Elementary Particle Physics Seminar, University of Warwick   30th November 2023

The (theory) puzzles of g-2

UNIVERSITY OF LIVERPOOL

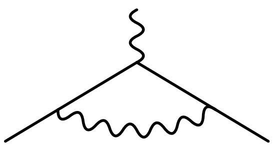
Thomas Teubner

- Introduction & overview, a_e vs. a_μ
- **Data-driven HVP evaluation:** basic ingredients, main features
- **The most important 2π channel**, other channels, total HVP
- Recent new data, **one more puzzle**
- Outlook, new analyses & pathways to solving the puzzles

Introduction: it all started with the electron…

- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with g = 2.00229 ± 0.00008

- 1948: Schwinger calculates the famous radiative correction:



➠ g = 2 (1+a), with the **anomaly**

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \approx 0.001161$$

``If you can't join 'em, beat 'em"

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

- In terms of an effective Lagrangian, the anomaly is from the Pauli term:

$$\delta\mathcal{L}_{\rm eff}^{\rm amm} = -\frac{Qe}{4m}\, a\, \bar\psi_L \sigma^{\mu\nu}\psi_R F_{\mu\nu} \;+\; ({\rm L} \leftrightarrow {\rm R})$$

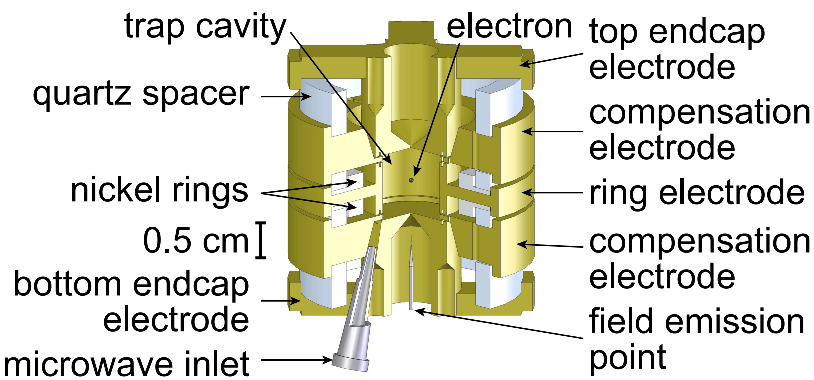
Note: This is a dimension 5 operator and NOT part of the fundamental (QED) Lagrangian, but occurs through radiative corrections and is **calculable in** (Standard Model) **theory**:

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadronic}$$

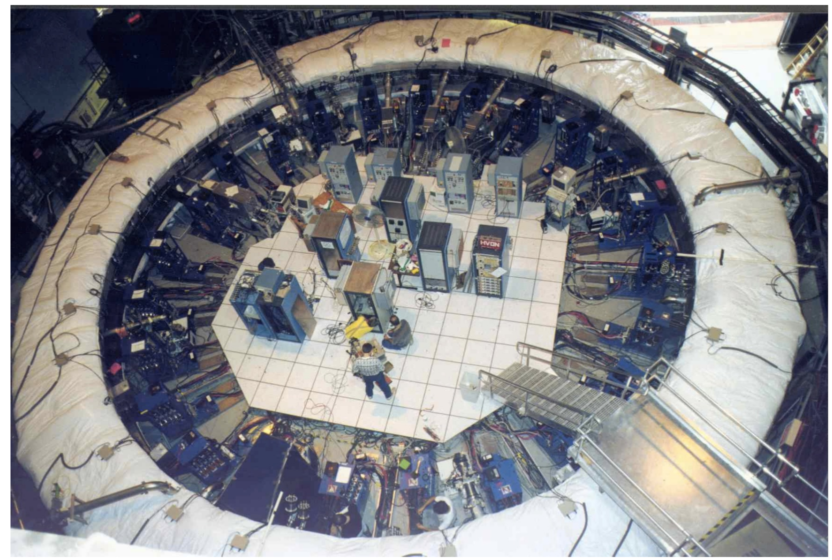
a_e vs. a_μ : why we want to study the muon

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]
Hanneke et al., PRL 100(2008)120801 @ Harvard

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]
Bennet et al., PRD 73(2006)072003 @ BNL



one-electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales

- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{NP} \sim \mathcal{C}\, m_\ell^2/\Lambda_{NP}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, a_e `determines' α, tests QED & low scales

[Note: τ too short-lived for storage-rings]

a_e current status (exp @ Northwestern): *PRL* 130 (2023) 7, 071801

Measurement of the Electron Magnetic Moment

[arXiv:2209.13084]

X. Fan,[1,2,*] T. G. Myers,[2] B. A. D. Sukra,[2] and G. Gabrielse[2,†]

[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *Center for Fundamental Physics, Northwestern University, Evanston, Illinois 60208, USA*

(Dated: September 28, 2022)

The electron magnetic moment in Bohr magnetons, $-\mu/\mu_B = 1.001\,159\,652\,180\,59\,(13)$ [0.13 ppt], is consistent with a 2008 measurement and is 2.2 times more precise. The most precisely measured property of an elementary particle agrees with the most precise prediction of the Standard Model (SM) to 1 part in 10^{12}, the most precise confrontation of all theory and experiment. The SM test will improve further when discrepant measurements of the fine structure constant α are resolved, since the prediction is a function of α. The magnetic moment measurement and SM theory together predict $\alpha^{-1} = 137.035\,999\,166\,(15)$ [0.11 ppb]

