Destigmatising modified gravity

extending gravity as a particle physicist

Peter Millington (UKRI Future Leaders Fellow, University of Manchester)
Elementary Particle Physics Seminars, University of Warwick, 30 June 2022

Lovelock's theorem

a local gravity action in (3+1)D containing only 2nd-order derivatives of the metric $g_{\mu\nu}$ necessarily leads to the **Einstein field equations**

$$\underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\text{Einstein Tensor}} + \underbrace{g_{\mu\nu}\Lambda}_{\text{CC}} = \underbrace{\kappa T_{\mu\nu}}_{\text{SM}}$$

Lovelock '71

Modifying gravity means

adding: new degrees of freedom (this talk)

higher-order derivatives

extra spatial dimensions

abandoning: locality

the action principle

Modifying gravity means

adding: new degrees of freedom (this talk)

higher-order derivatives (actually … also this talk)

extra spatial dimensions

abandoning: locality

the action principle

Modifying gravity means

adding: new degrees of freedom (this talk)

higher-order derivatives (actually … also this talk)

extra spatial dimensions (actually … also this talk)

abandoning: locality

the action principle

Why do it?

References from original slide: Di Valentino et al. 2103.01183; Right (x2): Planck Collaboration 1807.06205

Geodesic equation

connection encodes the geometry

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{d\lambda} \frac{\mathrm{d}x^\beta}{d\lambda} = 0$$

$g_{\alpha\beta} \dfrac{\mathrm{d}x^\alpha}{d\lambda} \dfrac{\mathrm{d}x^\beta}{d\lambda}$ can be **timelike**, **spacelike** or **null** (i.e., zero)

Weyl rescaling

$$g_{\mu\nu} \to \Omega^2(x) g_{\mu\nu} \qquad \text{and} \qquad \frac{\mathrm{d}}{\mathrm{d}\lambda} \to \Omega^{-2}(x) \frac{\mathrm{d}}{\mathrm{d}\lambda}$$

$$\underbrace{\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{d\lambda} \frac{\mathrm{d}x^\beta}{d\lambda}}_{\text{geodesic motion}} \underbrace{- g_{\alpha\beta} \frac{\mathrm{d}x^\alpha}{d\lambda} \frac{\mathrm{d}x^\beta}{d\lambda} \frac{\partial}{\partial x_\mu} \ln \Omega}_{\text{“fifth force”}} = 0$$

Jordan versus Einstein frame

Jordan frame:

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{F(\phi)}{2}R - \frac{Z^{\mu\nu}(\phi, \partial\phi, \ldots)}{2}\partial_\mu\phi\partial_\nu\phi - V(\phi) + \mathscr{L}_{\mathrm{SM}}(g_{\alpha\beta})\right]$$

Einstein frame, via $g_{\mu\nu} = M_{\mathrm{Pl}}^2 F^{-1}(\phi)\tilde{g}_{\mu\nu} = M_{\mathrm{Pl}}^2 A^2(\tilde{\phi})\tilde{g}_{\mu\nu}$:

$$S = \int \mathrm{d}^4x \sqrt{-\tilde{g}}\left[\frac{M_{\mathrm{Pl}}^2}{2}\tilde{R} - \frac{\tilde{Z}^{\mu\nu}(\tilde{\phi}, \partial\tilde{\phi}, \ldots)}{2}\partial_\mu\tilde{\phi}\partial_\nu\tilde{\phi} - \tilde{V}(\tilde{\phi}) + \mathscr{L}_{\mathrm{SM}}(A^2(\tilde{\phi})\tilde{g}_{\alpha\beta})\right]$$

see also the series of papers by Finn, Karamitsos and Pilaftsis on frame covariance of the quantum effective action

Jordan versus Einstein frame

Don't @ me!

Jordan frame:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{F(\phi)}{2} R - \frac{Z^{\mu\nu}(\phi, \partial\phi, \ldots)}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \mathscr{L}_{\mathrm{SM}}(g_{\alpha\beta}) \right]$$

Einstein frame, via $g_{\mu\nu} = M_{\mathrm{Pl}}^2 F^{-1}(\phi) \tilde{g}_{\mu\nu} = M_{\mathrm{Pl}}^2 A^2(\tilde{\phi}) \tilde{g}_{\mu\nu}$:

$$S = \int \mathrm{d}^4x \sqrt{-\tilde{g}} \left[\frac{M_{\mathrm{Pl}}^2}{2} \tilde{R} - \frac{\tilde{Z}^{\mu\nu}(\tilde{\phi}, \partial\tilde{\phi}, \ldots)}{2} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \tilde{V}(\tilde{\phi}) + \mathscr{L}_{\mathrm{SM}}(A^2(\tilde{\phi}) \tilde{g}_{\alpha\beta}) \right]$$

see also Finn, Karamitsos and Pilaftsis 1910.06661

Einstein frame

$$S_{\rm SM}[A^2(\tilde\phi)\tilde g_{\mu\nu},\{\psi\}] = S_{\rm SM}[\tilde g_{\mu\nu},\{\psi\}] + \frac{1}{2}[A^2(\tilde\phi) - 1]\tilde T_{\rm SM} + \dots$$

