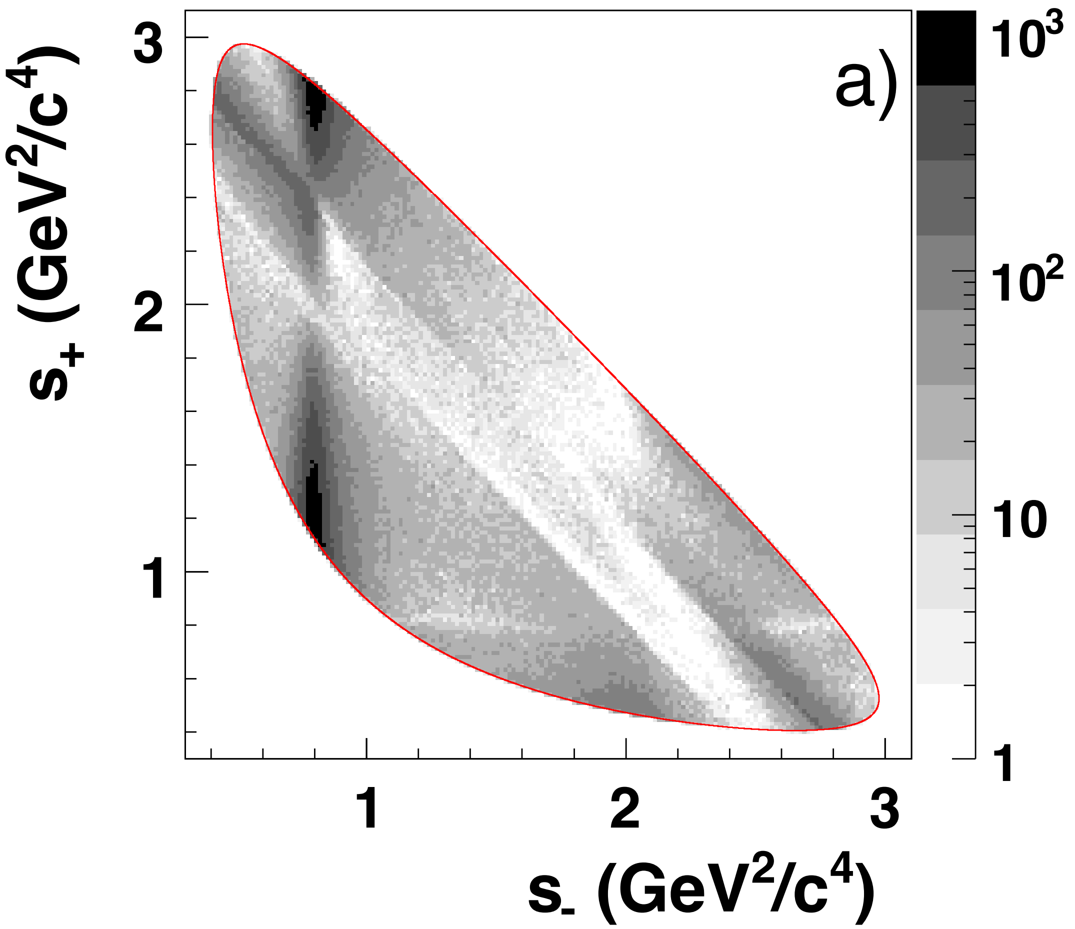
→ But in reality…….
not all of them are clearly present

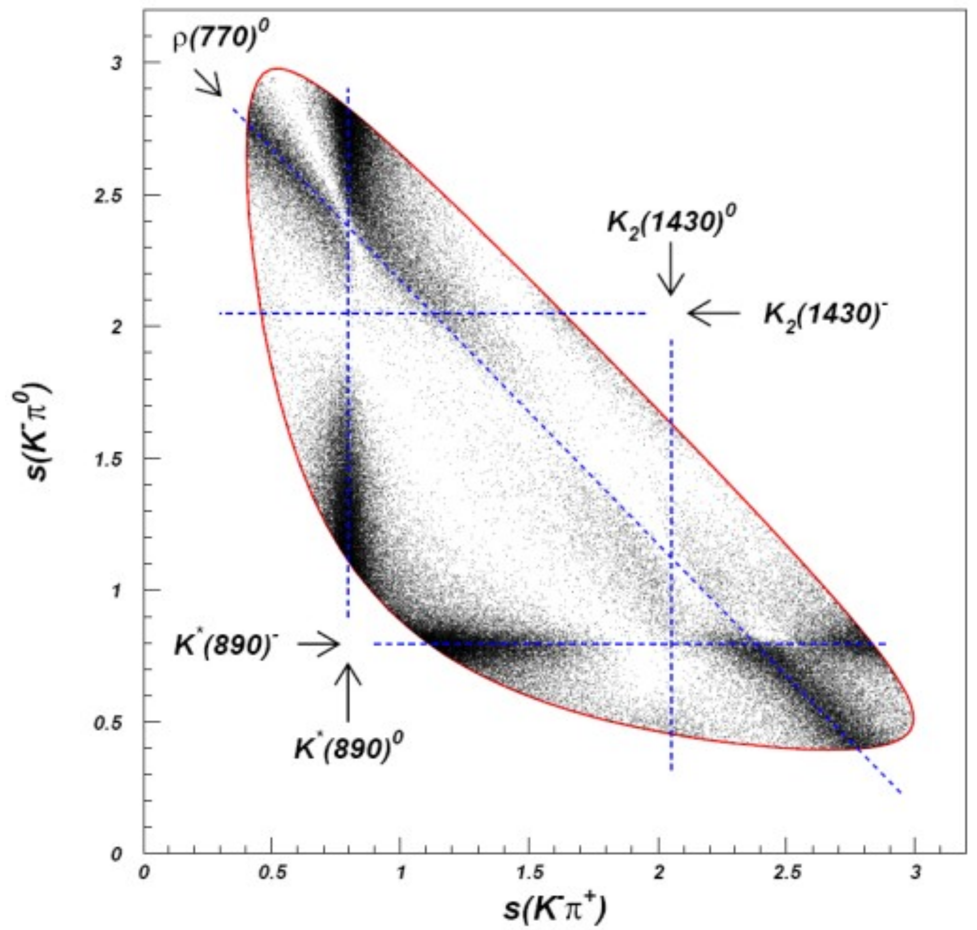


BABAR Phys.Rev. Lett. 105 (2010) 081803

Three-body kinematics : DALITZ plot

- $D^0 \to K_s \pi^- \pi^+$
- $D^0 \to K^- \pi^+ \pi^0$





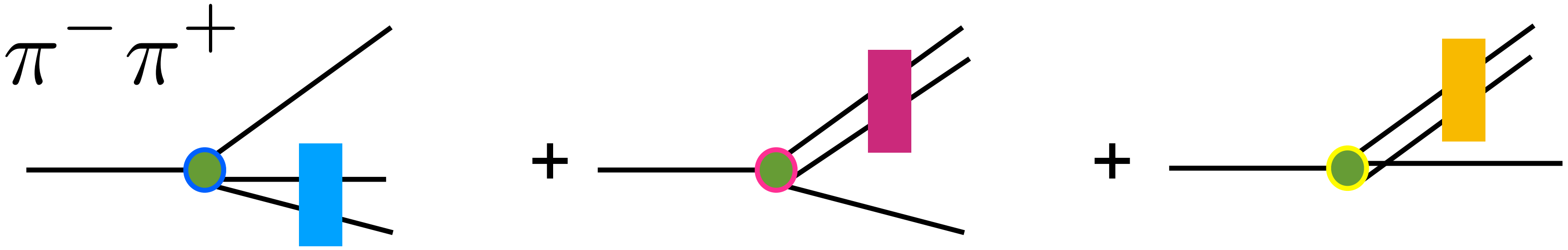
credit:Brian Meadows

⟶ Similar final state but different interference pattern

↳ different dynamics to be understood

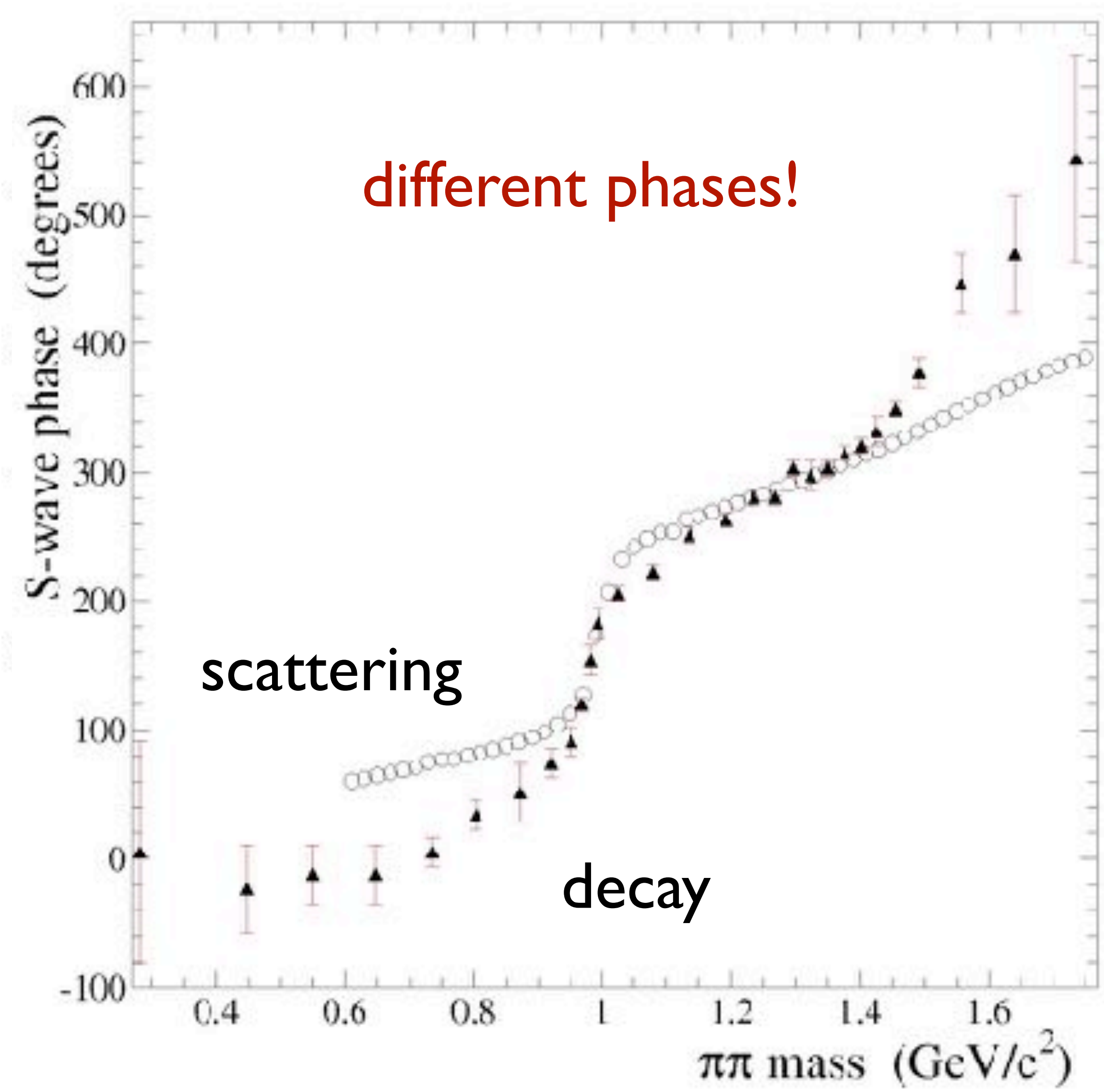
⟶ to disentangle the interference we need amplitude analysis

- $D_s^+ \to \pi^+\pi^-\pi^+$



- If this is the "nature" picture ⟶ once it only contain 2-body information, decay phase should be the same as scattering

→ Is not as simple as it look like!

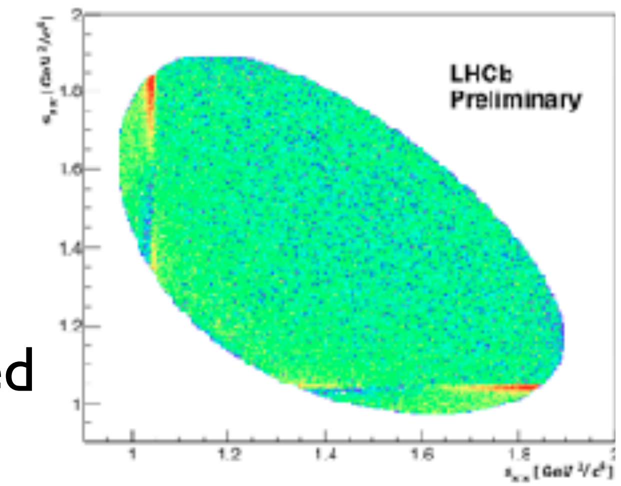


Phys.Rev. D 79 (2009) 032003

- Quantum numbers:
  - 2-body amplitude: spin and isospin well defined!
  - 3-body data: only spin! and $\neq$ dynamics

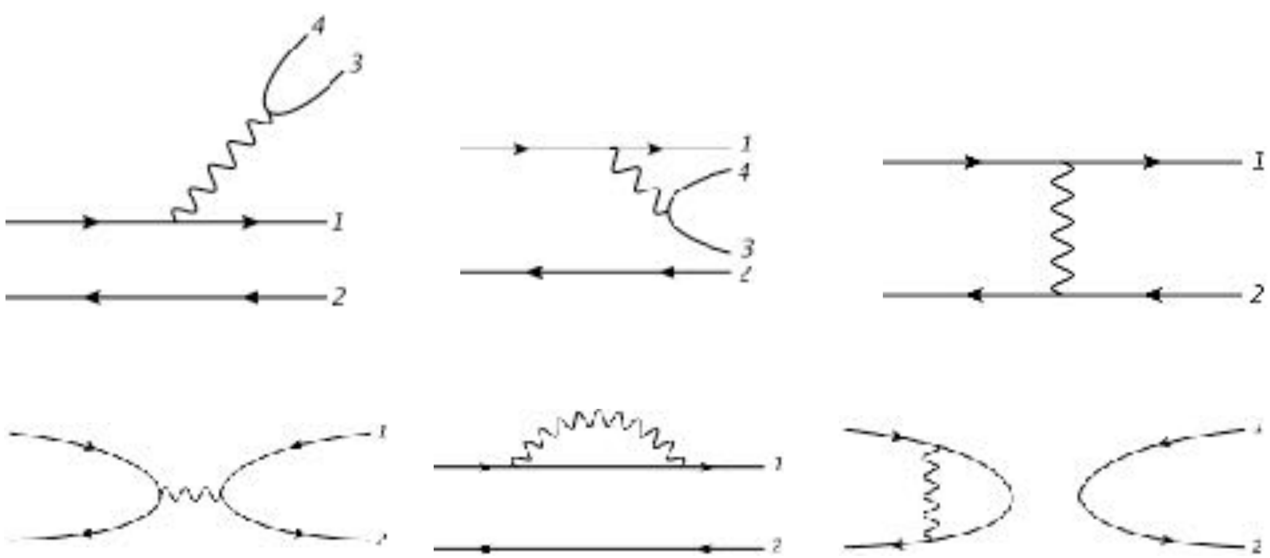
Three-body heavy meson decay

- dynamics $D^+ \to K^- K^+ K^-$

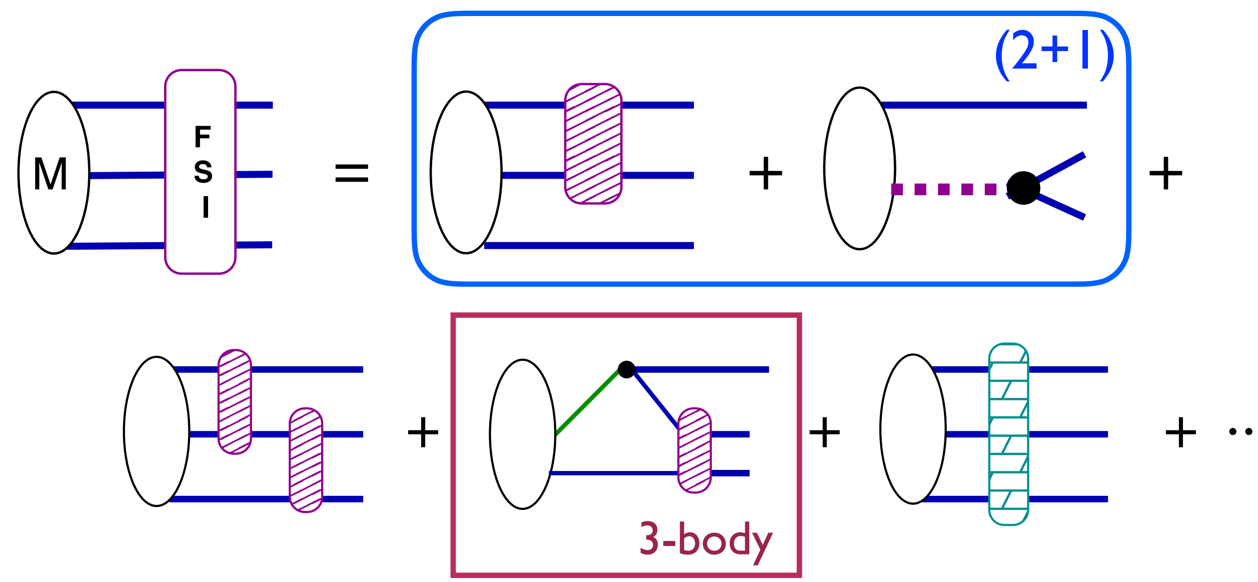


primary vertex - weak -

Final State Interactions - strong -

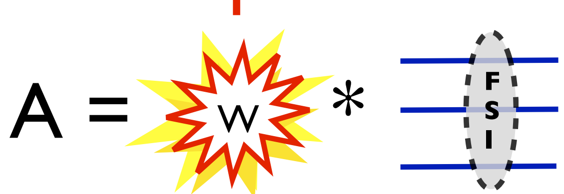


QCD, CKM coupling and phase



2-body is crucial!!!!

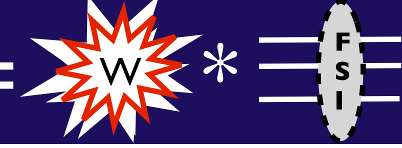
To extract information from data we need an amplitude MODEL



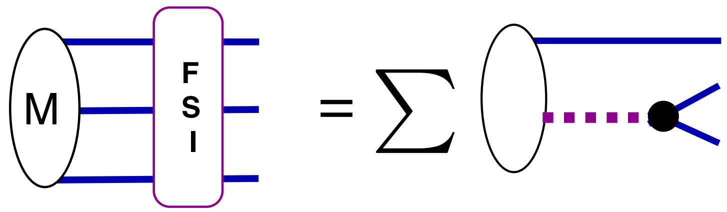
$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12}, s_{23})|^2$$

dynamics

Models available

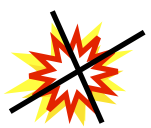


- isobar model: widely used by experimentalists
  - (2+1) approximation:



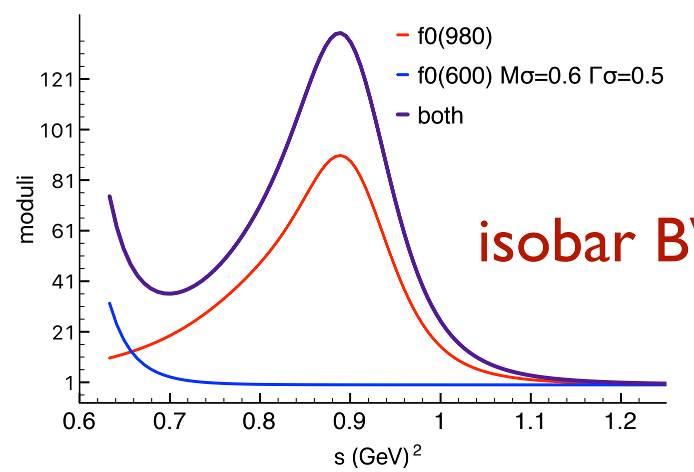
→ ignore the 3rd particle (bachelor)

$A = \sum c_k A_{k;}$ + NR { non-resonant as constant or exponential! / each resonance as Breit-Wigner $\mathrm{BW}(s_{12}) = \frac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})}$,



weak vertex is not considered explicitly

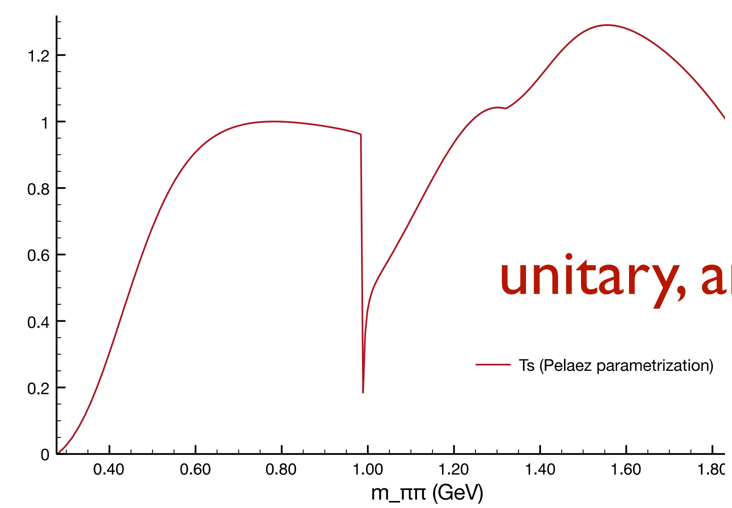
- worst problems: ππ S-wave



isobar BW

→ fit could change this interference

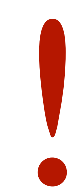
more than 2 scalars ←

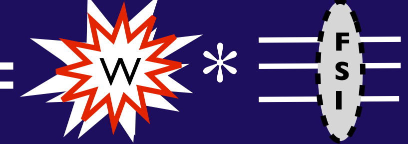


unitary, analytic,...

Pelaez, Yndurain PRD71(2005) 074016

- sum of BW violates two-body unitarity (2 res in the same channel);
- do NOT include rescattering and coupled-channels;
- free parameters are not connected with theory !



Models available A = 

- movement to use better 2-body (unitarity) inputs in data analysis 

  - "K-matrix" : ππ S-wave 5 coupled-channel modulated by a production amplitude

    ↳ used by Babar, LHCb, BES II — Anisovich PLB653(2007)

  - rescattering $\pi\pi \to KK$ contribution in LHCb $\begin{cases} B^\pm \to \pi^+\pi^-\pi^\pm \\ B^\pm \to K^-K^+\pi^\pm \end{cases}$ [arXiv:1909.05212; 1909.05211] [arXiv:1905.09244]

    Pelaez, Yndurain PRD71(2005) 074016

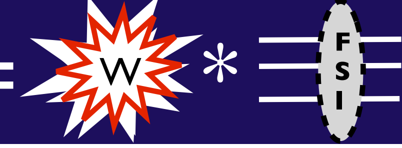
    ↳ new parametrization Pelaez, and Rodas EPJ. C78 (2018) 11, 897

→ other scalar and vector form factors available

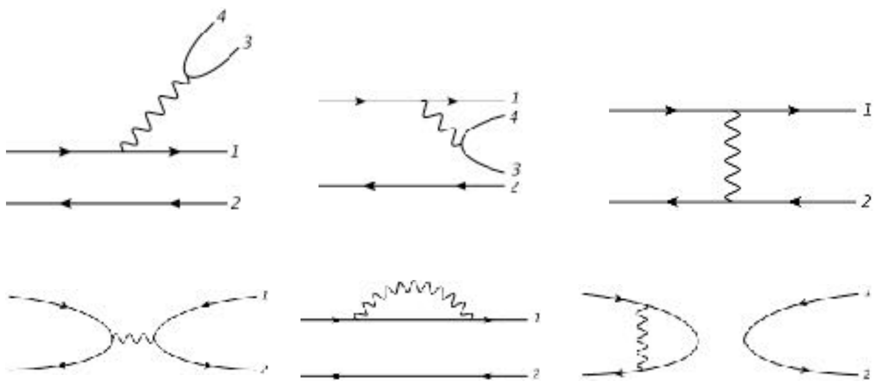
Limited to low E (2 GeV)!

$< \pi\pi \| 0 >$	scalar	Moussallam EPJ C 14, 111 (2000); Daub, Hanhart, and B. Kubis JHEP 02 (2016) 009.
	vector	Hanhart, PL B715, 170 (2012); Dumm and Roig EPJ C 73, 2528 (2013).
$< K\pi \| 0 >$	scalar	Moussallam EPJ C 53, 401 (2008); Jamin, Oller and Pich, PRD 74, 074009 (2006)
	vector	Boito, Escribano, and Jamin EPJ C 59, 821 (2009).
$< KK \| 0 >$ (no data)	Fit from 3-body data	PCM, Robilotta + LHCb JHEP 1904 (2019) 063
	extrapolate from unitarity model	Albaladejo and Moussallam EPJ C 75, 488 (2015).
	quark model with isospin symmetry	Bruch, Khodjamirian, and Kühn , EPJ C 39, 41 (2005)

Models available

A = 

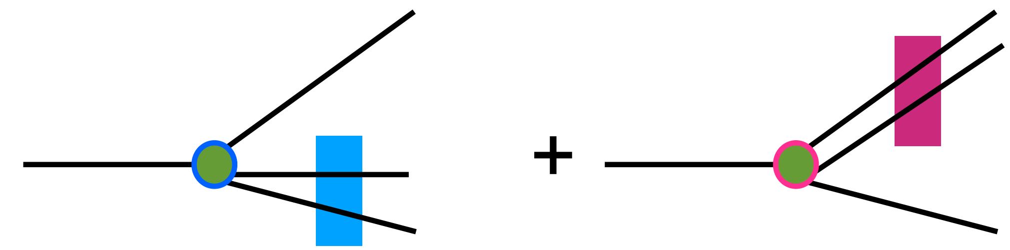
- QCD factorization approach → factorize the quark currents



Chau [Phys. Rep. 95,1(1983)]

- challenging for 3-body
- not all FSI and 3-body NR
- scale issue with charm

!



$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \left[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right] + \text{h.c.} ,$$

→ ex: $B^+ \to \pi^+ \pi^- \pi^+$ — how to describe it?

$$A \sim \underbrace{\langle [\pi^+(p_2)\pi^-(p_3)]}_{R} \; |(\bar{u}b)_{V-A}|B^-\rangle \langle \pi^-(p_1)|(\bar{d}u)_{V-A}|0\rangle + \langle \pi^-(p_1)|(\bar{d}b)_{sc-ps}|B^-\rangle \underbrace{\langle [\pi^+(p_2)\pi^-(p_3)] \; |(\bar{d}d)_{sc+ps}|0\rangle}_{FF}$$

- naive factorization {
  - intermediate by a resonance R;
  - FSI with scalar and vector form factors FF

↳ parametrizations for B and D→3h Boito et al. PRD96 113003 (2017)

- modern QDC factorization: improvement to include "long distance" still developing

Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)

QCDF predictions

Branching Fraction (tree dominated decays)

	Theory I		Theory II		Experiment
$B^- \to \pi^-\pi^0$	$5.43^{+0.06\,+1.45}_{-0.06\,-0.84}$	$(\star)$	$5.82^{+0.07\,+1.42}_{-0.06\,-1.35}$	$(\star)$	$5.59^{+0.41}_{-0.40}$
$\bar{B}^0_d \to \pi^+\pi^-$	$7.37^{+0.86\,+1.22}_{-0.69\,-0.97}$	$(\star)$	$5.70^{+0.70\,+1.16}_{-0.55\,-0.97}$	$(\star)$	5.16 ± 0.22
$\bar{B}^0_d \to \pi^0\pi^0$	$0.33^{+0.11\,+0.42}_{-0.08\,-0.17}$		$0.63^{+0.12\,+0.64}_{-0.10\,-0.42}$		1.55 ± 0.19
			BELLE CKM 14:		0.90 ± 0.16
$B^- \to \pi^-\rho^0$	$8.68^{+0.42\,+2.71}_{-0.41\,-1.56}$	$(\star\star)$	$9.84^{+0.41\,+2.54}_{-0.40\,-2.52}$	$(\star\star)$	$8.3^{+1.2}_{-1.3}$
$B^- \to \pi^0\rho^-$	$12.38^{+0.90\,+2.18}_{-0.77\,-1.41}$	$(\star)$	$12.13^{+0.85\,+2.23}_{-0.73\,-2.17}$	$(\star)$	$10.9^{+1.4}_{-1.5}$
$\bar{B}^0 \to \pi^+\rho^-$	$17.80^{+0.62\,+1.76}_{-0.56\,-2.10}$	$(\star)$	$13.76^{+0.49\,+1.77}_{-0.44\,-2.18}$	$(\star)$	15.7 ± 1.8
$\bar{B}^0 \to \pi^-\rho^+$	$10.28^{+0.39\,+1.37}_{-0.39\,-1.42}$	$(\star\star)$	$8.14^{+0.34\,+1.35}_{-0.33\,-1.49}$	$(\star\star)$	7.3 ± 1.2
$\bar{B}^0 \to \pi^\pm\rho^\mp$	$28.08^{+0.27\,+3.82}_{-0.19\,-3.50}$	$(\dagger)$	$21.90^{+0.20\,+3.06}_{-0.12\,-3.55}$	$(\dagger)$	23.0 ± 2.3
$\bar{B}^0 \to \pi^0\rho^0$	$0.52^{+0.04\,+1.11}_{-0.03\,-0.43}$		$1.49^{+0.07\,+1.77}_{-0.07\,-1.29}$		2.0 ± 0.5
$B^- \to \rho^-_L\rho^0_L$	$18.42^{+0.23\,+3.92}_{-0.21\,-2.55}$	$(\star\star)$	$19.06^{+0.24\,+4.59}_{-0.22\,-4.22}$	$(\star\star)$	$22.8^{+1.8}_{-1.9}$
$\bar{B}^0_d \to \rho^+_L\rho^-_L$	$25.98^{+0.85\,+2.93}_{-0.77\,-3.43}$	$(\star\star)$	$20.66^{+0.68\,+2.99}_{-0.62\,-3.75}$	$(\star\star)$	$23.7^{+3.1}_{-3.2}$
$\bar{B}^0_d \to \rho^0_L\rho^0_L$	$0.39^{+0.03\,+0.83}_{-0.03\,-0.36}$		$1.05^{+0.05\,+1.62}_{-0.04\,-1.04}$		$0.55^{+0.22}_{-0.24}$

Theory I: $f_+^{B\pi}(0) = 0.25 \pm 0.05$, $A_0^{B\rho}(0) = 0.30 \pm 0.05$, $\lambda_B(1\,\mathrm{GeV}) = 0.35 \pm 0.15$ GeV
Theory II: $f_+^{B\pi}(0) = 0.23 \pm 0.03$, $A_0^{B\rho}(0) = 0.28 \pm 0.03$, $\lambda_B(1\,\mathrm{GeV}) = 0.20^{+0.05}_{-0.00}$ GeV

→ good agreement for Br

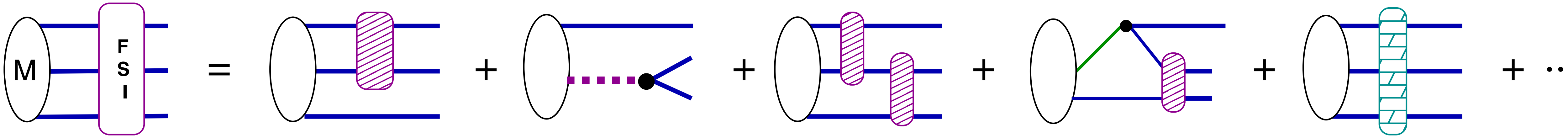
Acp (penguin dominante decays)

f	NLO	NNLO	NNLO + LD	Exp
$\pi^-\bar{K}^{*0}$	$1.36^{+0.25\,+0.60}_{-0.26\,-0.47}$	$1.49^{+0.27\,+0.69}_{-0.29\,-0.56}$	$0.27^{+0.05\,+3.18}_{-0.05\,-0.67}$	-3.8 ± 4.2
π^0K^{*-}	$13.85^{+2.40\,+5.84}_{-2.70\,-5.86}$	$18.16^{+3.11\,+7.79}_{-3.52\,-10.57}$	$-15.81^{+3.01\,+69.35}_{-2.83\,-15.39}$	-6 ± 24
π^+K^{*-}	$11.18^{+2.00\,+9.75}_{-2.15\,-10.62}$	$19.70^{+3.37\,+10.54}_{-3.80\,-11.42}$	$-23.07^{+4.35\,+86.20}_{-4.05\,-20.64}$	-23 ± 6
$\pi^0\bar{K}^{*0}$	$-17.23^{+3.33\,+7.59}_{-3.00\,-12.57}$	$-15.11^{+2.93\,+12.34}_{-2.65\,-10.64}$	$2.16^{+0.39\,+17.53}_{-0.42\,-36.80}$	-15 ± 13
$\delta(\pi\bar{K}^*)$	$2.68^{+0.72\,+5.44}_{-0.67\,-4.30}$	$-1.54^{+0.45\,+4.60}_{-0.58\,-9.19}$	$7.26^{+1.21\,+12.78}_{-1.34\,-20.65}$	17 ± 25
$\Delta(\pi\bar{K}^*)$	$-7.18^{+1.38\,+3.38}_{-1.28\,-5.35}$	$-3.45^{+0.67\,+9.48}_{-0.59\,-4.95}$	$-1.02^{+0.19\,+4.32}_{-0.18\,-7.86}$	-5 ± 45
$\rho^-\bar{K}^0$	$0.38^{+0.07\,+0.16}_{-0.07\,-0.27}$	$0.22^{+0.04\,+0.19}_{-0.04\,-0.17}$	$0.30^{+0.06\,+2.28}_{-0.06\,-2.39}$	-12 ± 17
ρ^0K^-	$-19.31^{+3.42\,+13.95}_{-3.61\,-8.96}$	$-4.17^{+0.75\,+19.26}_{-0.80\,-19.52}$	$43.73^{+7.07\,+44.00}_{-7.62\,-137.77}$	37 ± 11
ρ^+K^-	$-5.13^{+0.95\,+6.38}_{-0.97\,-4.02}$	$1.50^{+0.29\,+8.69}_{-0.27\,-10.36}$	$25.93^{+4.43\,+25.40}_{-4.90\,-75.63}$	20 ± 11
$\rho^0\bar{K}^0$	$8.63^{+1.59\,+2.31}_{-1.65\,-1.69}$	$8.99^{+1.66\,+3.60}_{-1.71\,-7.44}$	$-0.42^{+0.08\,+19.49}_{-0.08\,-8.78}$	6 ± 20
$\delta(\rho\bar{K})$	$-14.17^{+2.80\,+7.98}_{-2.96\,-5.39}$	$-5.67^{+0.96\,+10.86}_{-1.01\,-9.79}$	$17.80^{+3.15\,+19.51}_{-3.01\,-62.44}$	17 ± 16
$\Delta(\rho\bar{K})$	$-8.75^{+1.62\,+4.78}_{-1.66\,-6.48}$	$-10.84^{+1.98\,+11.67}_{-2.09\,-9.09}$	$-2.43^{+0.46\,+4.60}_{-0.42\,-19.43}$	-37 ± 37

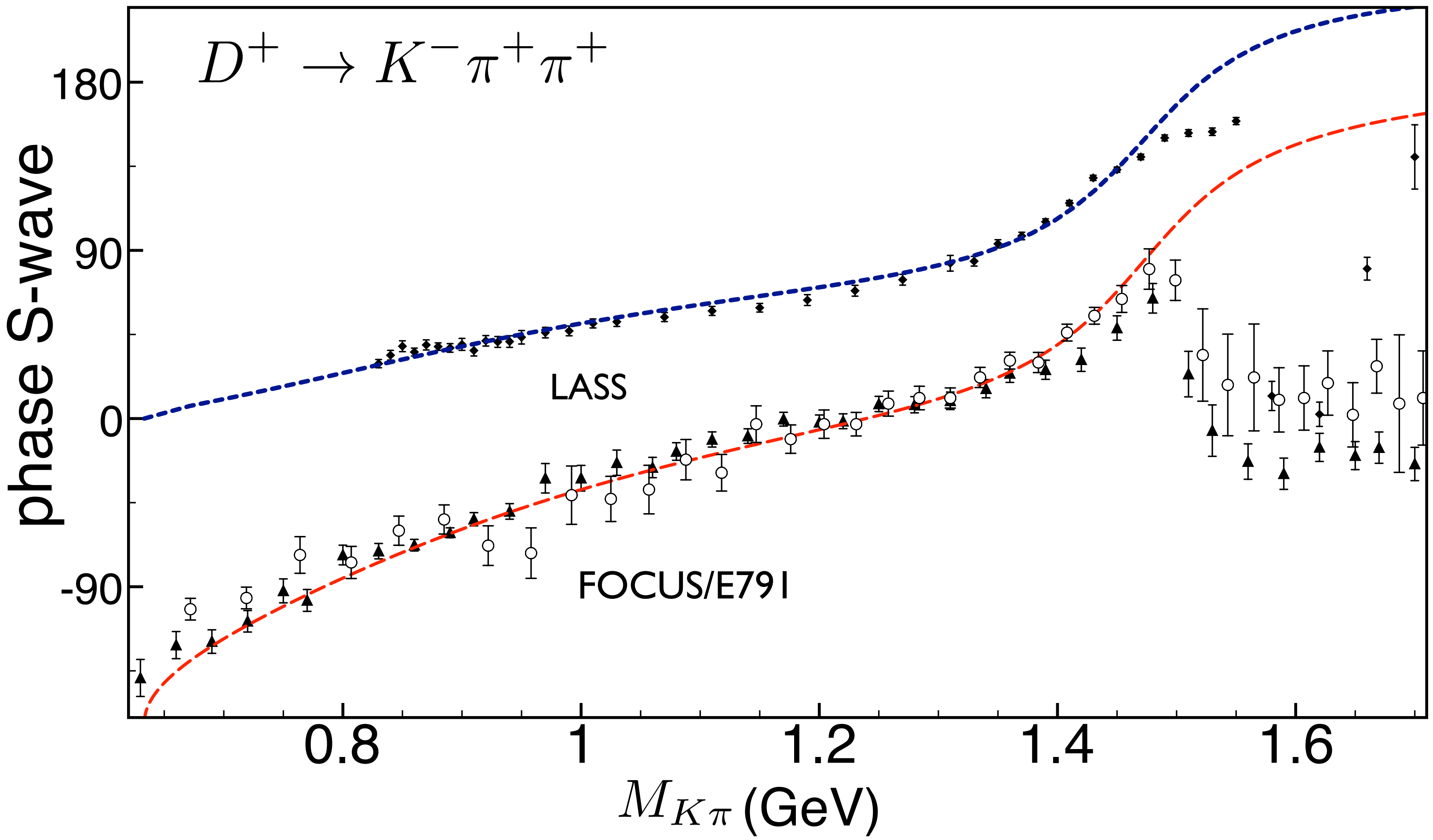
not good agreement for Acp ←

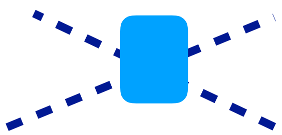
Beneke Seminar at "Future Challenges in Non-Leptonic B Decays", Bad Honnef, 2016

- Three-body FSI (beyond 2+1)