"universal" coupling to SM

EFT

Test particle experiences a **fifth force**: $\overrightarrow{F}/m = -\overrightarrow{\nabla}\ln A(\tilde\phi)$

Fifth force

Classical EoM for perturbations $\delta\tilde{\varphi} = \left\langle \tilde{\phi} \right\rangle - \tilde{\varphi}$:

$$\tilde{Z}(\tilde{\varphi})(\ddot{\delta\tilde{\varphi}} - c_s^2(\tilde{\varphi})\,\nabla^2\delta\tilde{\varphi}) + m^2(\tilde{\varphi})\delta\tilde{\varphi} = -\frac{1}{2}\frac{\mathrm{d}A^2(\tilde{\varphi})}{\mathrm{d}\tilde{\varphi}}\tilde{T}_{\mathrm{SM}}$$

Yukawa potential around a point source $\tilde{T} = -A^{-1}(\tilde{\varphi})\mathscr{M}\delta^3(\mathbf{x})$:

$$\tilde{U}(r) \supset -\frac{1}{\tilde{Z}(\tilde{\varphi})c_s^2(\tilde{\varphi})}\left[\frac{\mathrm{d}A(\tilde{\varphi})}{\mathrm{d}\tilde{\varphi}}\right]^2\frac{1}{4\pi r}\exp\left[-\frac{m(\tilde{\varphi})r}{\tilde{Z}^{1/2}(\tilde{\varphi})c_s(\tilde{\varphi})}\right]\mathscr{M}$$

Joyce, Jain, Khoury, Trodden 1407.0059

Screening

Exploit **non-linearity** of the field equations and the coupling to **matter sources** to suppress the fifth force by:

- increasing the **mass** (chameleon)

Khoury, Weltman astro-ph/0309300

- suppressing the matter **coupling** (symmetron/Damour-Polyakov)

see e.g., Gessner ’92; Damour, Polyakov hep-th/9401069; Hinterbichler, Khoury 1001.4525

- modifying the **kinetic** energy (Vainshtein)

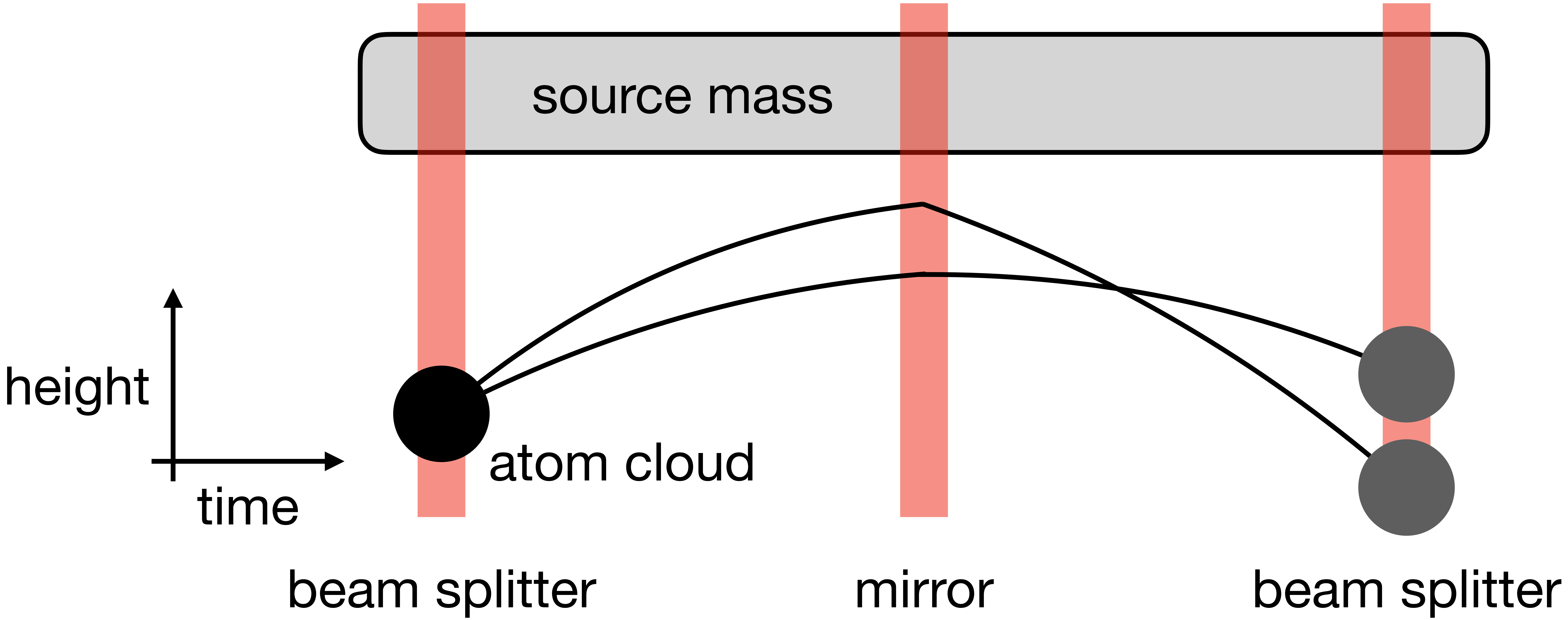
Vainshtein ‘72

Weak equivalence principle

= the universality of free fall

and modifications to GR can lead to effective violations

(Mach-Zehnder) atom interferometer



see, e.g., Jaffe et al. 1612.05171

Open quantum dynamics

Consider the light scalar composes a **weakly coupled environment**:

$$\partial_t \rho(\mathbf{p}, \mathbf{p}', t) = i \lim_{x'^0 \to t^+,\ y'^0 \to 0^-} \int_{\mathbf{k},\mathbf{k}'} e^{i(E_{\mathbf{k}} - E_{\mathbf{k}'})t} \rho(\mathbf{k}, \mathbf{k}', t) \int_{\mathbf{x},\mathbf{x}',\mathbf{y},\mathbf{y}'} e^{-i(\mathbf{p}\cdot\mathbf{x} - \mathbf{p}'\cdot\mathbf{x}' - \mathbf{k}\cdot\mathbf{y} - \mathbf{k}'\cdot\mathbf{y}')}$$

$$\times \text{ LSZ reduction of} \int \mathscr{D}\phi^{\pm}\, e^{iS_\phi[\phi^+,\phi^-]} \phi_x^+ \phi_{x'}^- \partial_t S_{\text{eff}}[\phi^+, \phi^-; t] \phi_y^+ \phi_{y'}^-$$

density operator
matrix element

inc. influence action via Feynman-Vernon

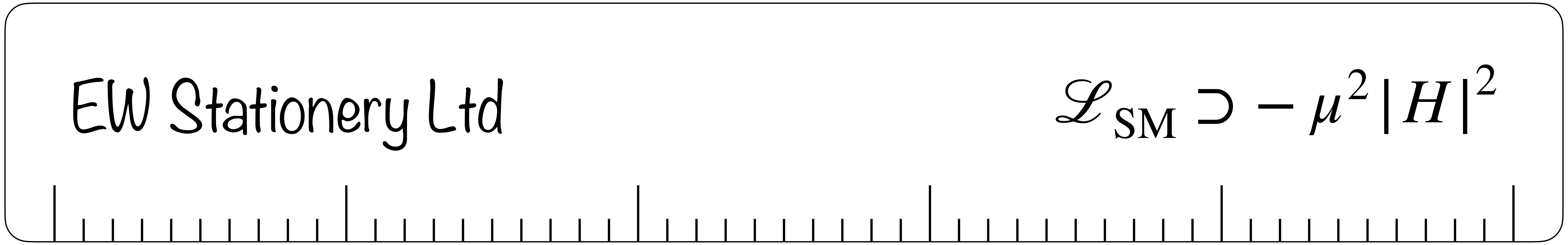
Burrage, Käding, PM, Minář 1812.12301

See Community Proposal for ESA Road-Map for Cold Atoms in Space, 2201.07789

Weyl invariance

This is not the full story:

The SM Lagrangian is **scale invariant** with the **exception** of the **quadratic term** in the **Higgs potential**.



Beyond GR or Beyond SM?