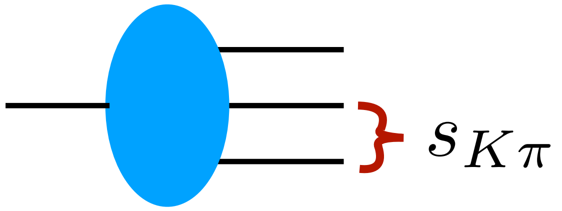
- shown to be relevant on charm sector





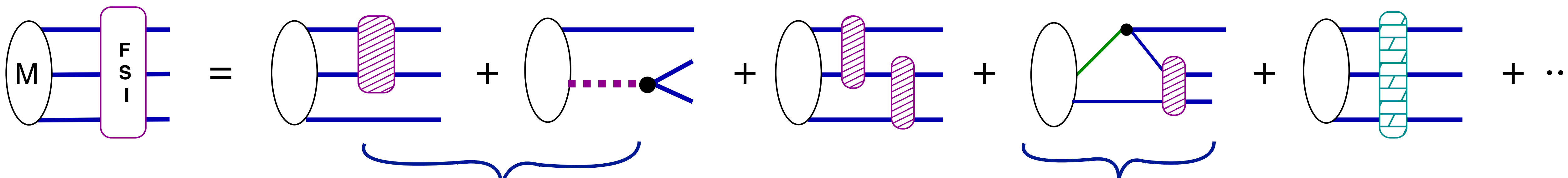
Scattering

Decay projected in one pair mass



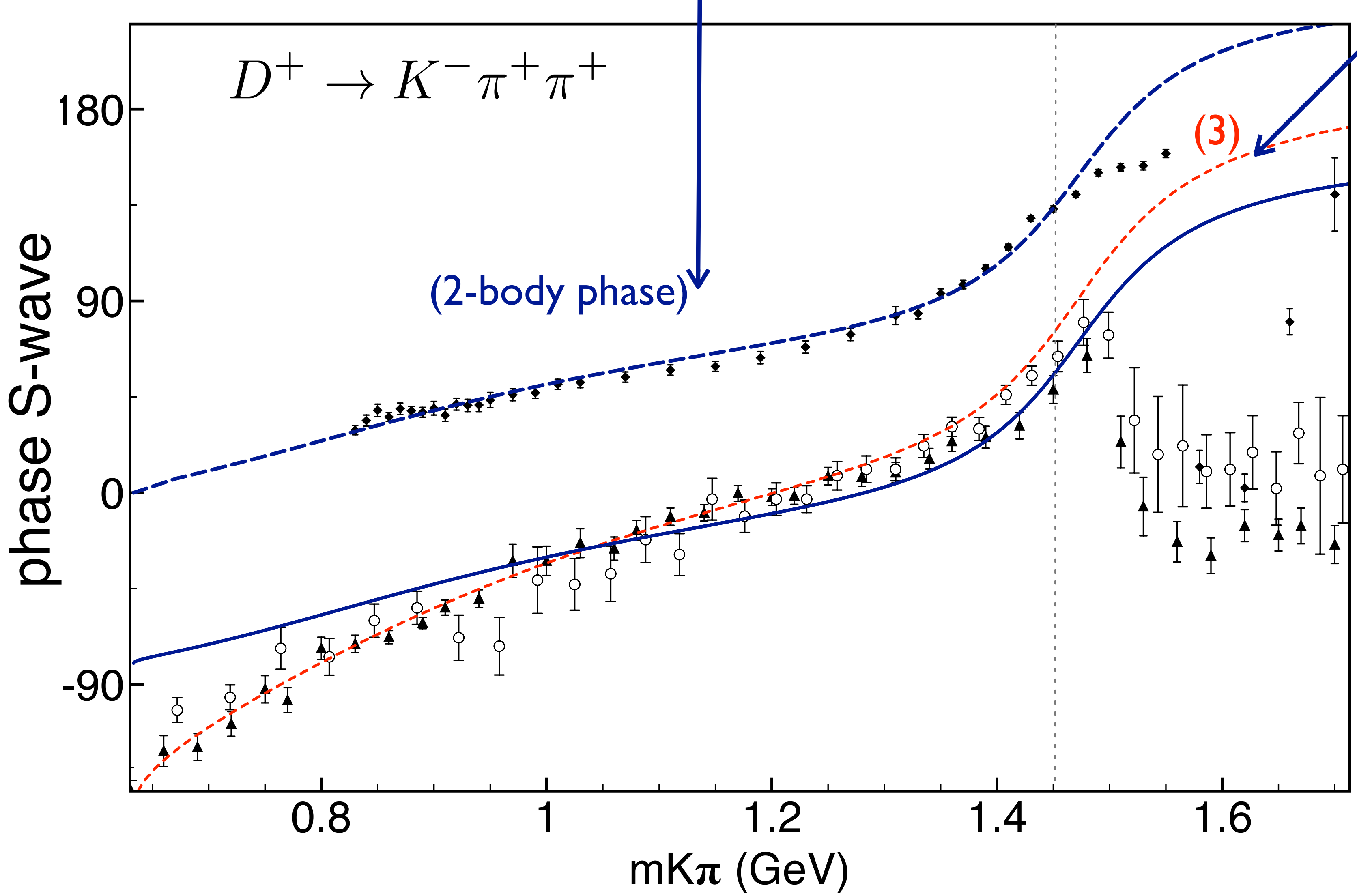
Models available

- Three-body FSI (beyond 2+1)



- shown to be relevant on charm sector

PRD92 094005 (2015)



- 3-body approaches
  PCM et.al: PRD84 094001 (2011),
  S.Nakamura PRD93 014005 (2016)
  Niecknig, Kubis, JHEP10 142 (2015)

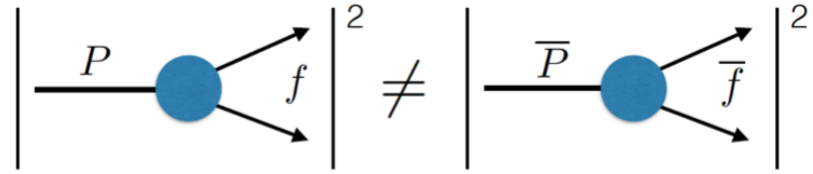
↳ 3-body FSI play a role

↳ data analysis…

Final State Interaction in B decays as a source of CP violation

- Charge Parity Violation

$$\Gamma(M \to f) \neq \Gamma(\bar{M} \to \bar{f})$$



  - condition to CPV

    → $2 \neq$ amplitudes, SAME final state with strong (δ_i) and weak (ϕ_i) phase

$$\langle f \,|\, T \,|\, M \rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

↓ CP

$$\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

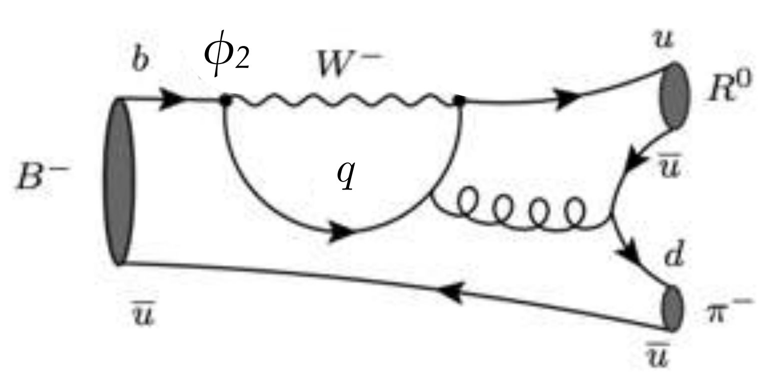
$$\therefore\ \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f \,|\, T \,|\, M \rangle|^2 - |\langle \bar{f} \,|\, T \,|\, \bar{M} \rangle|^2 = -4 A_1 A_2 \sin(\delta_1 - \delta_2)\, \sin(\phi_1 - \phi_2)$$

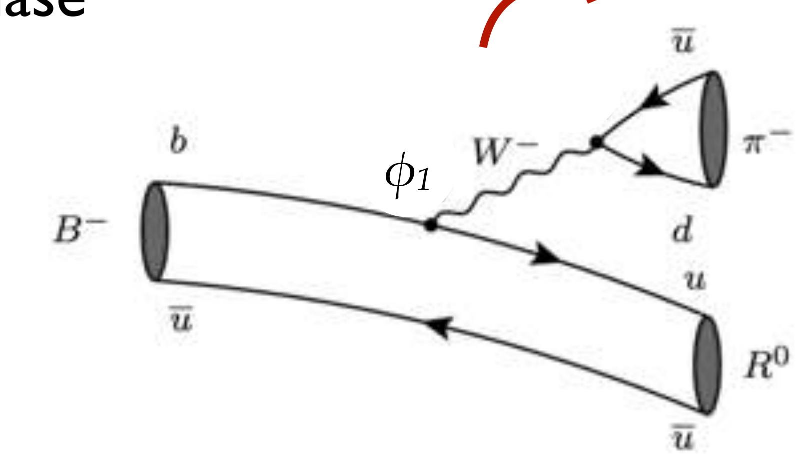
- BSS model 

Bander Silverman & Soni PRL 43 (1979) 242

strong phase

weak phase: CKM Vub

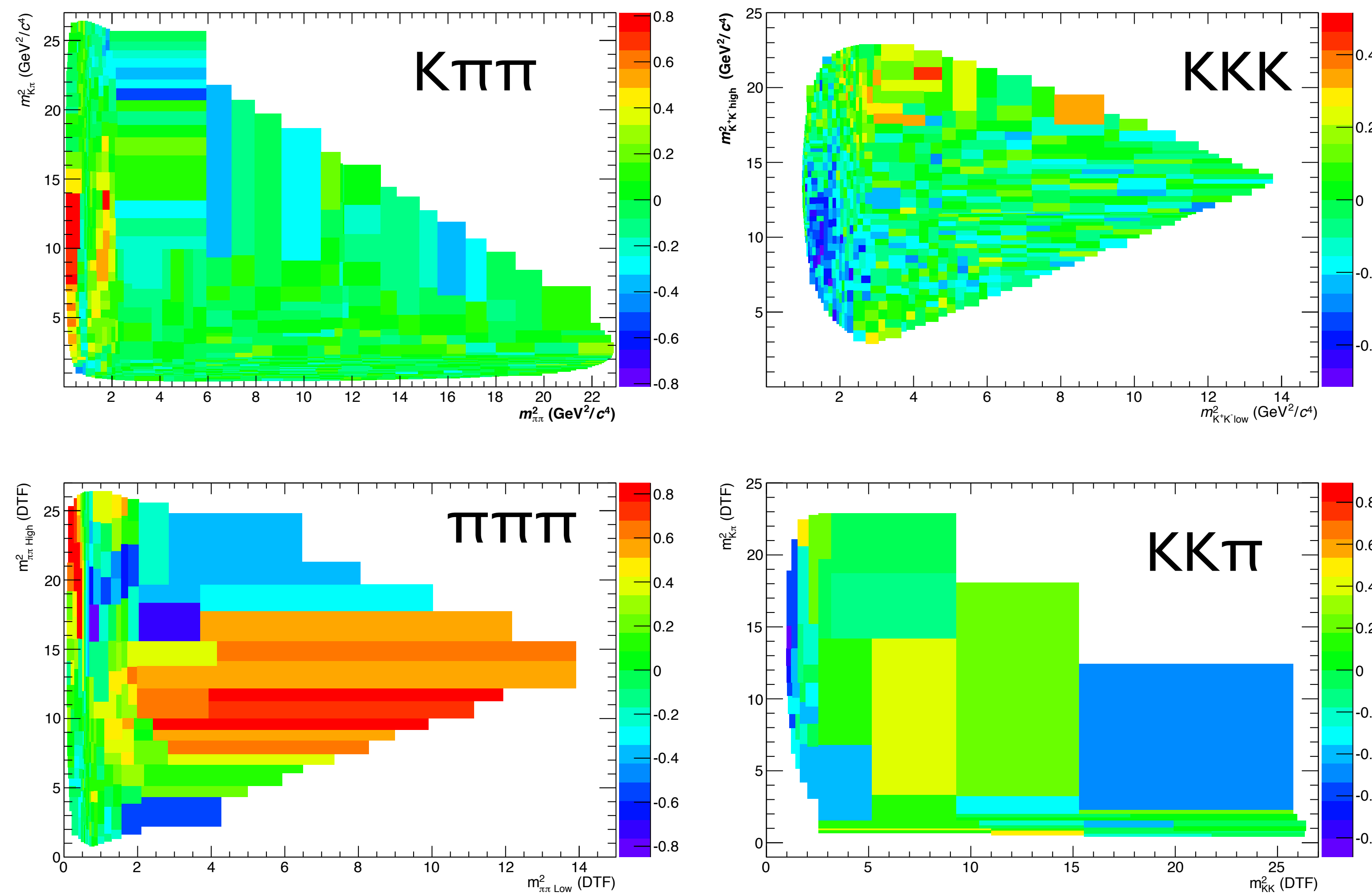




CPV on data: Puzzle!

- $B^{\pm} \to h^{\pm}h^{-}h^{+}$ massive localized Acp
  - suggest dynamic effect
  - middle looks “empty" → CPV

$$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$



LHCb PRD90 (2014) 112004

- BSS model  + 

not enough!!

- hadronic interactions → strong phase

- FSI → strong phase

Wolfenstein PRD43 (1991) 151

- $B^{\pm} \to h^{\pm}\pi^{-}\pi^{+}$ and $B^{\pm} \to h^{\pm}K^{-}K^{+}$ low-energy CPV with opposite signs

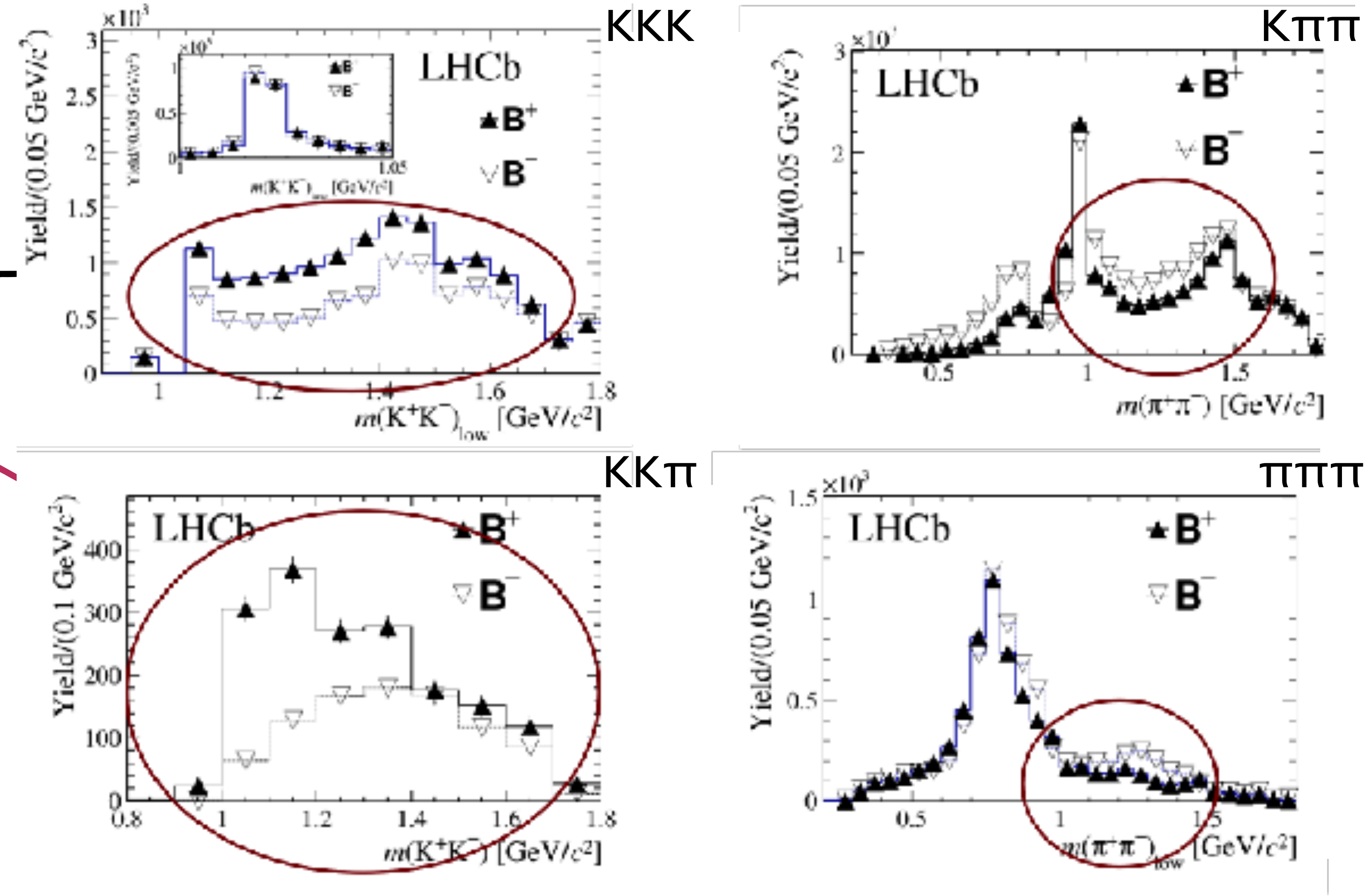
↳ $\pi\pi \to KK$

Frederico, Bediaga, Lourenço PRD89(2014)094013

CPV on data

- low-energy CPV [1 - 2] GeV

$$\pi\pi \rightarrow KK$$



- CPT:

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \bar{\Gamma}_{total}$

$$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$$

$$\bar{\Gamma}_{total} = \bar{\Gamma}_1 + \bar{\Gamma}_2 + \bar{\Gamma}_3 + \bar{\Gamma}_4 + \bar{\Gamma}_5 + \bar{\Gamma}_6 + \ldots$$

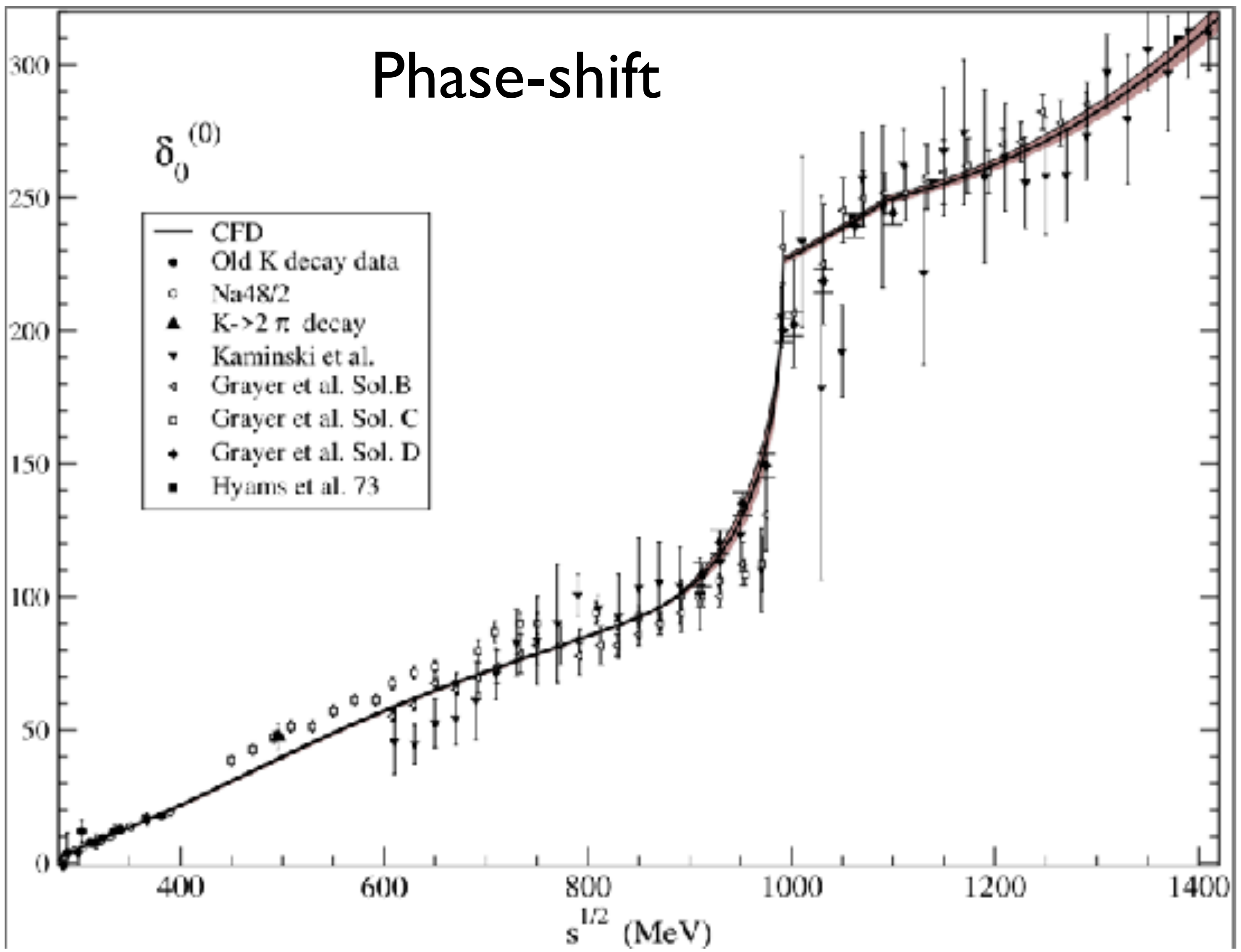
→ CPV in one channel should be compensated by another one with opposite sign

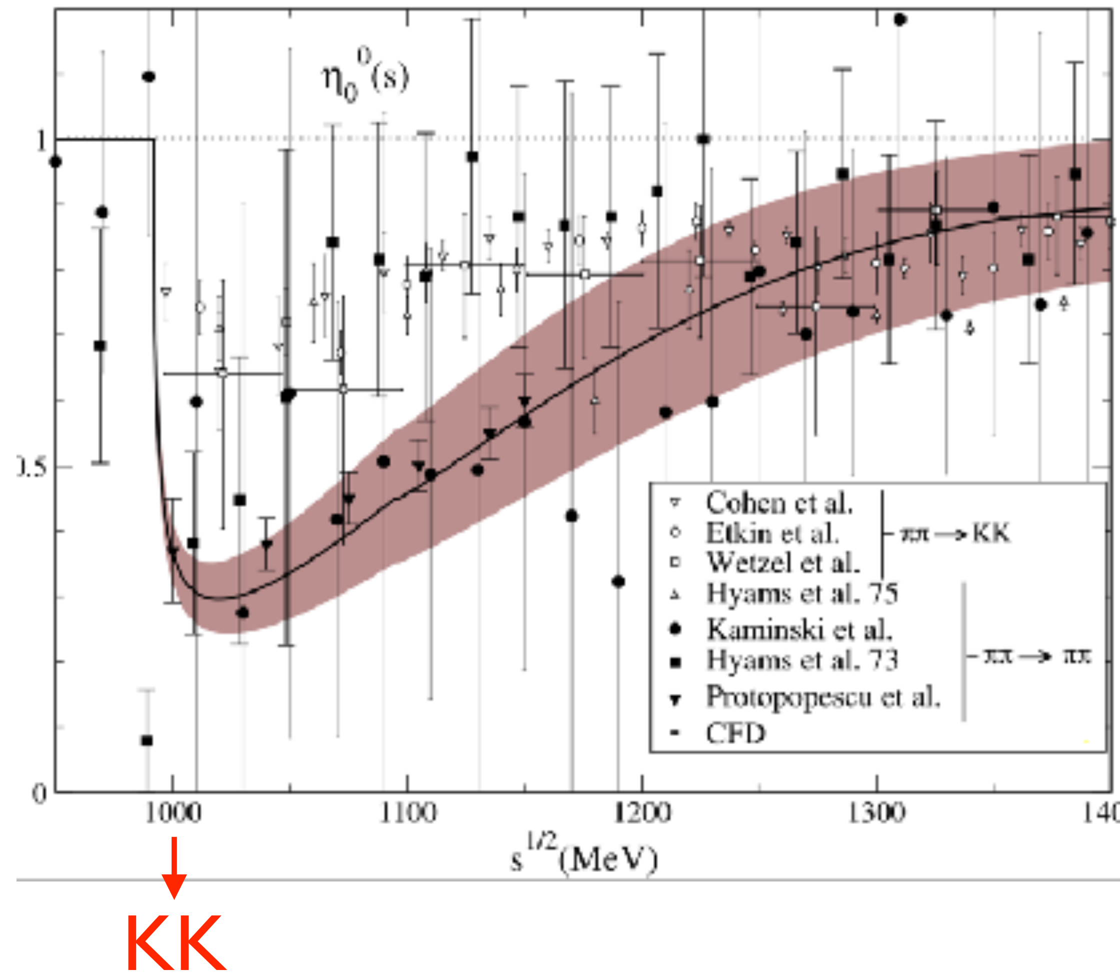
- $\pi\pi$ scattering data S-Wave

Pelaez, Yndurain PRD71(2011) 074016

- amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$

elasticity



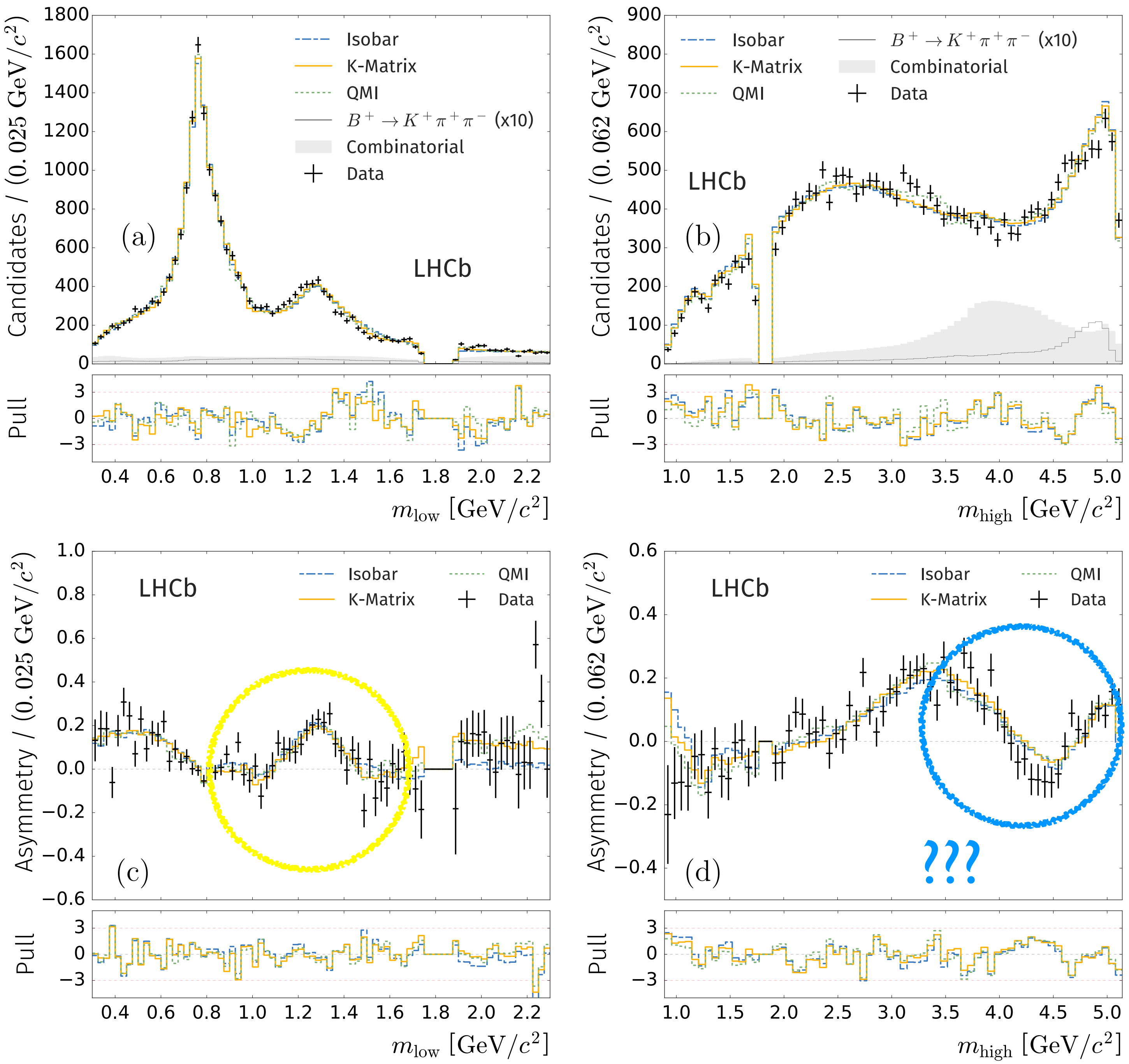


$$\sigma_l^{\rm el} = \frac{1}{2}\left\{\frac{1+\eta_l^2}{2} - \eta\cos 2\delta_l\right\},$$

elasticity: one minus the probability of losing signal (1==elastic)

CPV: amplitude analysis $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$

- LHCb recent Amplitude analysis $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$ [arXiv:1909.05212(PRD); 1909.05211(PRL)]
  - $(\pi^{-}\pi^{+})_{S-Wave}$ 3 different model:
    - ↪ σ as BW (!) + rescattering;
    - ↪ P-vector K-Matrix;
    - ↪ binned freed lineshape (QMI);



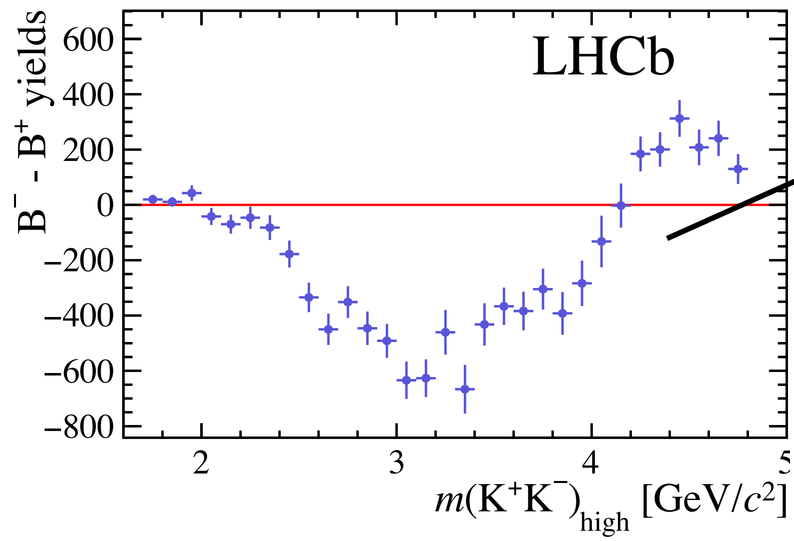
Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

- ANA for $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$ [arXiv:1905.09244]

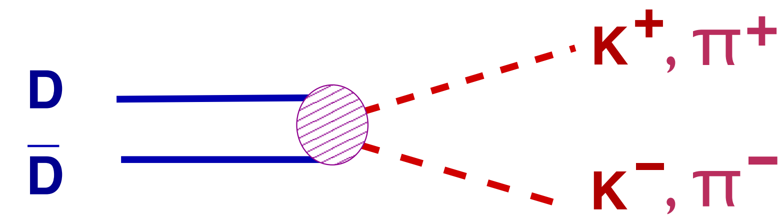
Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[°] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$ $1.06 \pm 0.04 \pm 0.02$	0 (fixed) 0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$ $0.82 \pm 0.09 \pm 0.10$	$-176 \pm 10 \pm 16$ $136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$ $1.97 \pm 0.12 \pm 0.20$	$-138 \pm 7 \pm 5$ $166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$ $1.92 \pm 0.10 \pm 0.07$	$-175 \pm 10 \pm 15$ $140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$ $1.13 \pm 0.08 \pm 0.05$	$-106 \pm 11 \pm 10$ $-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$ $0.86 \pm 0.07 \pm 0.04$	$-56 \pm 12 \pm 18$ $-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$ $0.22 \pm 0.06 \pm 0.04$	$-52 \pm 23 \pm 32$ $107 \pm 33 \pm 41$

CPV high energy

- CPV high mass?



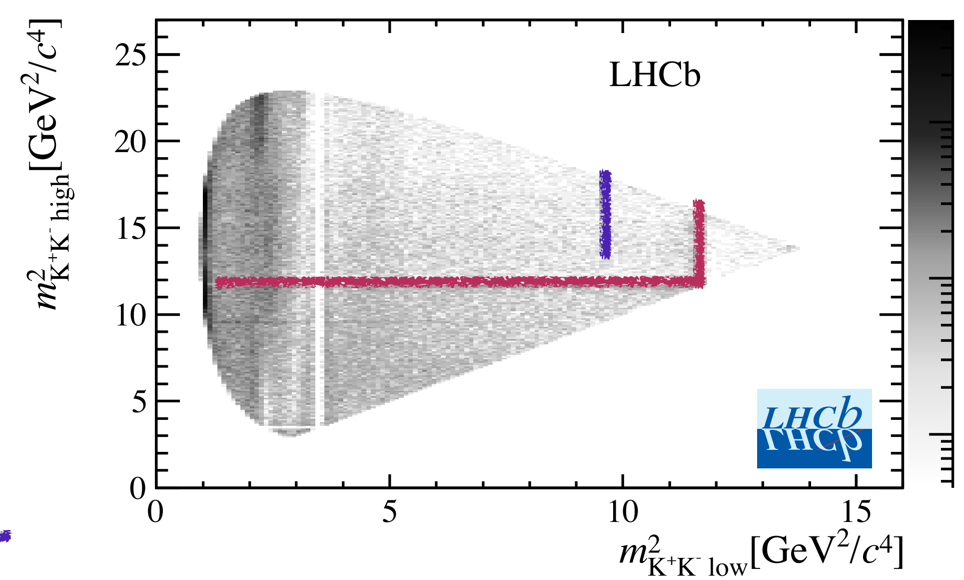
~ $D\bar{D}$ open channel →

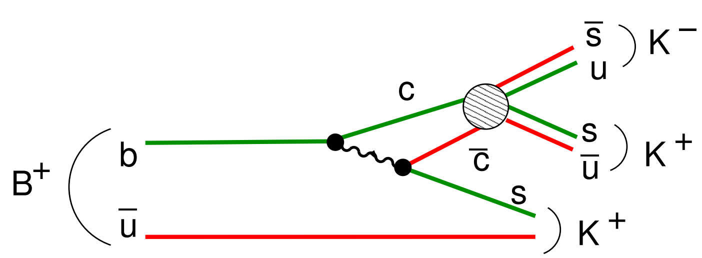


same observed in coupled-channels

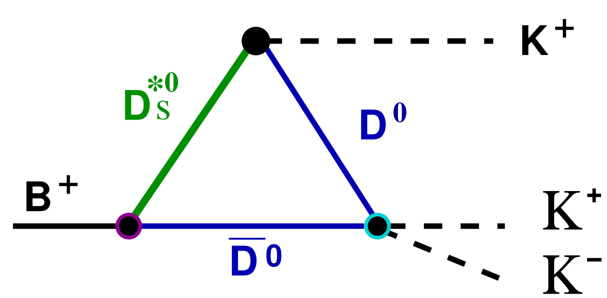
→ **charm intermediate processes as source of strong phase**

- $B^+ \to K^- K^+ K^+$
  - high statistic 109k
  - nonresonant → all phase-space
  - presence of charm resonances: χ_{c0} J/ψ
  - dominated by penguin





→

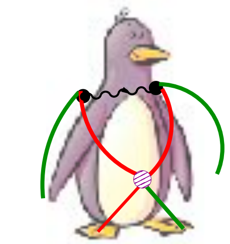


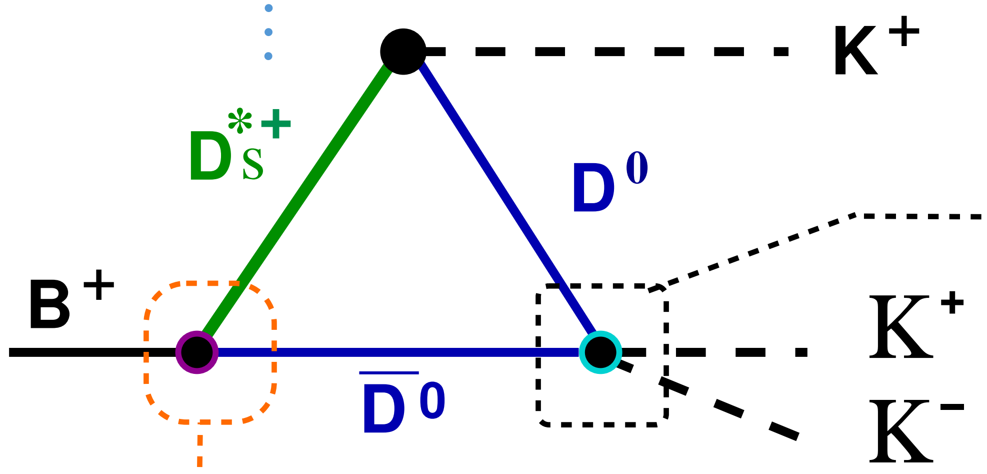
charm rescattering!

I. Bediaga, PCM, T Frederico
PLB 780 (2018) 357

hadronic loop

I. Bediaga, PCM, T Frederico
PLB 780 (2018) 357





form factor for $W^+ \to D^0 K^+$

$D^0 \bar{D}^0 \to K^+ K^-$ scattering amplitude

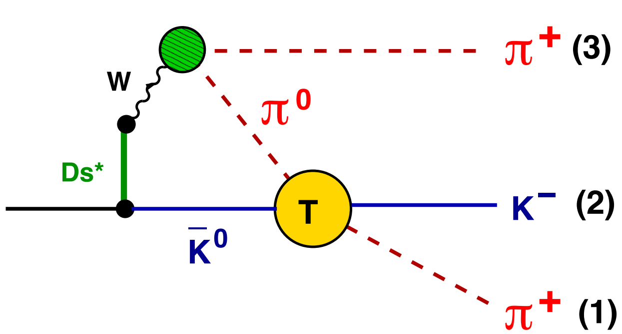
phenomenological:
S- matrix unitarity + Regge theory

weak transition $B^+ \to W^+ \bar{D}^0$ → $C_0 \times$ form factor to regulate

- $Br\,[B \to D D_s^*]$ ~1% → 1000 x $Br\,[B \to KKK]$
- hadronic loop → three-body FSI - introduce new complex structures

- $B^+ \to \pi^+ \pi^- \pi^+$
- $D^+ \to \pi^+ K^- \pi^+$

PCM & I Bediaga
arXiv:1512.09284

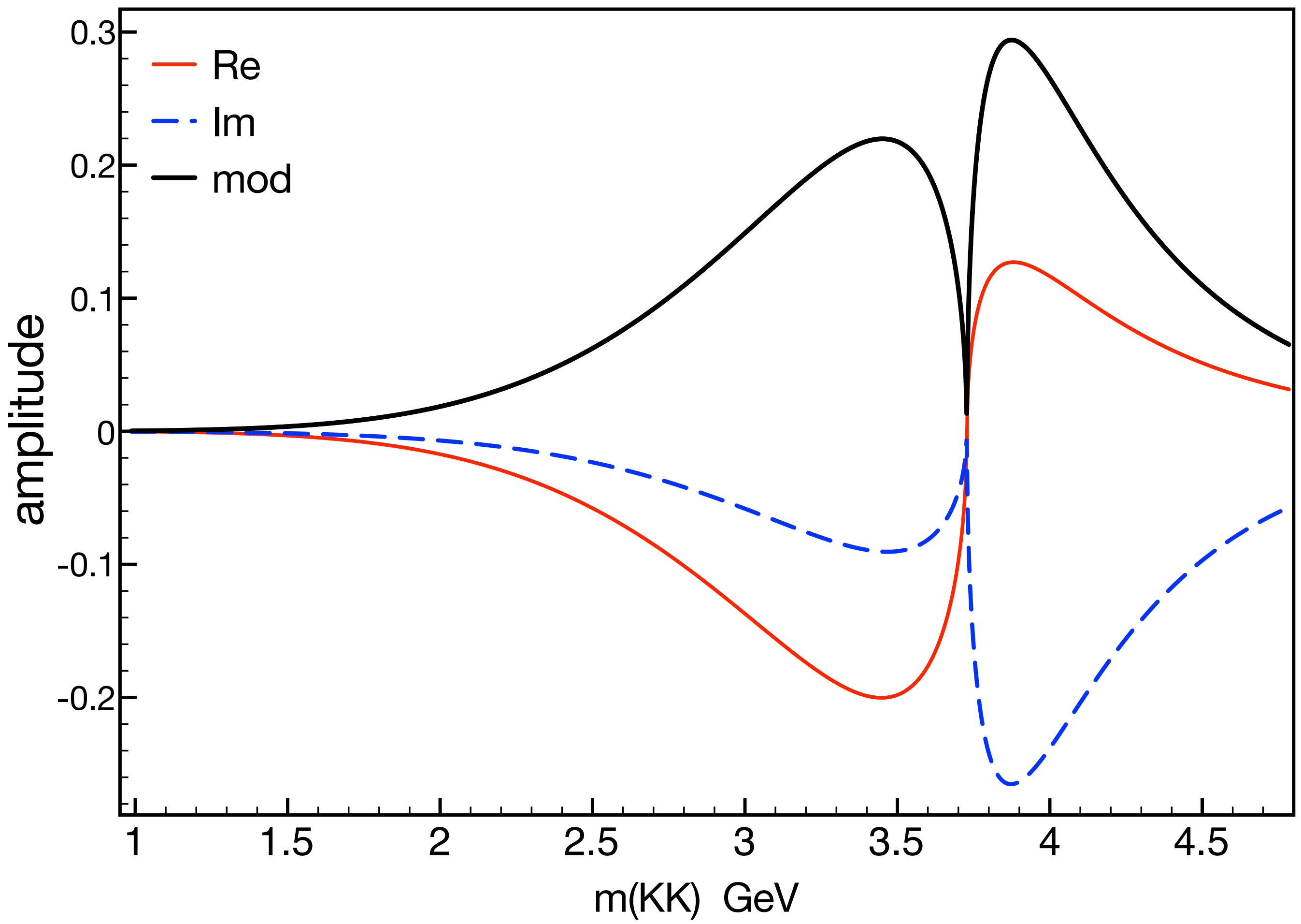


PCM & M Robilotta
PRD 92 094005 (2015) [arXiv:1504.06346]
PCM et al
PRD 84 094001 (2011) [arXiv:1105.5120]