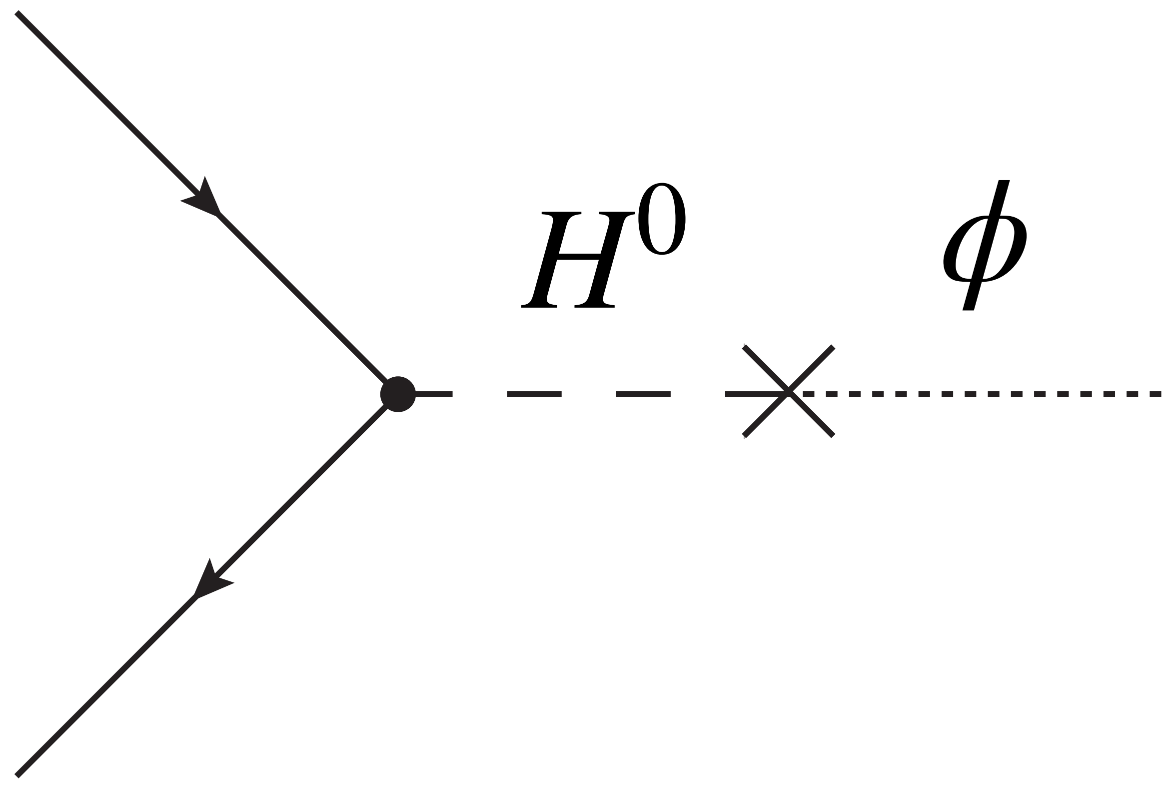
scalar-tensor extension of GR

$$\phi^2 R \qquad \text{versus} \qquad \phi^2 |H|^2$$

Higgs-portal extension of the SM

Burrage, Copeland, PM, Spannowsky 1804.07180

Coupling to leptons



$$H^0 = \frac{h}{\sqrt{2}} + \frac{v}{\sqrt{2}}\frac{\zeta}{M}$$

$$\mathscr{L}_{\text{eff}} \supset -\frac{2\mu^2}{m_H^2}\frac{m_L}{M}\bar{\psi}_L \zeta \psi_L \quad \text{where} \quad \frac{2\mu^2}{m_H^2} = 1 \quad \text{for the SM}$$

Burrage, Copeland, PM, Spannowsky 1804.07180

Scale symmetry breaking

Fifth forces from $F(\phi)R$ terms depend on the origin of **scale breaking** in the electroweak and QCD sectors.

explicit scale breaking $\Rightarrow$ fifth force coupling

dynamical scale breaking $\Rightarrow$ **no** fifth force coupling

Higgs-Dilaton

$$M_{\mathrm{Pl}}^2 R \to (\xi_H H^2 + \xi_\phi \phi^2) R \quad \text{and} \quad -\mu^2 H^2 + \frac{\lambda}{4} H^4 \to \frac{\lambda}{4}\left(H^2 - \frac{\beta}{\lambda}\phi^2\right)^2$$