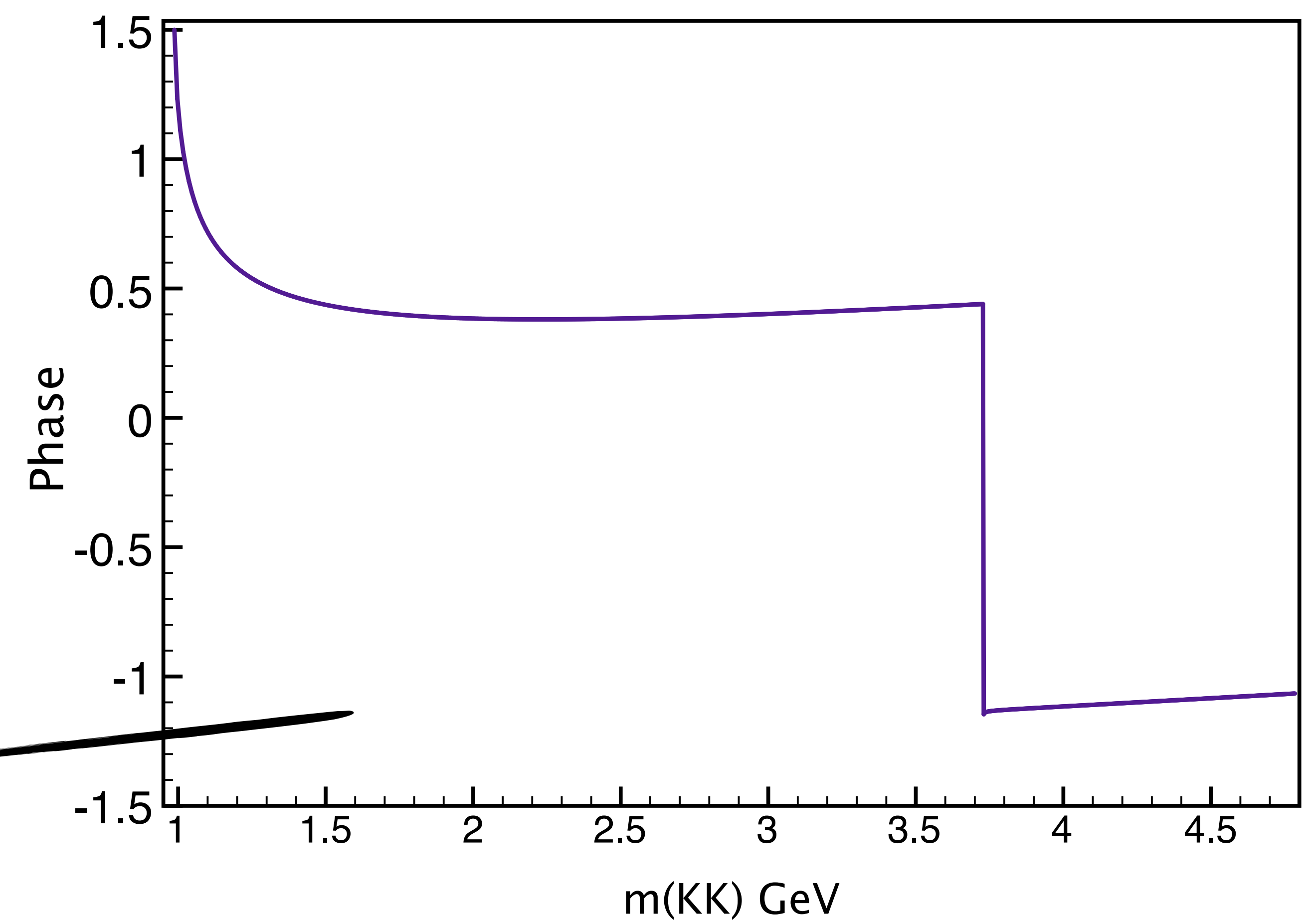
$D^0\bar{D}^0 \rightarrow K^+K^-$ scattering amplitude

- $$T_{\bar{D}^0 D^0 \to KK}(s) = \frac{s^\alpha}{s^\alpha_{th\,D\bar{D}}}\,\frac{2\kappa_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi+\alpha}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}+\kappa_2}{\frac{1}{a}-\kappa_2}\right)\right]^{\frac{1}{2}},\quad s < s_{th\,D\bar{D}}$$
$$= -i\,\frac{2\,k_2}{\sqrt{s_{th\,D\bar{D}}}}\left(\frac{s_{th\,D\bar{D}}}{s+s_{QCD}}\right)^{\xi}\left(\frac{m_0}{s-m_0}\right)^{\beta}\left[\left(\frac{c+bk_1^2-ik_1}{c+bk_1^2+ik_1}\right)\left(\frac{\frac{1}{a}-ik_2}{\frac{1}{a}+ik_2}\right)\right]^{\frac{1}{2}},\; s \geq s_{th\,D\bar{D}}$$

→ parameters fix by data!



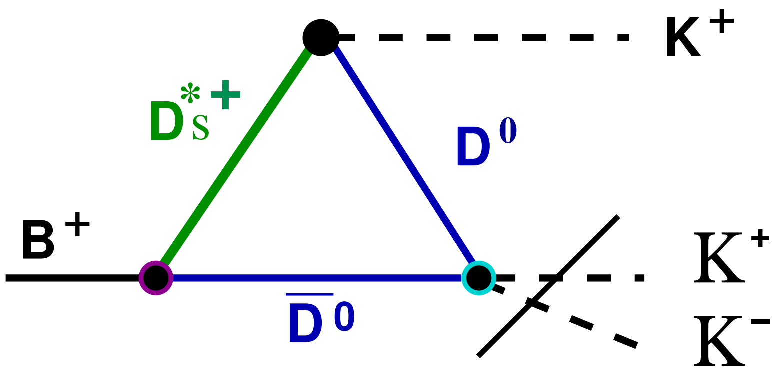
→ zero at threshold

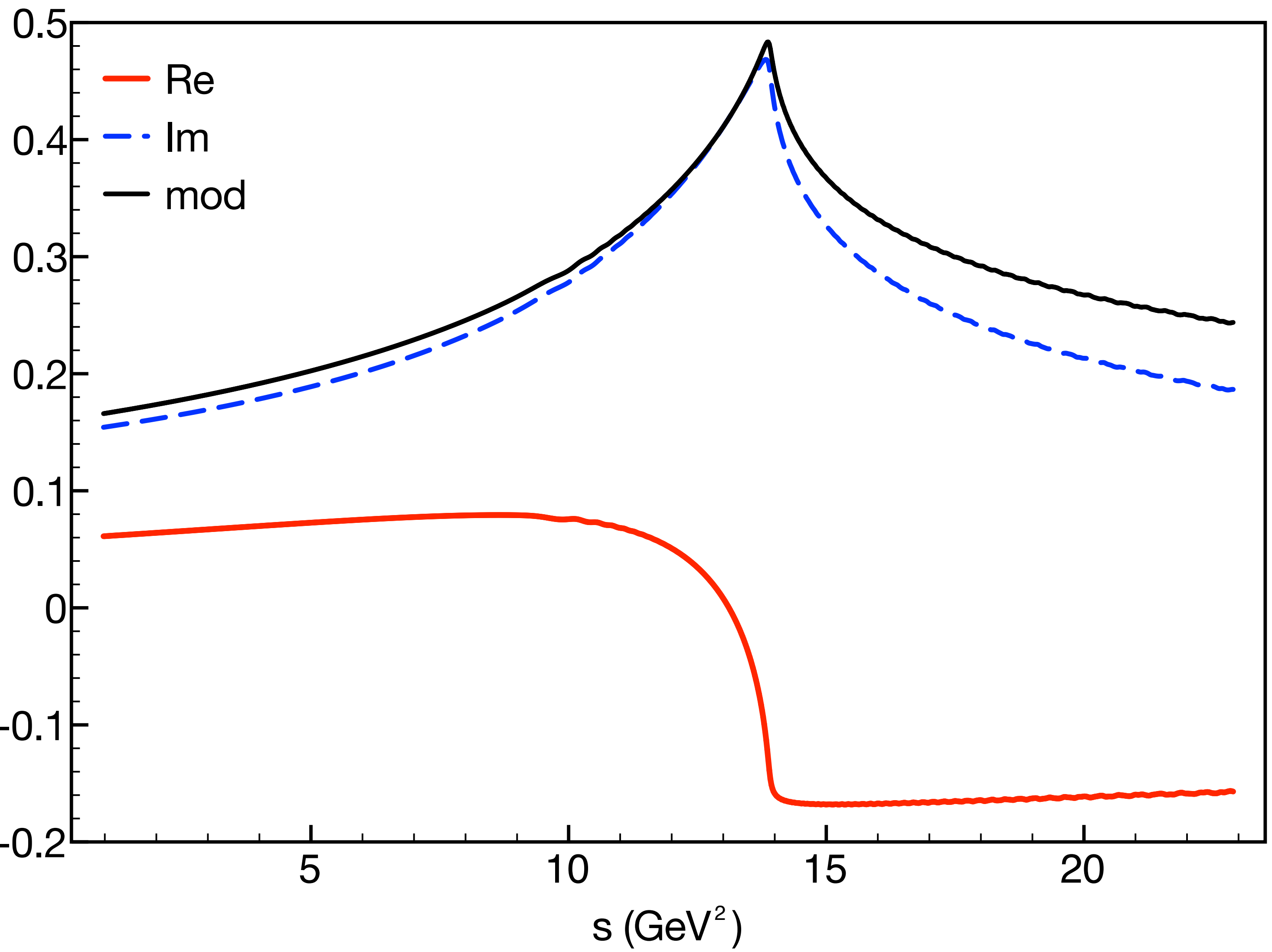


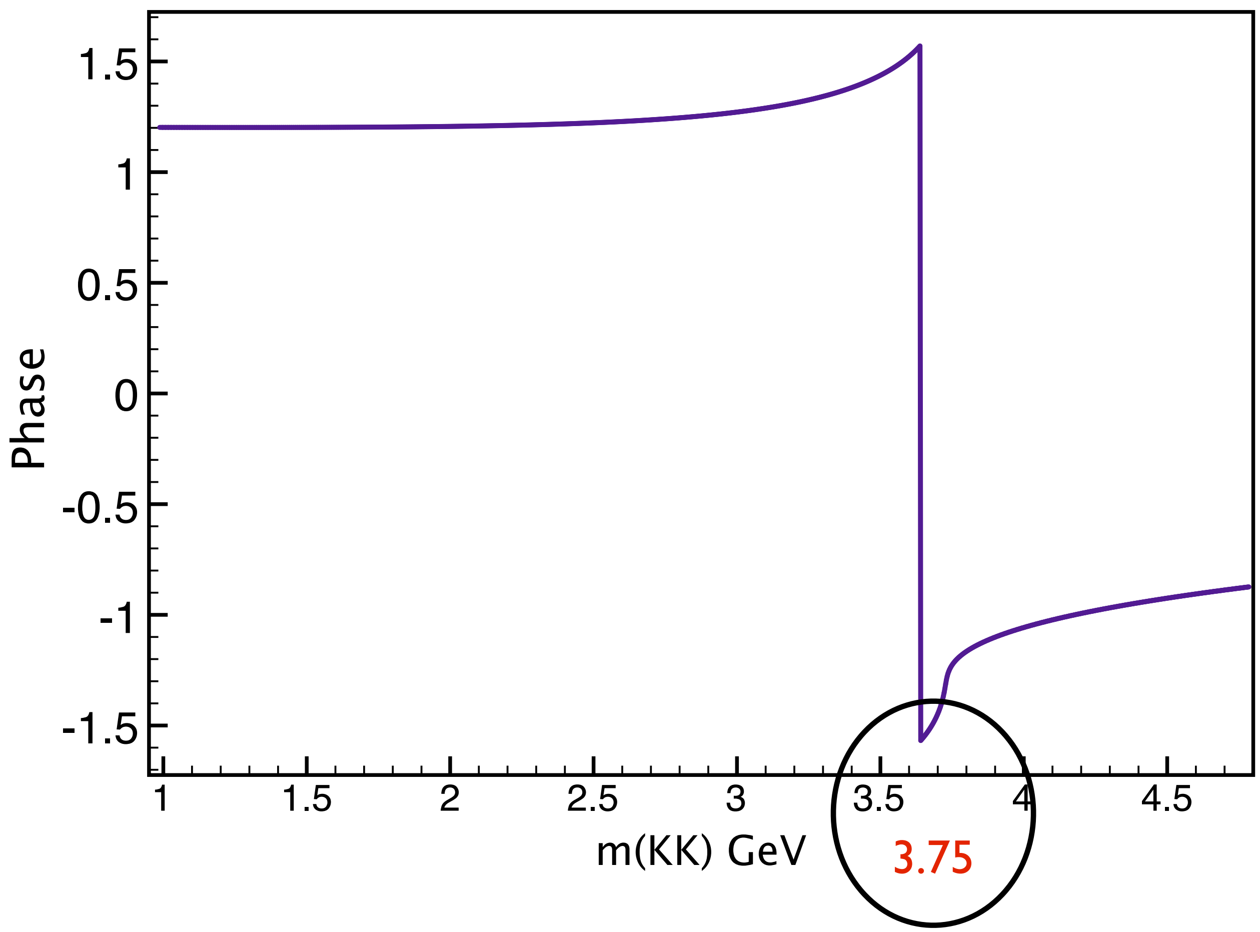
discontinuity at threshold ←

hadronic loop

- $$Loop = i \int \frac{d^4\ell}{(2\pi)^4} \, \frac{\Delta_{D^0} + 2\,\Delta_{\bar{D}^0} - 2\,s_{23} + 3\,M_K^2 + M_B^2 - l^2}{\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_{D*}\,[l^2 - m_{B^*}]}$$



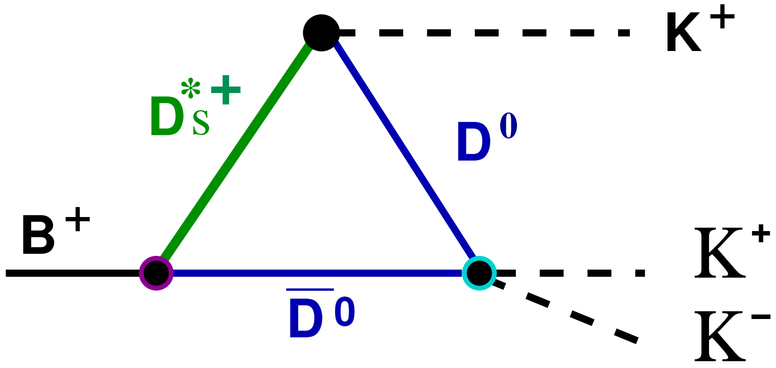


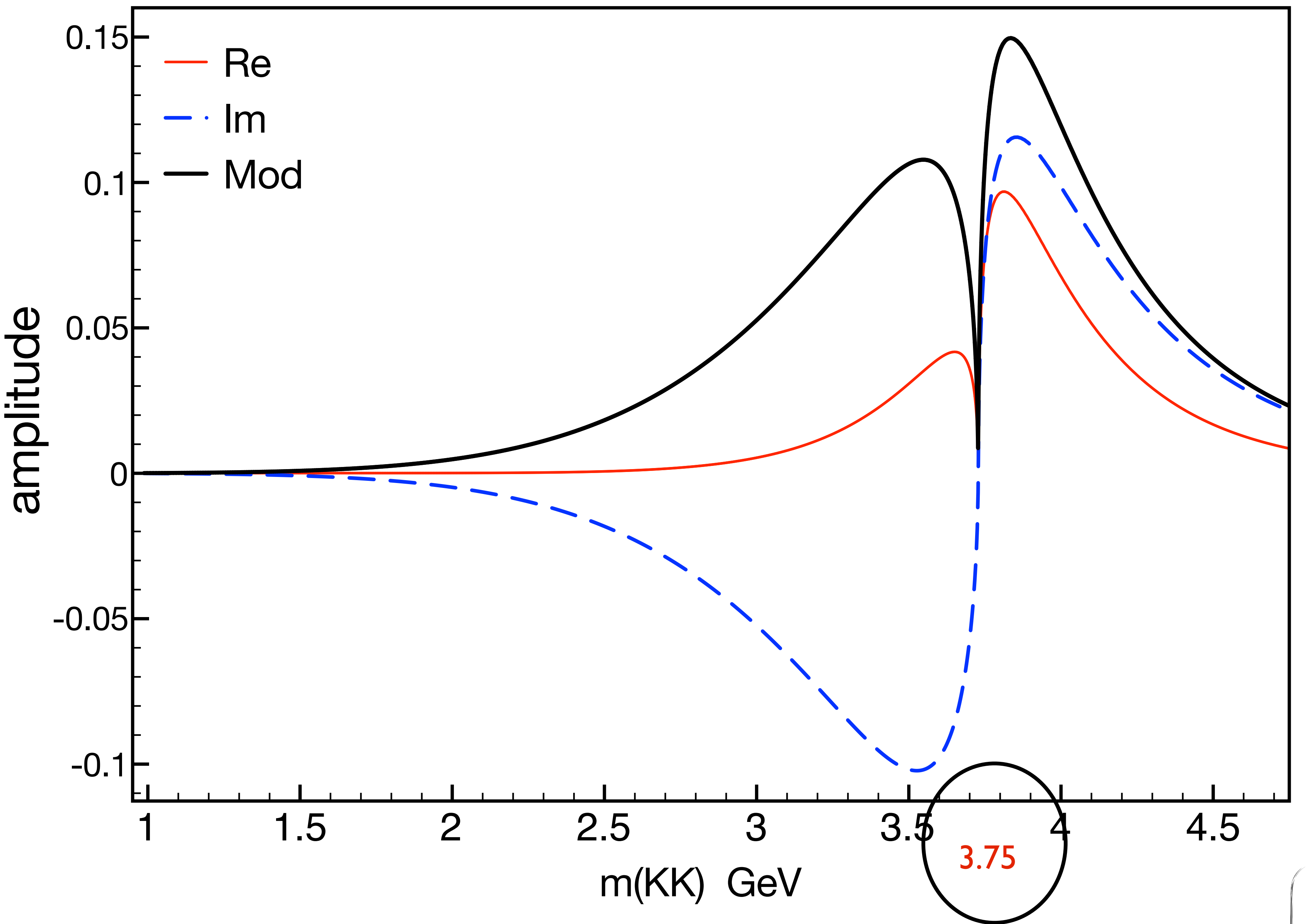


discontinuity at threshold

→ change sign at threshold

- $$A = iC\ m_a^2 \int \frac{d^4\ell}{(2\pi)^4}\ \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23})\,[-2\,p_3' \cdot (p_2' - p_1)]}{\Delta_{D^{+*}}\,\Delta_{D^0}\,\Delta_{\bar{D}^0}\,\Delta_a}\ ,$$