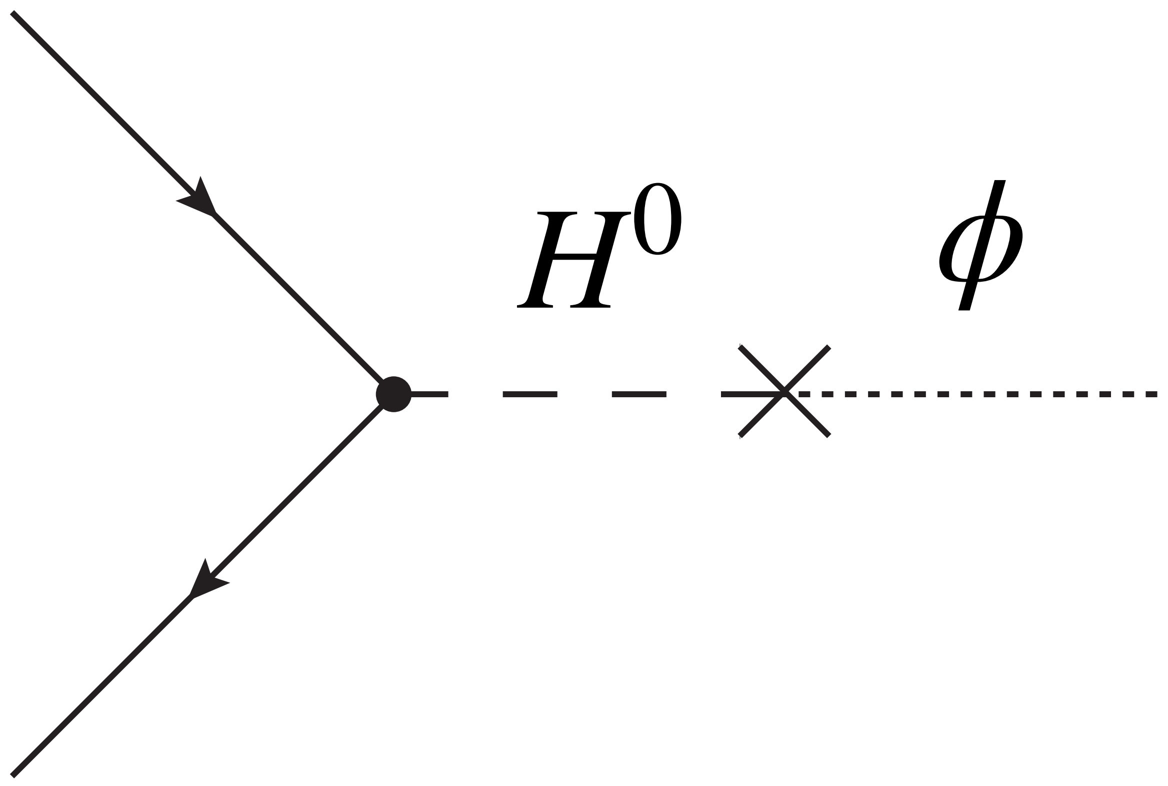
There is a **conserved dilatation current**, and a **massless Goldstone mode**

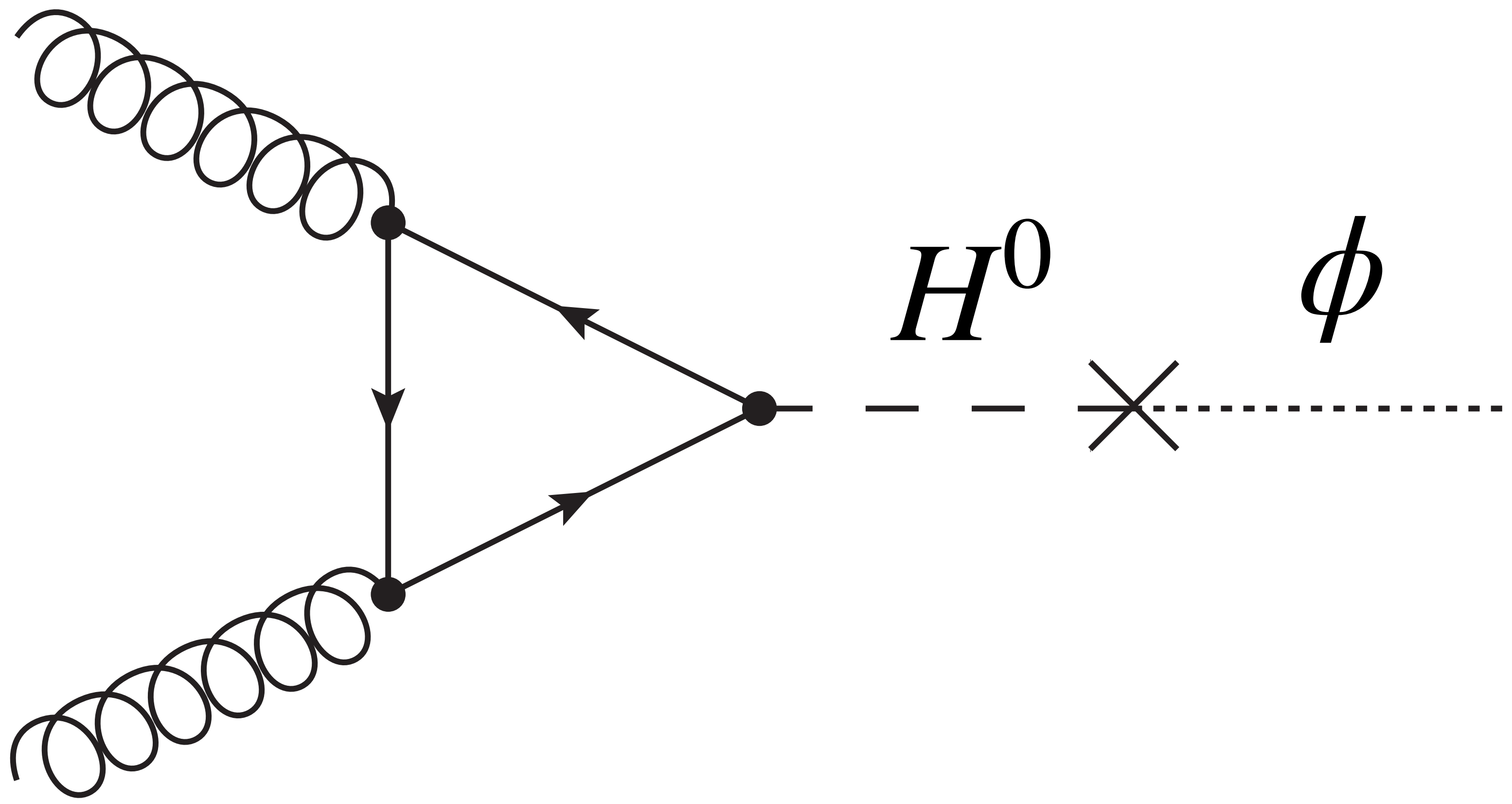
$$\sigma \propto \ln\left[(6\xi_\phi + 1)\phi^2 + (6\xi_H + 1)H^2\right]$$