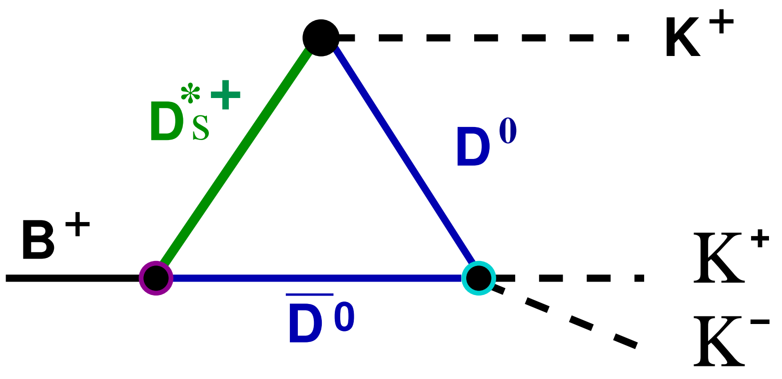


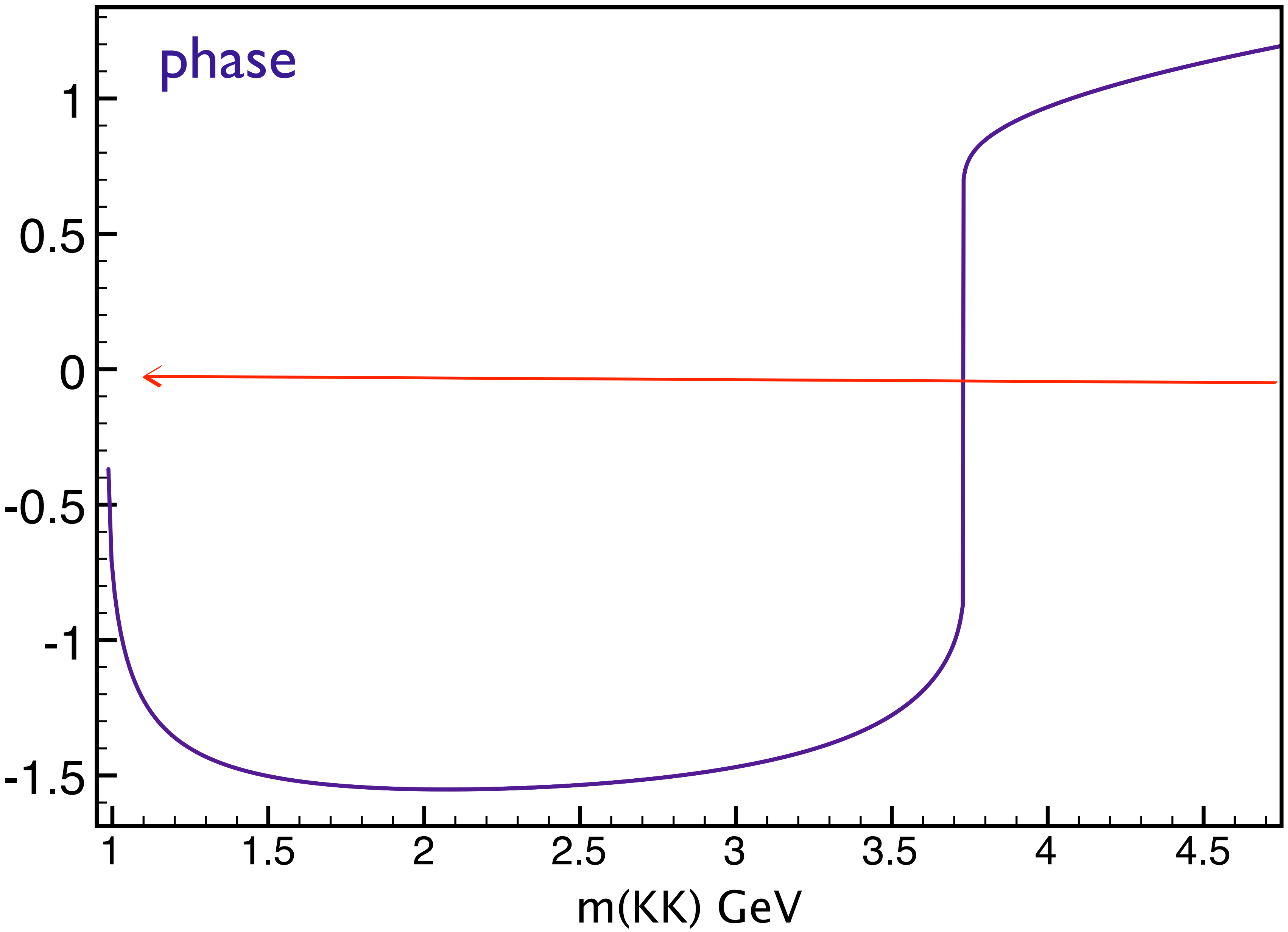


→ zero in between two bumps

rescattering $D^0\bar{D}^0 \to K^+K^-$ play a major role

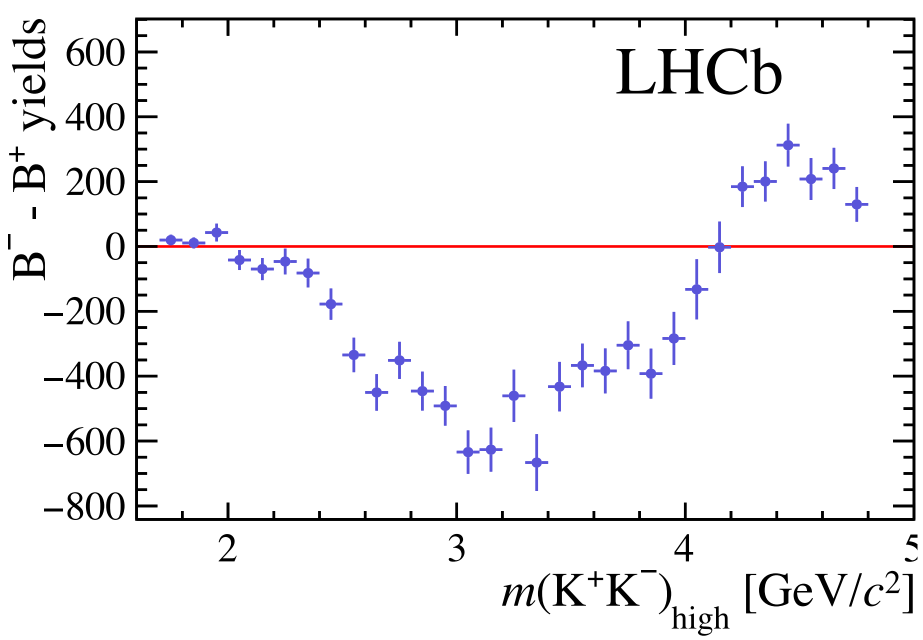
- $$A = iC \; m_a^2 \int \frac{d^4\ell}{(2\pi)^4} \frac{T_{\bar{D}^0 D^0 \to KK}(s_{23}) \left[-2\, p'_3 \cdot (p'_2 - p_1)\right]}{\Delta_{D^{+*}} \Delta_{D^0} \Delta_{\bar{D}^0} \Delta_a} ,$$





→ Phase change signal in the same region as Acp data

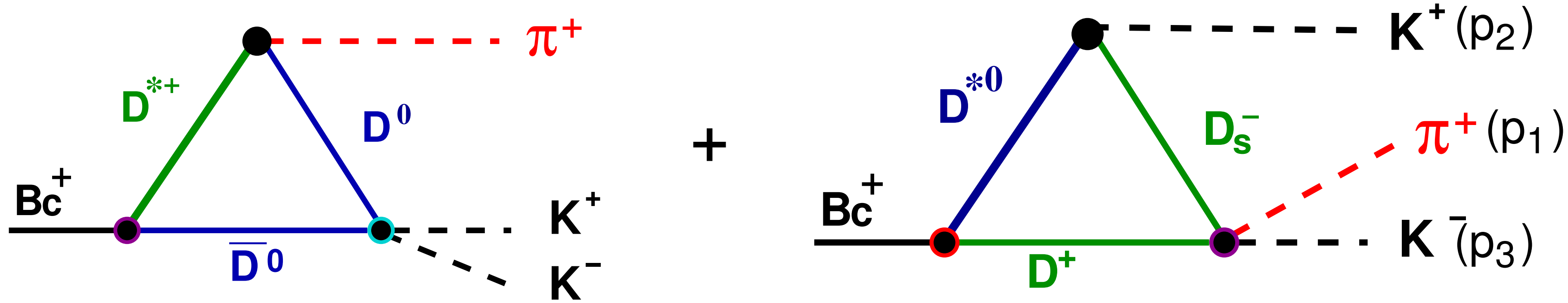
Promising mechanism !



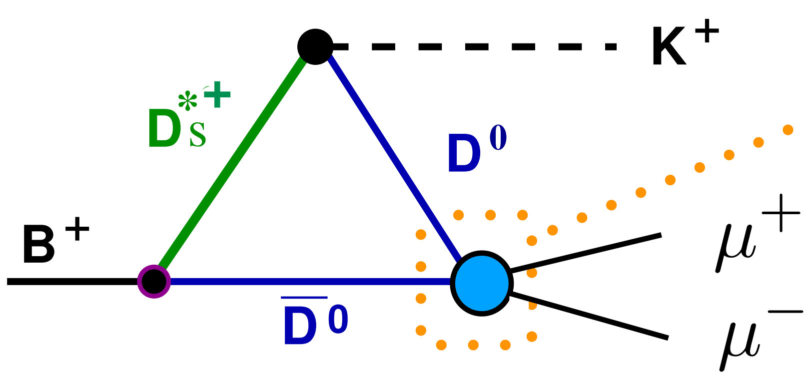
→ can explain change CPV signal in DP!!!

charm rescattering to $B_c^+ \to K^- K^+ \pi^+$

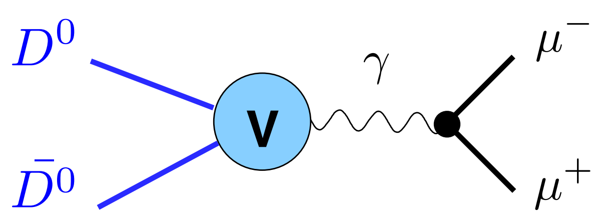
I. Bediaga, PCM, T Frederico
PLB 785 (2018) 581



Next: investigating Hadronic effect in $B \to K\mu\mu$



$\bar{D}^0 D^0 \to \gamma \to \mu^+\mu^-$



vector meson dominance

V= all vector family psi, …

How much of the anomalies can be understood as hadronic effects?

Charm rescattering

- $B_c^+ \to K^- K^+ \pi^+$

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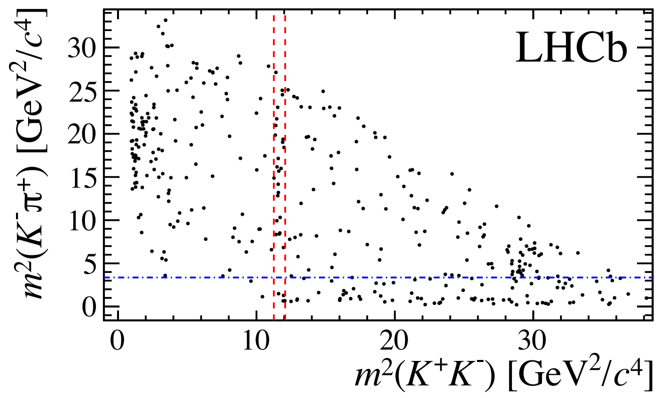
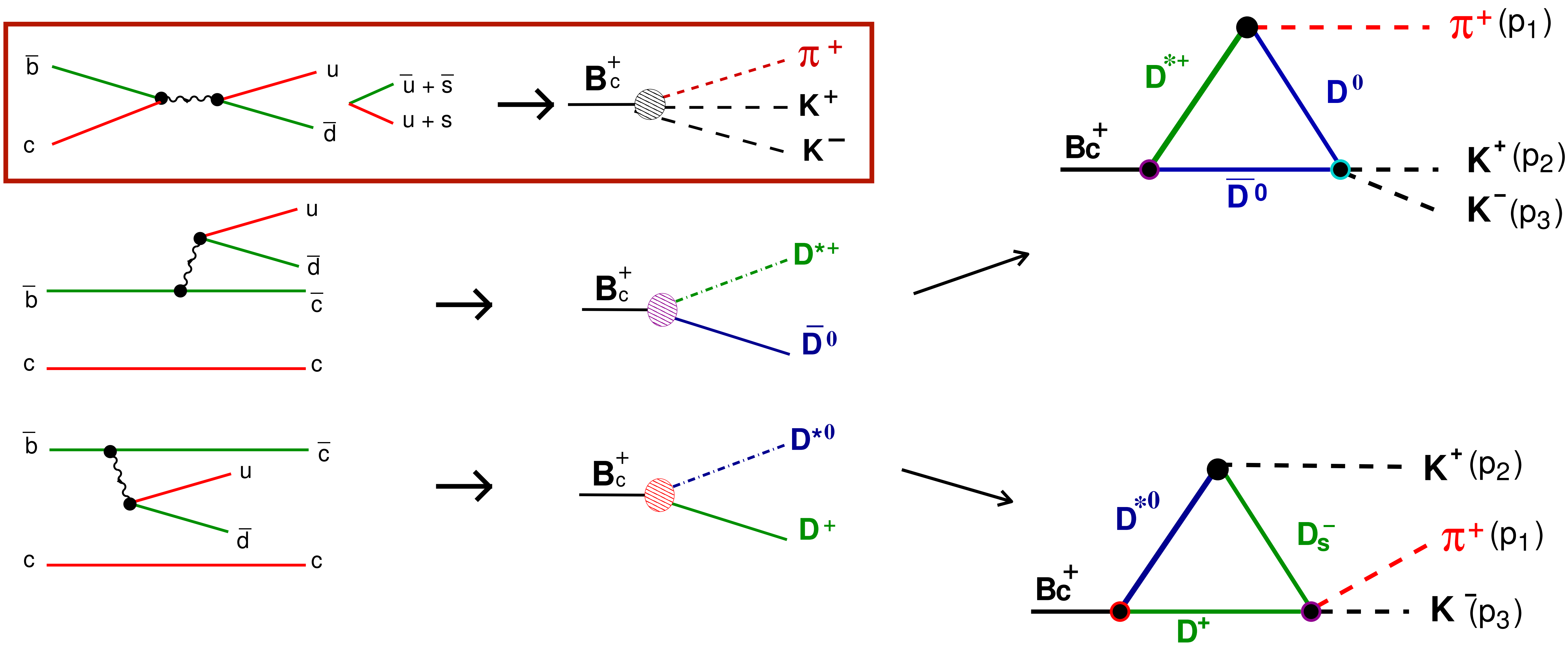
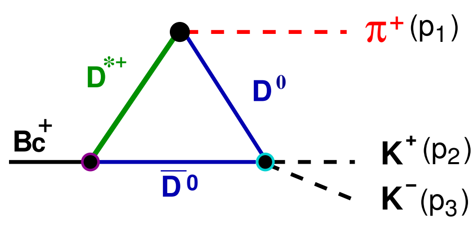


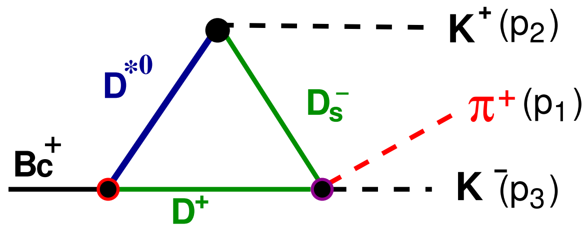
- very suppressed direct production (annihilation)
- charm rescattering can be the dominant mechanism

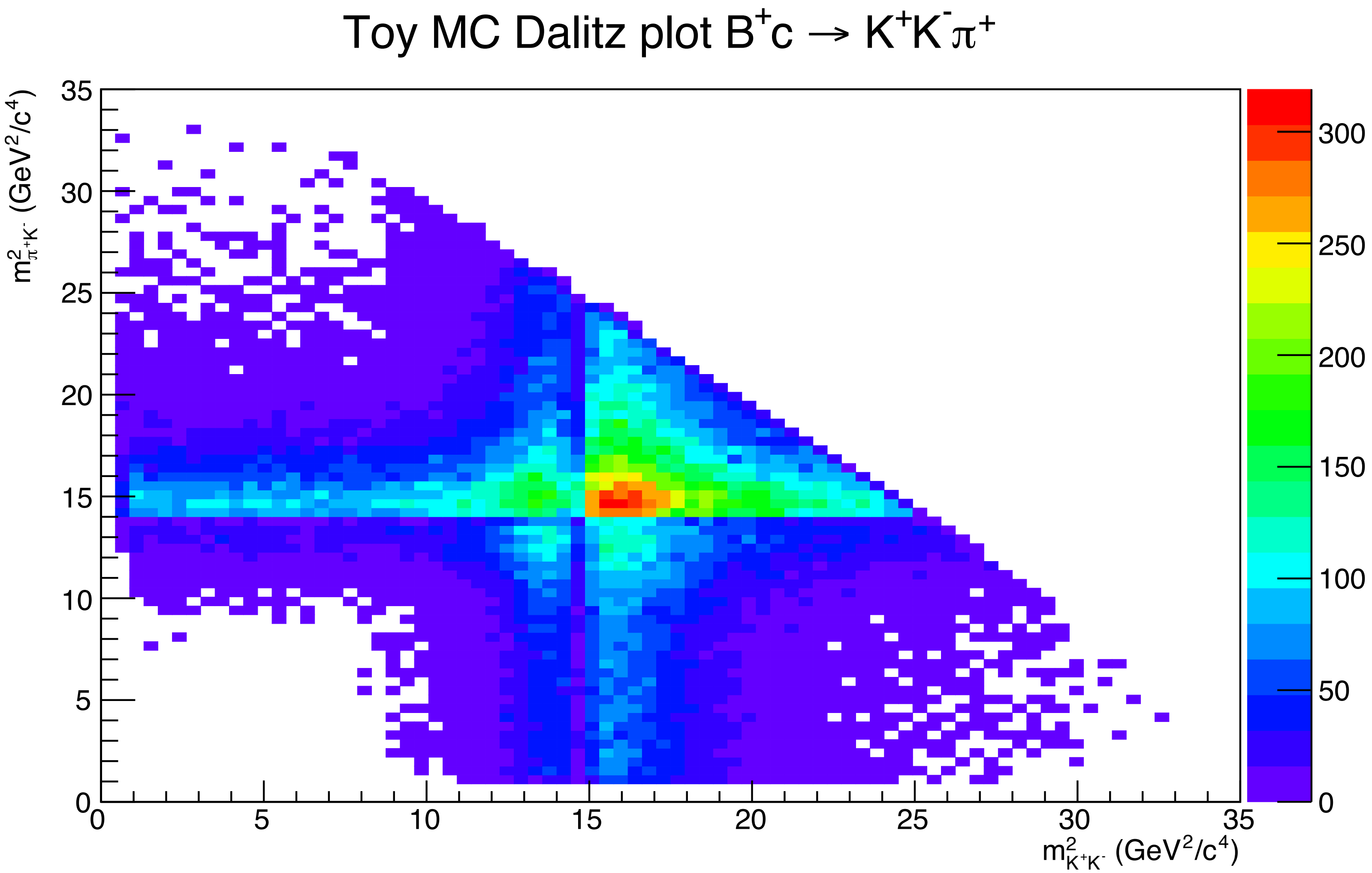


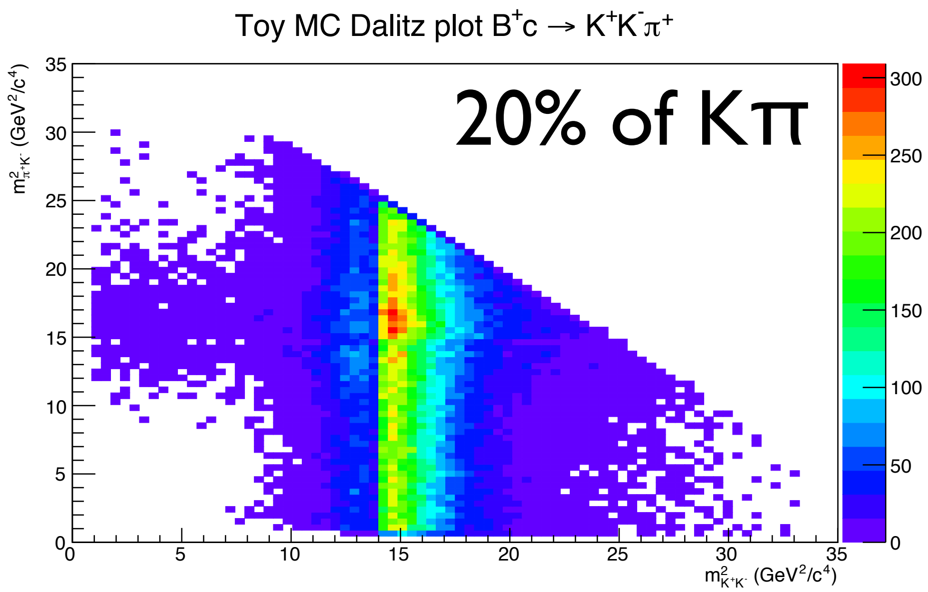
- toy Monte Carlo generator



&





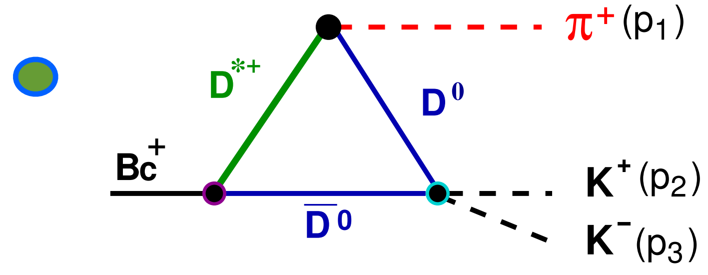


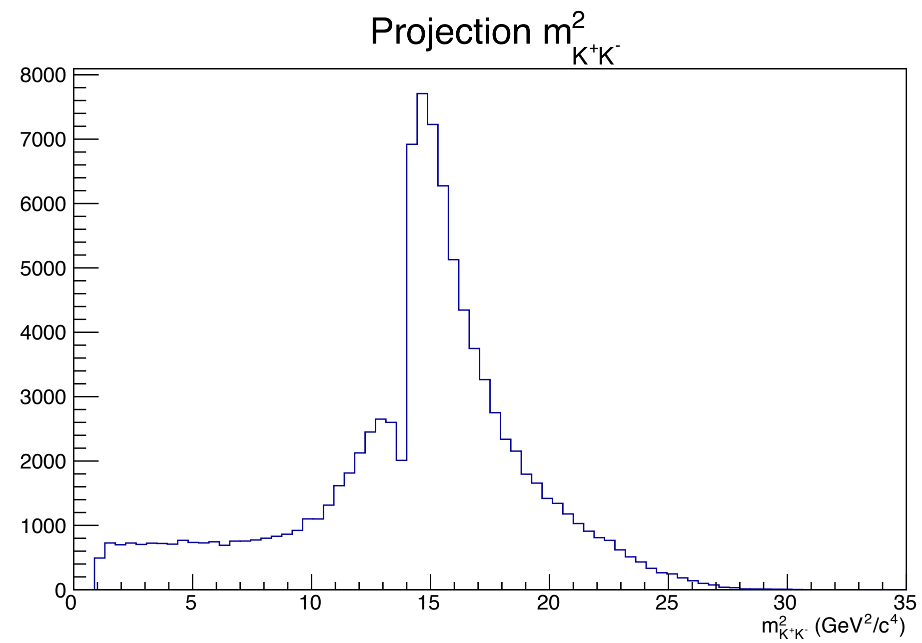
↓

change side bands but not the minimum position (thresholds)

→ leave a signature in the middle of the Dalitz plot

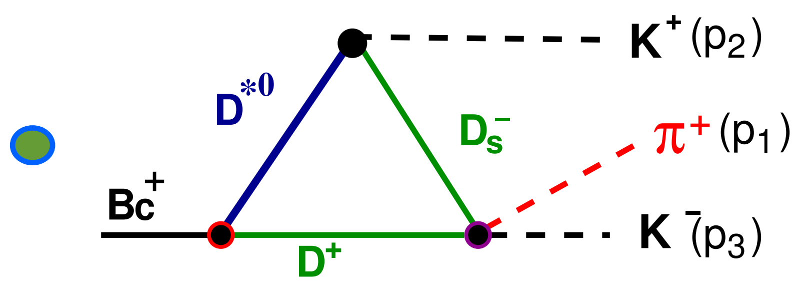
- Amplitudes projections

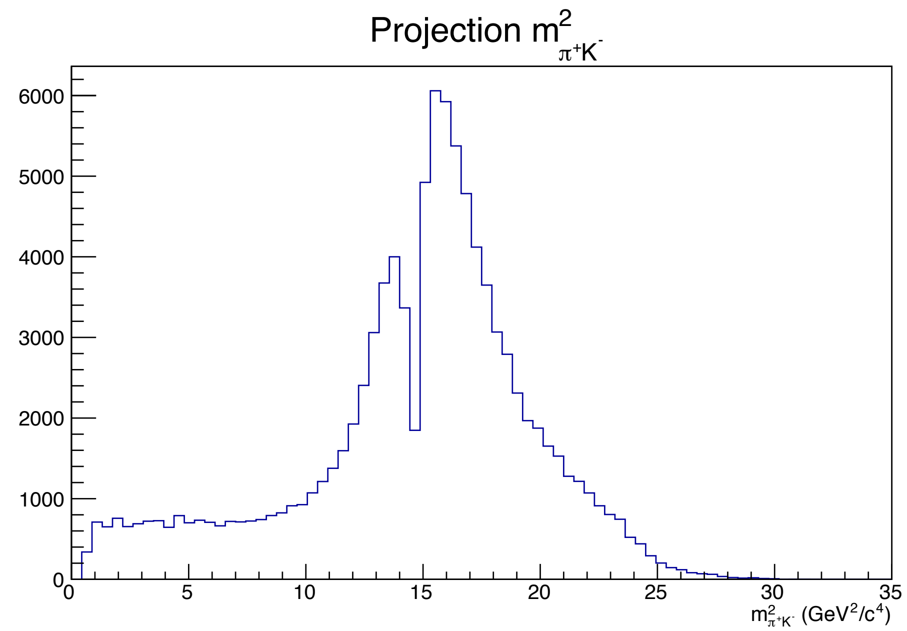




⟶ minima in different positions (≠ thresholds)

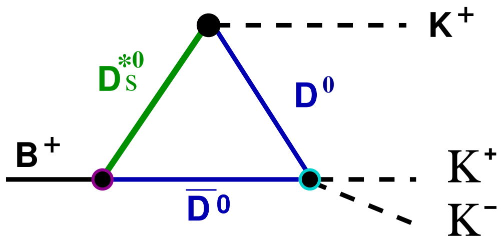
⟶ ≠ mass parameters inside triangle and rescattering amplitudes are relevant





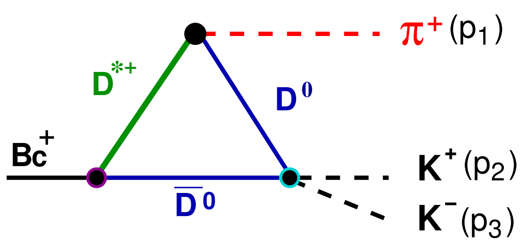
Triangle hadronic loop with charm FSI play an important role!

- $B^{\pm} \to K^{+}K^{-}K^{+}$



→ mechanism to produce CP asymmetry at high mass

- $B_c^{+} \to K^{-}K^{+}\pi^{+}$



→ main mechanism to produce this final state

- If direct production (annihilation) → expect resonances in KK and Kπ channels
  → not observed

- real data: interference between ≠ triangles & NR sources & resonances
  → canNOT change the signature of a minima between the bumps and phases!
  → LHCb run 2 to confirm

FSI are important and play a major role in hadronic 3-body decays!

→ superposition of resonant and non-resonant at low and high energy

→ Charm rescattering is under intense investigation : CPV on B, exotics, anomalies, ……

- Lots of theoretical limitations to be developed:
  - need to merge the short and long distance descriptions!
  - extend the meson-meson interaction to high E, …
- Successful examples of cooperation between theory and experiment !!!

→ Important tool !

Thank you very much!

image credit: unknown

Backup slides

$D^0\bar{D}^0 \to K^+K^-$ scattering amplitude

- not well understand on literature

Donoghue *et al.*, *PRL* 77(1996)2178;
Suzuki,Wolfenstein, PRD 60 (1999)074019;
Falk et al. PRD 57,4290(1998);
Blok, Gronau, Rosner, PRL 78, 3999 (1997).

- important as FSI in B two-body decays

- phenomenological amplitude

Antunes, Bediaga, Frederico, PCM
ICHEP2016 - proceedings

  - unitarity of the S-matrix $S = \begin{pmatrix} \eta\, e^{2i\alpha} & \sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} \\ -\sqrt{1-\eta^2}\, e^{i(\alpha+\beta)} & \eta\, e^{2i\beta} \end{pmatrix}$

  - inspired in the damping factor of the S matrix i.e. $\pi\pi \to KK$

$$\eta = \mathcal{N}\sqrt{s/s_{th}-1}/(s/s_{th})^{2.5}$$

KK: $e^{2i\alpha} = 1 - \dfrac{2ik_1}{\frac{c}{1-k_1/k_0} + ik_1}$, DD: $e^{2i\beta} = 1 - \dfrac{2ik}{\frac{1}{a} + ik}$

$k = \sqrt{\frac{s-s_{th}}{4}}$, $k_1 = \sqrt{\frac{s-s_{th1}}{4}}$ and $k_0 = \sqrt{\frac{s_0-s_{th}}{4}}$

$$S_{\beta,\alpha} = \delta_{\beta,\alpha} + it_{\beta,\alpha}$$

$$t_{\beta,\alpha} = \sqrt{1-\eta^2}e^{i(\alpha+\beta)}$$

FSI in three-body decay :

references

I. Bediaga, I., T. Frederico, T. and O. Louren   Phys. Rev. D89, 094013(2014),[arXiv:1307.8164]

J. H. Alvarenga Nogueira, I. Bediaga, A. B. R. Cavalcante, T. Frederico and O. Louren¸ Phys. Rev. D92, 054010 (2015) [ArXiv:1506.08332].

PC Magalhães and I Bediaga   arXiv:1512.09284;

P. C Magalhães and R.Robilotta, Phys. Rev. D92 094005 (2015) [arXiv:1504.06346] ; P.C.Magalhães et. al. Phys. Rev. D84 094001 (2011) [arXiv:1105.5120]; P.C. Magalhães and Michael C. Birse, PoS QNP2012, 144 (2012).

I. Caprini, Phys. Lett. B 638 468 (2006).

Bochao Liu, M. Buescher, Feng-Kun Guo, C. Hanhart, and Ulf-G. Meissner, Eur. Phys. J. C 63 93 (2009).

F Niecknig and B Kubis - JHEP 10 142 (2015) ArXiv:1509.03188

H. Kamano, S.X. Nakamura, T.-S.H. Lee and T. Sato, Phys. Rev. D 84, 114019 (2011).

S. X. Nakamura, arXiv:1504.02557 (2015).

J. -P. Dedonder, A. Furman, R. Kaminski, L. Lesniak, L. and B. Loiseau, Acta Phys. Polon. B42, 2013 (2011), [Arxiv: 1011.0960]

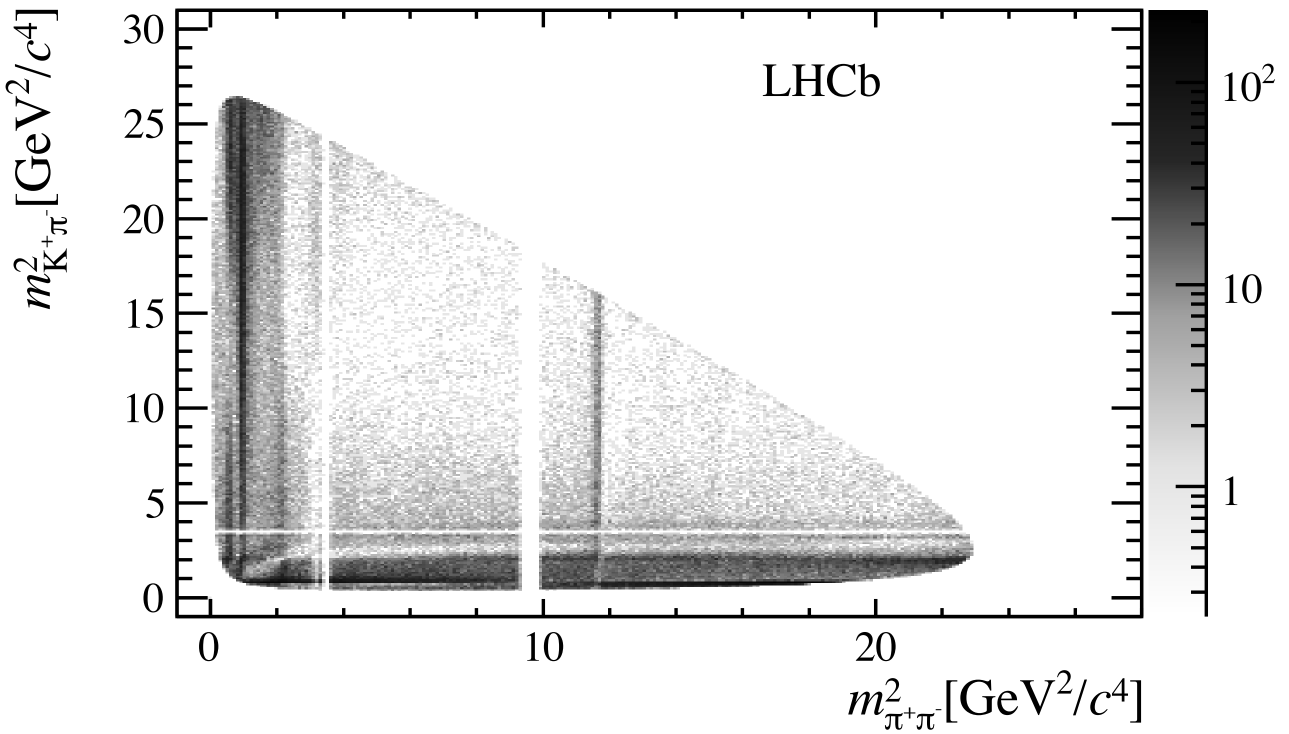
J.-P. Dedonder, R. Kaminski, L. Lesniak, and B. Loiseau, , Phys. Rev.D89, 094018 (2014).

Donoghue *et al.*, *Phys. Rev Letters* 77(1996) 2178;

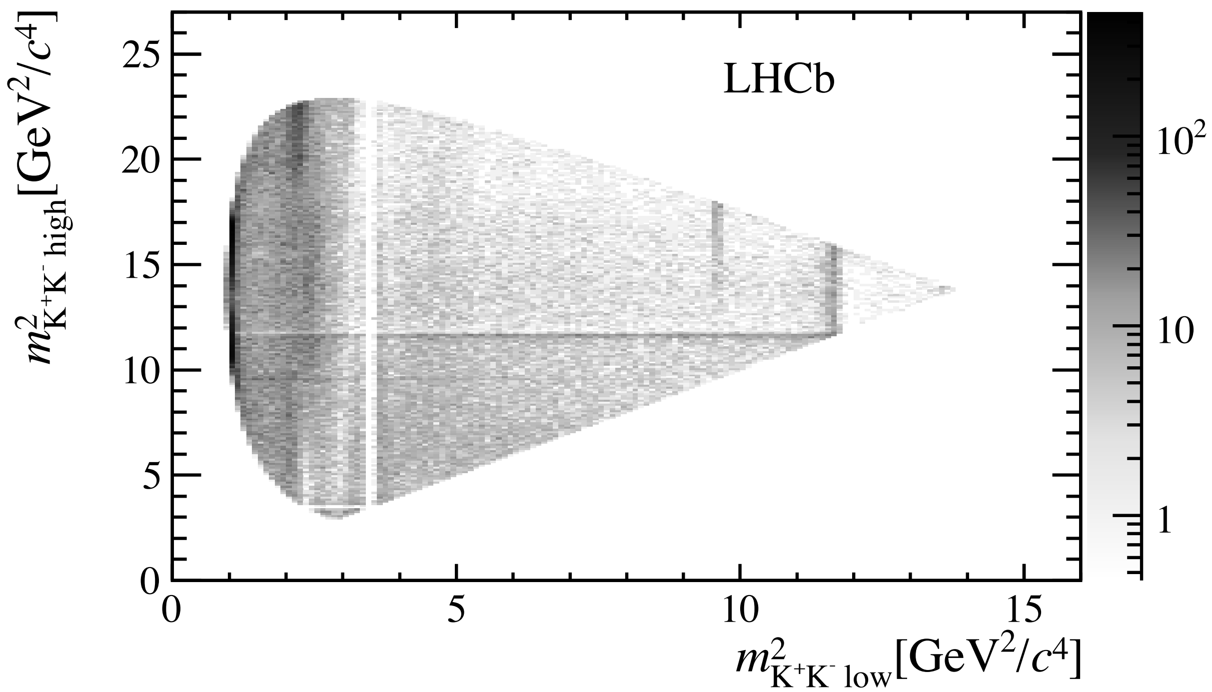
Suzuki,Wolfenstein, Phys. Rev. D 60 (1999)074019;
Falk et al. Phys. Rev. D 57,4290(1998);
Blok, Gronau, Rosner, *Phys. Rev Letters* 78, 3999 (1997).

many more ...

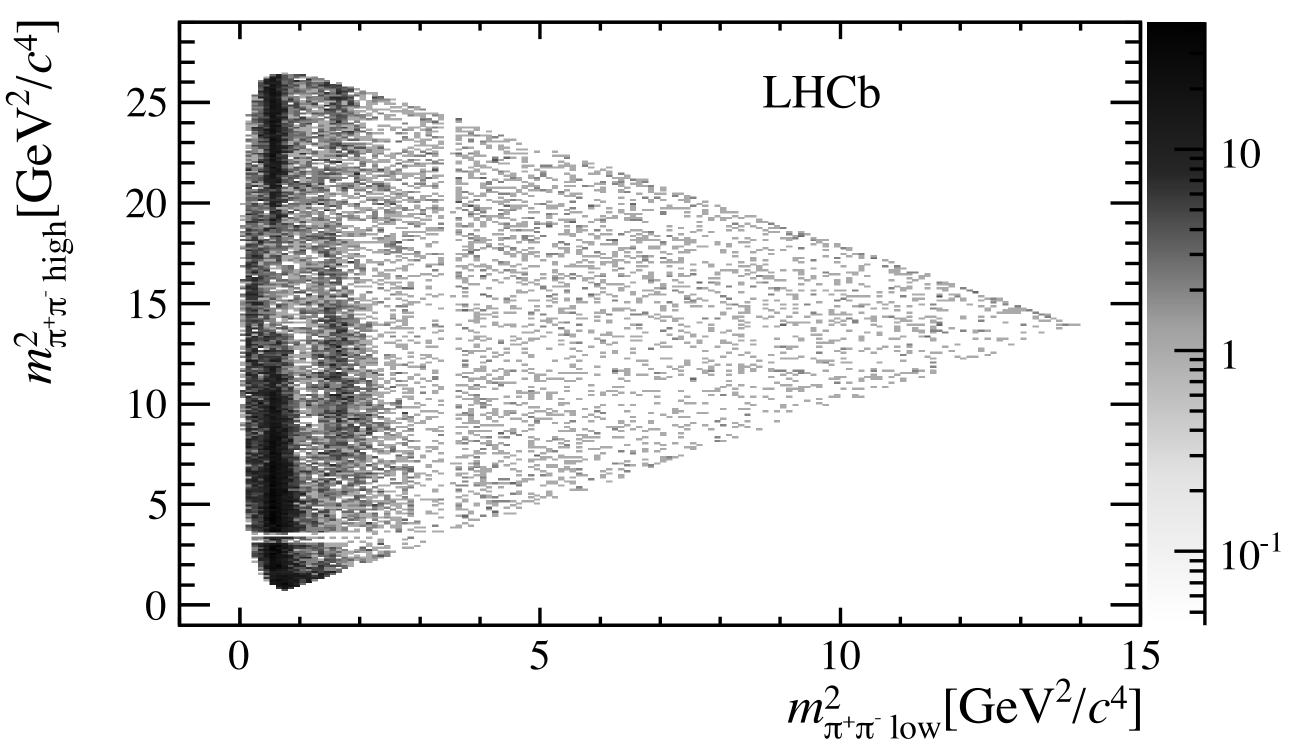
Kpp



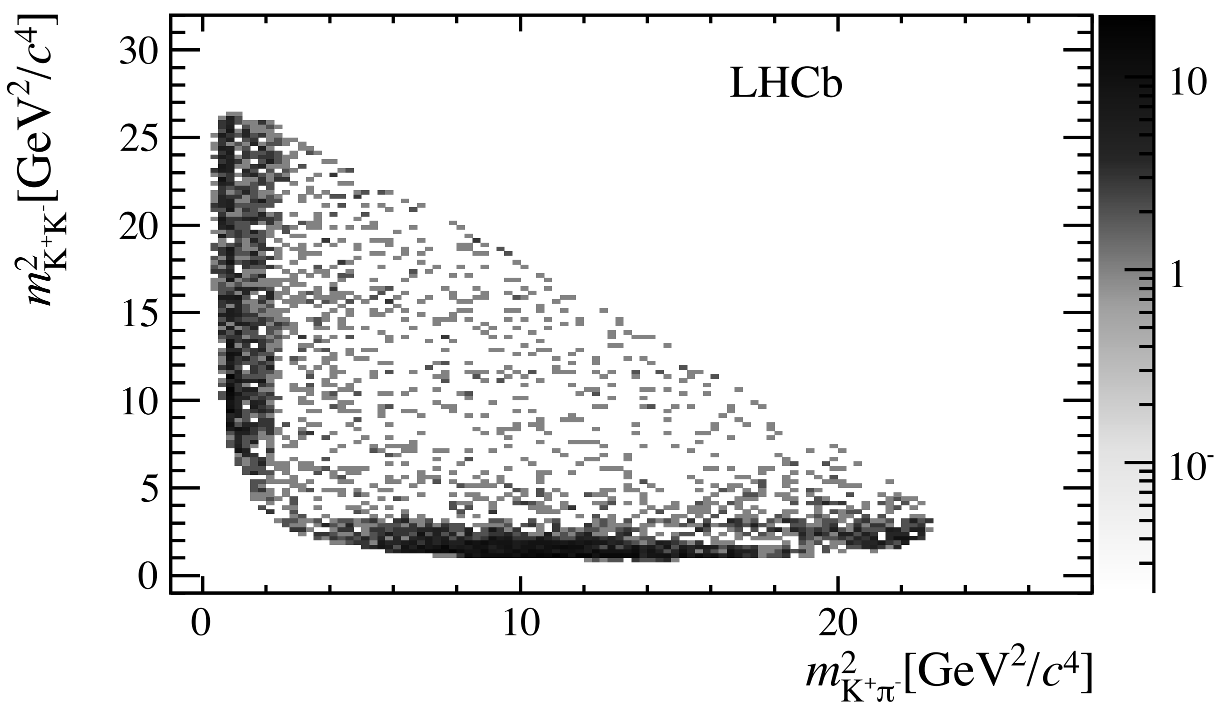
KKK



PPP



KKp



amplitude analysis for D decay

Theoretical model

PHYSICAL REVIEW D **98,** 056021 (2018)

arXiv:1805.11764 [hep-ph]

Multimeson model for the $D^+ \to K^+K^-K^+$ decay amplitude

R. T. Aoude,[1,2] P. C. Magalhães,[1,3,*] A. C. dos Reis,[1] and M. R. Robilotta[4]

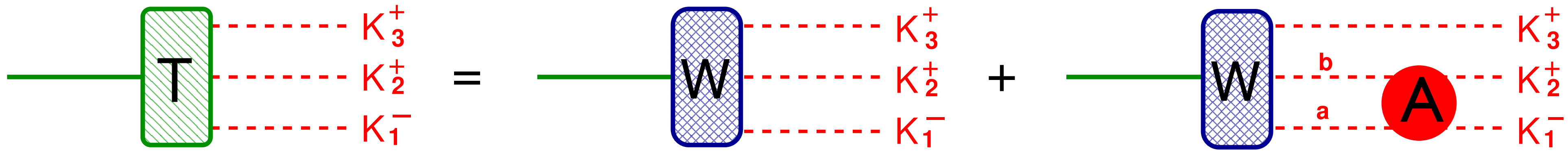
fitted to LHCb data

JHEP 1904 (2019) 063

KK scattering amplitude

multi meson model - $D^+ \to K^- K^+ K^+$

Triple-M

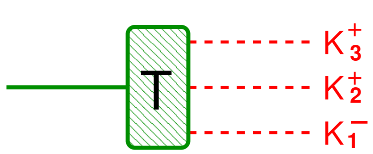


- alternative to isobar model in amplitude analysis

- depart from a fundamental theory → Chiral Lagrangian

  - track the ingredients we include in our model!