with at most derivative couplings to the Higgs boson ⇒ **no fifth forces**.

e.g., Shaposhnikov, Zenhäusern 0809.3395; Brax, Davis 1401.7281; Ferreira, Hill, Ross 1612.03157; Burrage, Copeland, PM, Spannowsky 1804.07180

Coupling to hadrons (for the SM)





$$H^0 = \frac{h}{\sqrt{2}} + \frac{v}{\sqrt{2}}\frac{\zeta}{M}$$

heavy quarks

$$\mathscr{L}_{\text{eff}} \supset -\eta \frac{m_N}{M}\bar{\psi}_N \zeta \psi_N \quad \text{with} \quad \eta \equiv \frac{2N_H}{3B} + \sum_{q\in\{u,d,s\}} f^N_{Tq}\left(1 - \frac{2N_H}{3B}\right) < 1$$

light quarks

Burrage, Copeland, PM, Spannowsky 1804.07180

Back to the Jordan frame

Quantum corrections generate couplings to the gravity sector:

$$\mathscr{L} \supset \xi |H|^2 R \to \beta_\xi = -\frac{1}{16\pi^2}\boxed{\left(\xi + \frac{1}{6}\right)}\left(12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2\right)$$

Herranen, Markkanen, Nurmi, Rajantie 1407.3141

Linearised gravity

Fifth forces in the Jordan frame via mixing with the graviton ($g \to \eta + g$):

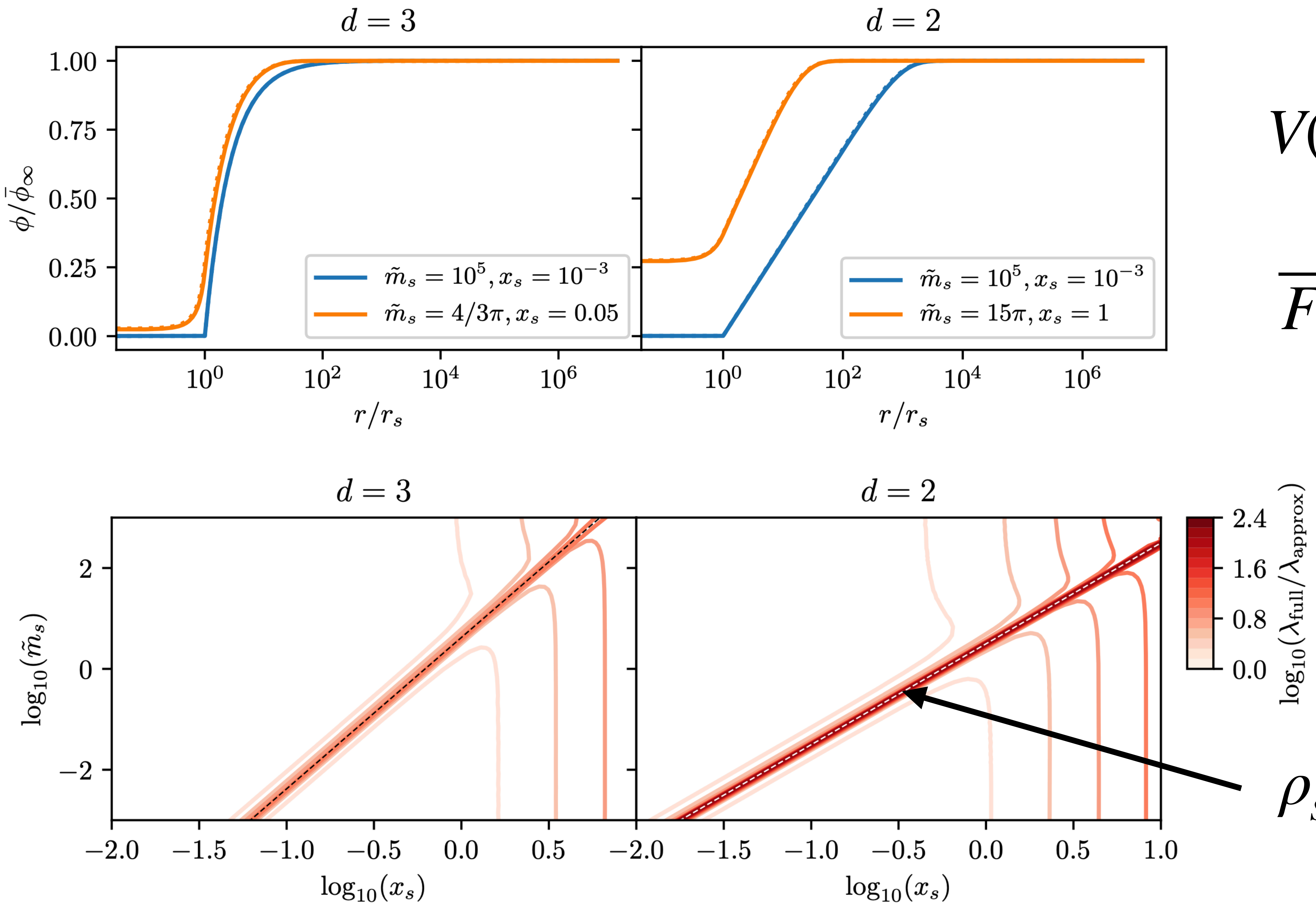
$$\mathscr{L} = \frac{F(\phi)}{4}\left[\frac{1}{4}\partial_\mu g \partial^\mu g - \frac{1}{2}\partial_\rho g_{\mu\nu}\partial^\rho g^{\mu\nu}\right] + \boxed{\frac{F'(\phi)}{4}\eta^{\mu\nu}\partial_\mu g \partial_\nu \phi}$$