  - A_{ab}^{JI} ⟶ unitary scattering amplitude for $ab \to K^+ K^-$
    ⟶ full FSI: coupled channel,

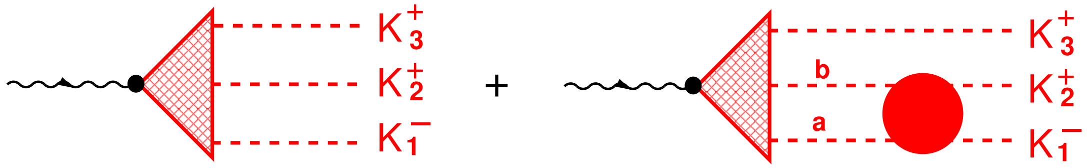


fitted to LHCb data → predict KK scattering amplitude

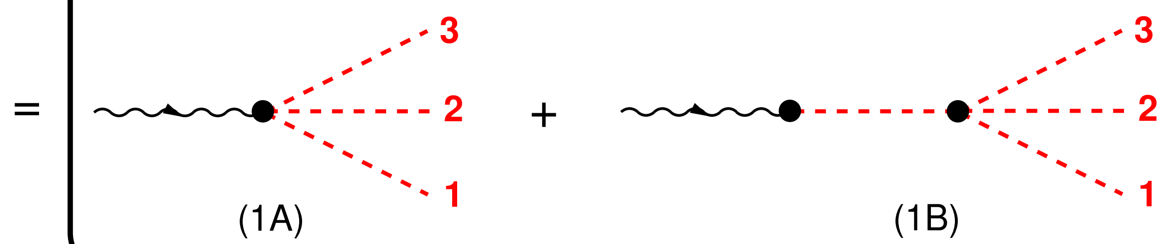
JHEP 1904 (2019) 063

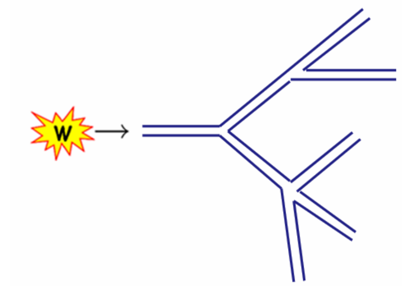
→ parameters have physical meaning: resonance masses and coupling constants

Triple - M



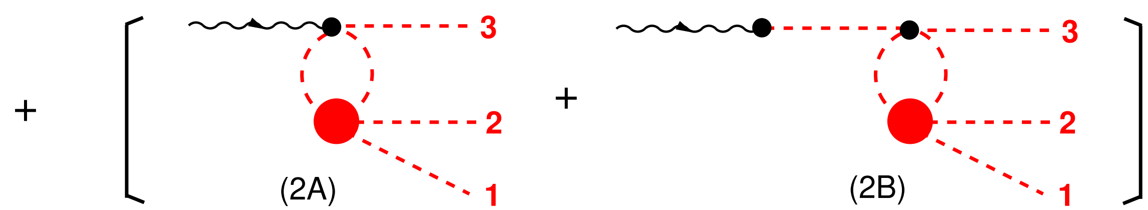
LO: non-resonant



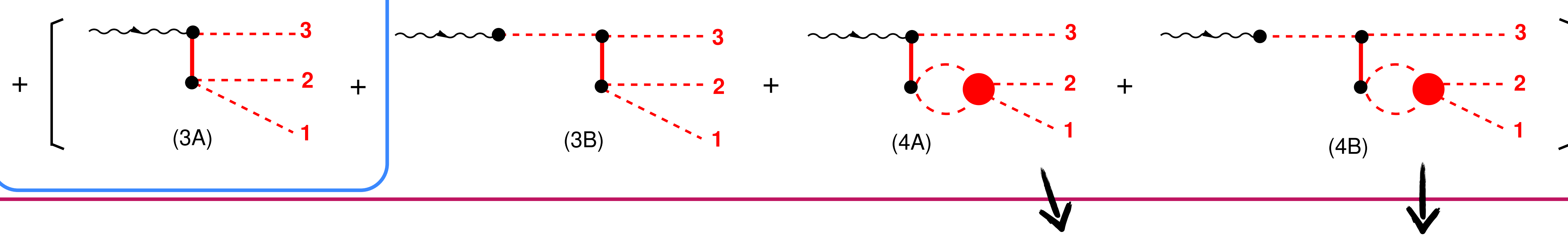


Chiral symmetry

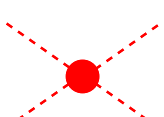
NLO
a_0, f_0, ρ, ϕ



isobar



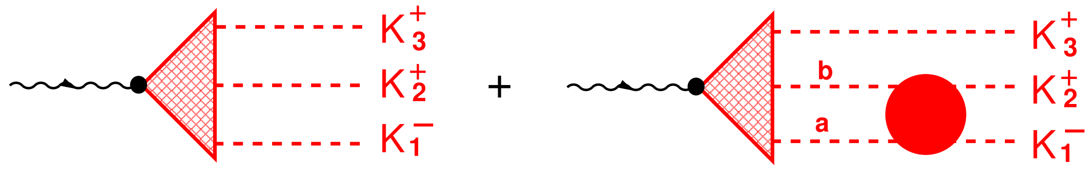
width obtained through dynamics



$K\bar{K}$ coupled-channel unitary amplitude
$\pi\pi,\ \eta\eta,\ \pi\eta,\ \rho\pi$

isospin decomposition $[J, I = (0,1), (0,1)]$

Triple M LHCb fit



$$T^S = T^S_{NR} + T^{00} + T^{01}$$

$$T^P = T^P_{NR} + T^{11} + T^{10}$$

parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5\,\text{MeV}$
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6\,\text{MeV}$
m_{S_o}	$992.0^{+8.5}_{-7.5} \pm 8.6\,\text{MeV}$
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1\,\text{MeV}$
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51\,\text{MeV}$
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9\,\text{MeV}$
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3\,\text{MeV}$
$\tilde{c}_d$	$-6.15^{+0.55}_{-0.54} \pm 0.19\,\text{MeV}$
$\tilde{c}_m$	$-10.8^{+2.0}_{-1.5} \pm 0.4\,\text{MeV}$

FF_{NR}	FF^{00}	FF^{01}	FF^{10}	FF^{11}	$\text{FF}_{\text{S-wave}}$
14 ± 1	29 ± 1	131 ± 2	7.1 ± 0.9	0.26 ± 0.01	94 ± 1

- strong destructive interference in S-wave

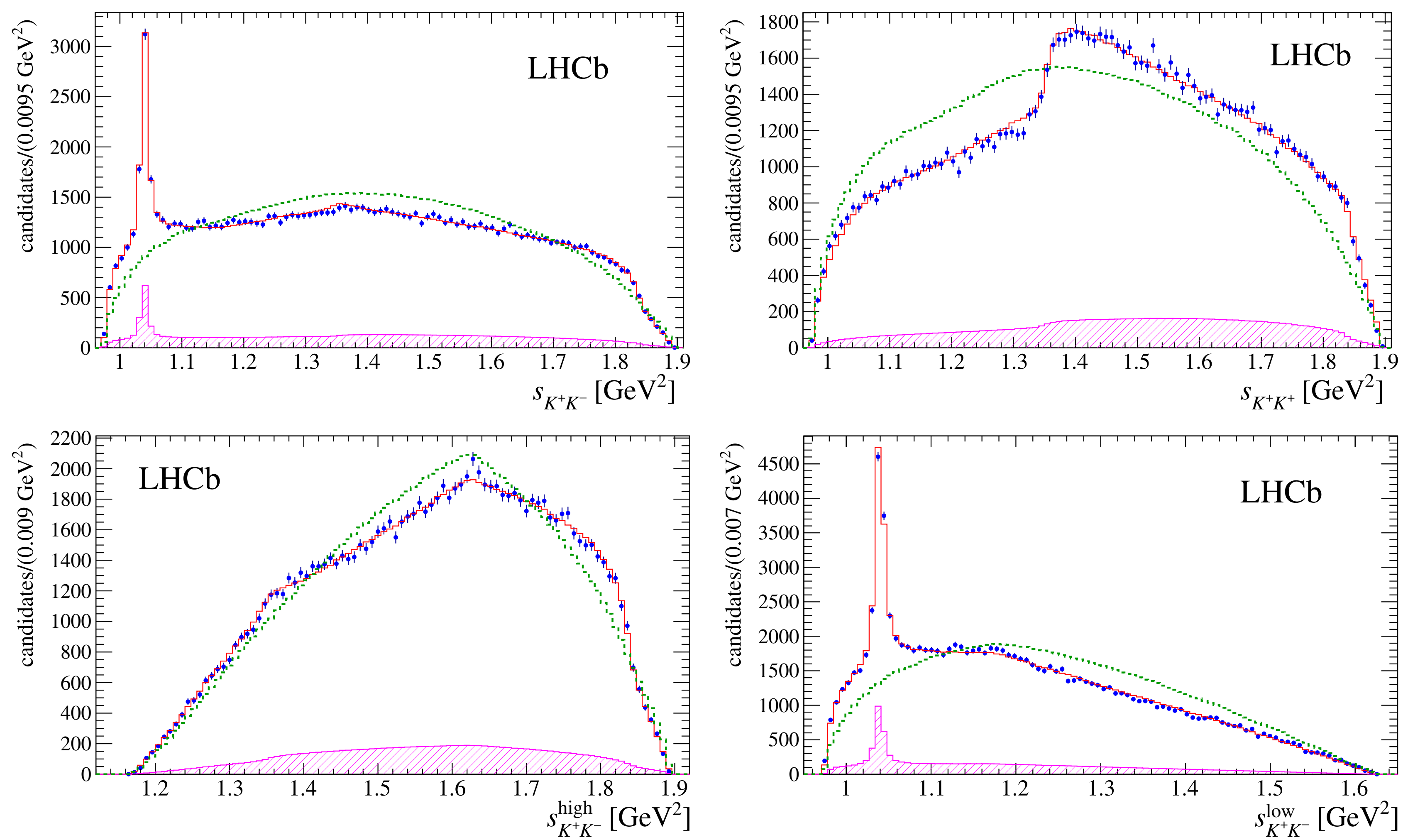
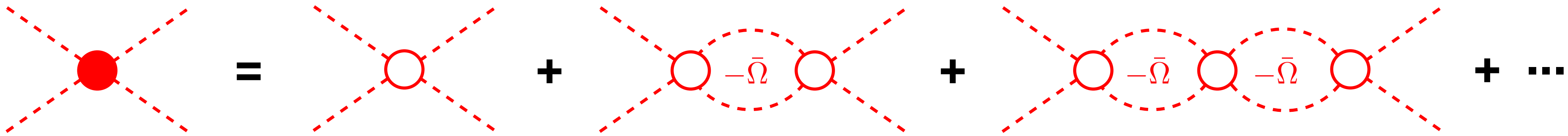


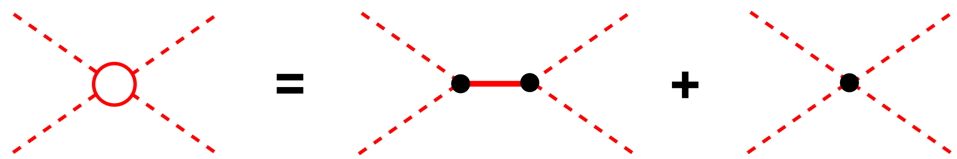
Figure 11. Projections of the Dalitz plot onto (top left) $s_{K^+K^-}$, (top right) $s_{K^+K^+}$, (bottom left) $s^{\text{high}}_{K^+K^-}$ and (bottom right) $s^{\text{low}}_{K^+K^-}$ axes, with the fit result with the Triple-M amplitude superimposed, whereas the dashed green line is the phase space distribution weighted by the efficiency. The magenta histogram represents the contribution from the background.

- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]



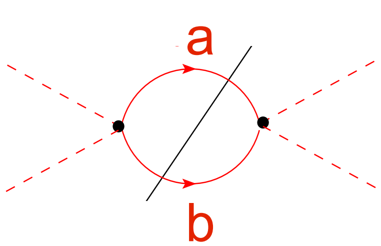
$$\mathcal{A}_{ab}^{JI} = \frac{\mathcal{K}_{ab\to cd}^{(JI)}}{1+\bar{\Omega}_{ab}\,\mathcal{K}_{ab\to cd}^{(JI)}}$$

- kernel $\mathcal{K}_{ab\to cd}^{(J,I)}$



resonance (NLO) + contact (LO)

- loops → K-matrix approximation: only on-shell



$$\{I_{ab};\, I_{ab}^{\mu\nu}\} = \int \frac{d^4\ell}{(2\pi)^4}\frac{\{1;\, \ell^\mu\,\ell^\nu\}}{D_a\,D_b}$$

$$D_a = (\ell+p/2)^2 - M_a^2 \qquad D_b = (\ell-p/2)^2 - M_b^2$$

→

$$\bar{\Omega}_{ab}^{S} = -\frac{i}{8\pi}\frac{Q_{ab}}{\sqrt{s}}\,\theta(s-(M_a+M_b)^2)$$

$$\bar{\Omega}_{aa}^{P} = -\frac{i}{6\pi}\frac{Q_{aa}^3}{\sqrt{s}}\,\theta(s-4\,M_a^2)$$

$$Q_{ab} = \frac{1}{2}\sqrt{s-2\,(M_a^2+M_b^2)+(M_a^2-M_b^2)^2/s}$$

- free parameters

- masses:

$m_\rho\,,\ m_{a_0}\,,\ m_{s0}\,,\ m_{s1}$ — $SU(3)$ singlet and octet

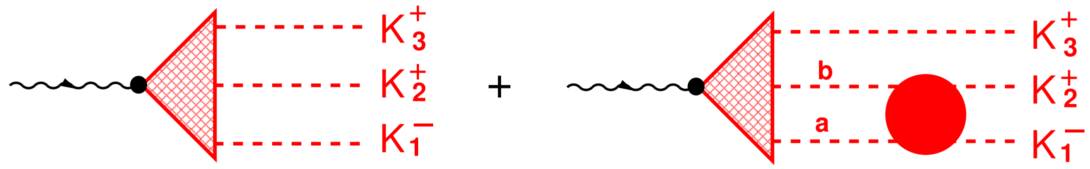
→ physical f_0 states are linear combination of $m_{s0}\,,\ m_{s1}$

- coupling constants:

$g_\rho\,,\ g_\phi$ — vector

$c_d\,,\ c_m\,,\ \tilde{c}_d\,,\ \tilde{c}_m$ — scalar

Triple M LHCb fit



$$T^S = T^S_{NR} + T^{00} + T^{01}$$

$$T^P = T^P_{NR} + T^{11} + T^{10}$$

- χ^2/ndof = 1.12 (Isobar 1.14-1.6)

parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5\,\text{MeV}$
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6\,\text{MeV}$
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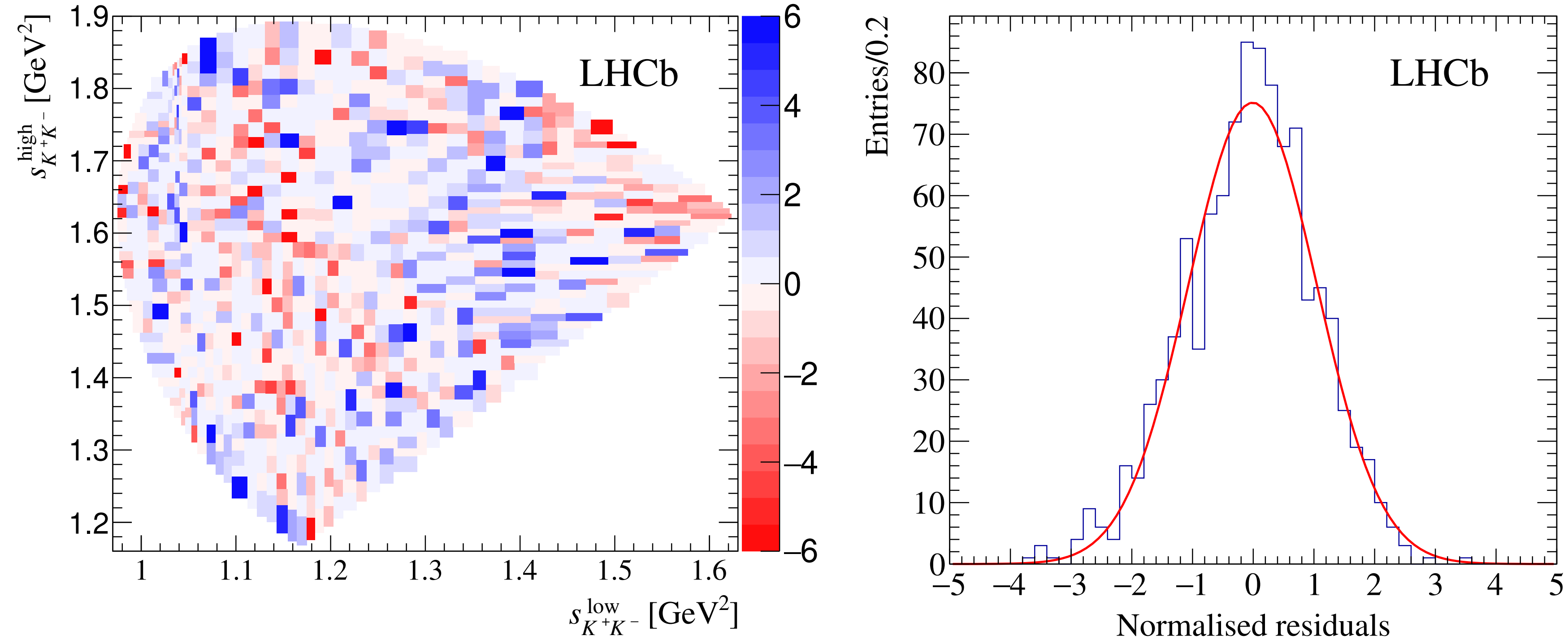

Figure 12. (left) Two-dimensional distribution of the normalised residuals for the Triple-M fit. (right) Distribution of normalised residuals of each bin.