$$- \frac{1}{2}\left[Z(\phi) + \frac{[F'(\phi)]^2}{2F(\phi)}\right]\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$

$$+ \frac{1}{2}g^{\mu\nu}T_{\mu\nu} + \mathscr{L}_{\mathrm{SM}}(\eta, \{\psi\})$$

Copeland, PM, Sevillano Muñoz 2111.06357

Back to (classical) fifth forces



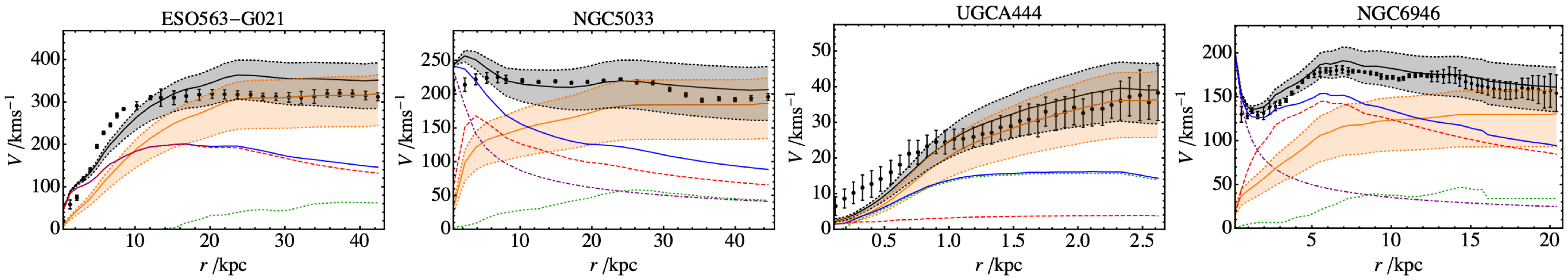
$$V(\phi) = \left[\frac{\rho_s}{M^2} - \mu_\phi^2\right]\frac{\phi^2}{2} + \frac{\lambda\phi^4}{4!}$$

$$\vec{F} \propto -\phi\,\vec{\nabla}\phi$$

$$\rho_s \sim \mu_\phi^2 M^2$$

Burrage, Elder, PM, Saadeh, Thrussell 2104.14564

Back to (classical) fifth forces



simulated data

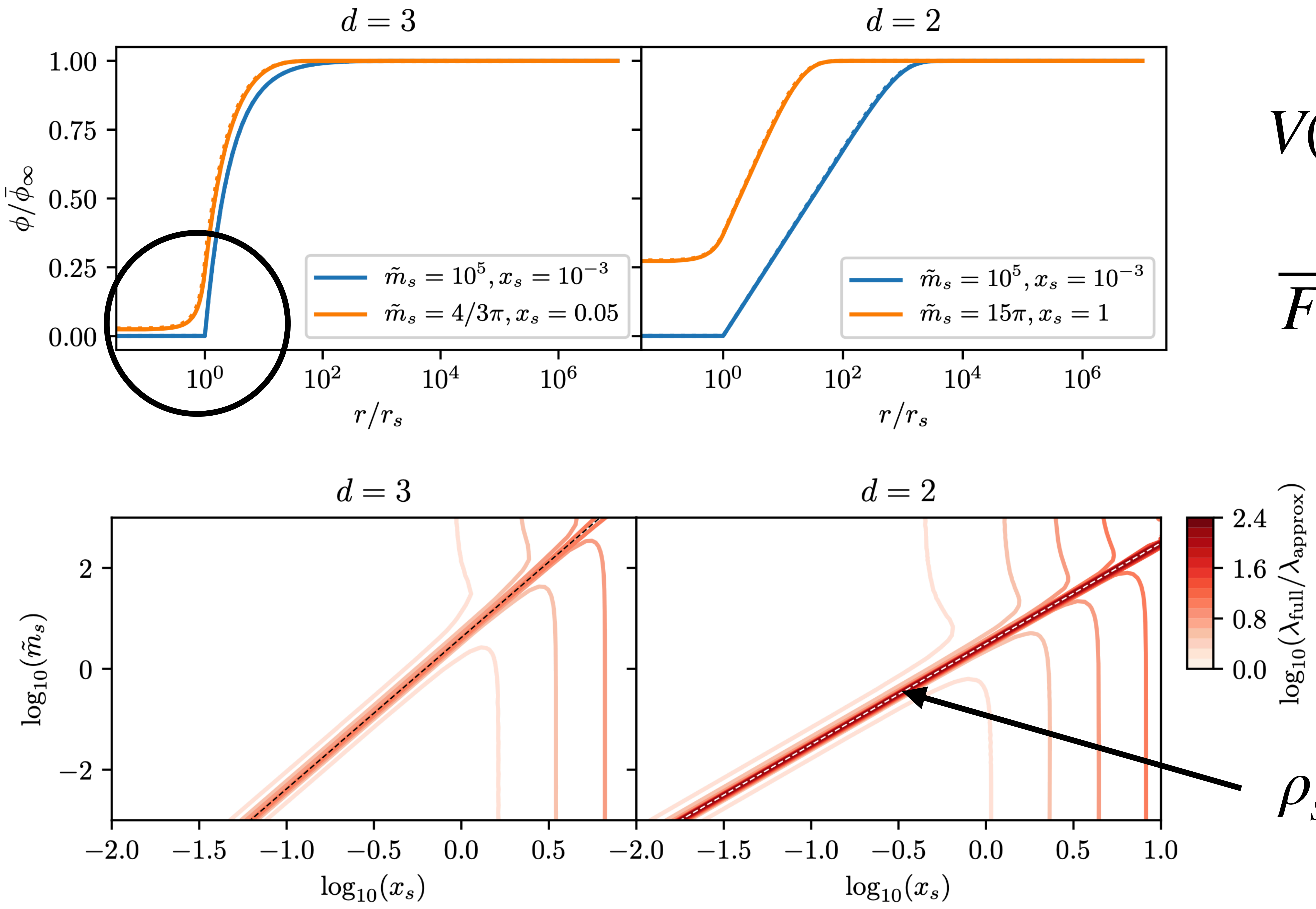
cherry picked from the SPARC dataset

See Fig 6. In O’Hare and Burrage, 1805.05226

$\rho_s \sim \mu_\phi^2 M^2$

Top: Burrage, Copeland, PM 1610.07529; Bottom: O’Hare, Burrage 1805.05226

Back to (classical) fifth forces



$$V(\phi) = \left[\frac{\rho_s}{M^2} - \mu_\phi^2\right]\frac{\phi^2}{2} + \frac{\lambda\phi^4}{4!}$$

$$\vec{F} \propto -\phi\,\vec{\nabla}\phi$$

$$\rho_s \sim \mu_\phi^2 M^2$$

Burrage, Elder, PM, Saadeh, Thrussell 2104.14564

Extending the class

Disformal couplings: $g_{\mu\nu} \to A(\phi) g_{\mu\nu} + B(\phi) \partial_\mu \phi \partial_\nu \phi$

Bekenstein gr-qc/9211017

Higher-order actions: Horndeski, beyond Horndeski and DHOST

Horndeski ’74; Gleyzes, Langlois, Piazza, Vernizzi 1304.4840; Zumalacárregui, García-Bellido 1308.4685; Langlois, Noui 1510.06930

We can think of all of these as a EFTs of the SM plus extra scalars, but with particular patterns of operators.

Summary

QFT prevents us from disentangling **scalar extensions of GR** from **scalar extensions of the SM**.

Screened fifth forces have rich phenomenology. Robust predictions means **non-linear QFT** in **curved spacetime** with **sources**.

Parallels between scalar-tensor theories of gravity and: **axion-like models**, **ultra-light dark matter models**, **Higgs-portal theories**, …


I liked your talk on MOND.
Modified gravity ≠ MOND.
imgflip.com