SM theory prediction depends on **α, but measurements with Cs and Rb disagree by 5.4σ :**

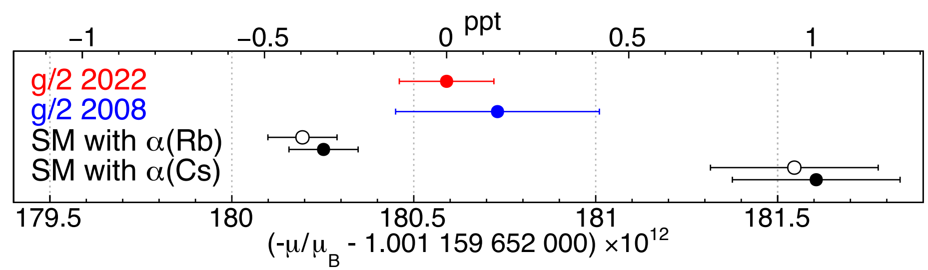
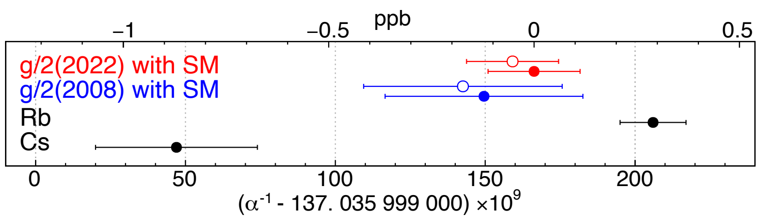


FIG. 1. This Northwestern measurement (red) and our 2008 Harvard measurement (blue) [26]. SM predictions (solid and open black points for slightly differing C_{10} [27, 28]) are functions of discrepant α measurements [29, 30]. A ppt is 10^{-12}.



⟵ Translation to derived value of **α**

Muon g-2 Theory Initiative est. 2017

https://muon-gm2-theory.illinois.edu

``… map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result.''

- Organised 9 int. workshops in 2017-2023, last plenary workshop 4-8.9.2023 in Bern
- **White Paper** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

  ``The anomalous magnetic moment of the muon in the Standard Model''
  [T. Aoyama et al., arXiv:2006.04822, *Phys. Rept.* 887 (2020) 1-166 > 1000 cites]

Group photo from the Bern workshop in September 2023

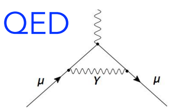
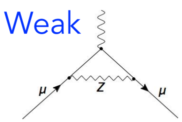
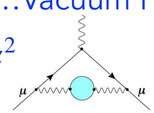
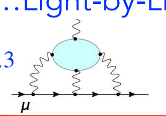
SM prediction from Theory Initiative vs. Experiment

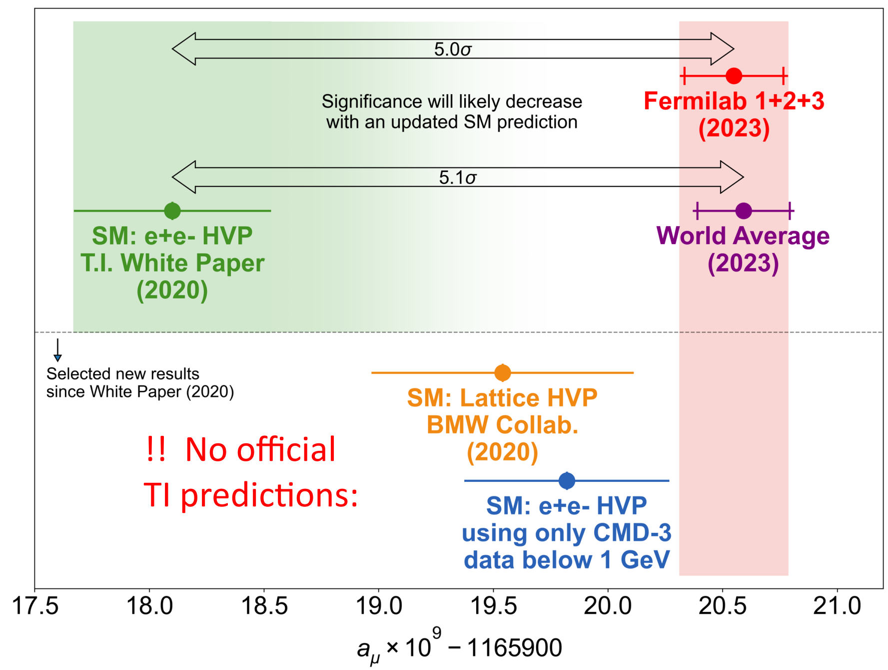
$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$$

White **P**aper [T. Aoyama et al., *Phys. Rept.* 887 (2020) 1-166]

Measurement of the Positive Muon Anomalous Magnetic Moment to **0.46 ppm** [*Phys. Rev. Lett.* 126 (2021) 14, 141801]

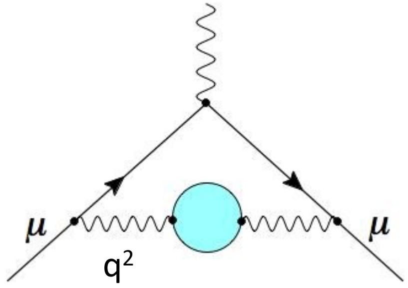
... **to 0.20 ppm** [*PRL* 131 (2023) 16, 161802]

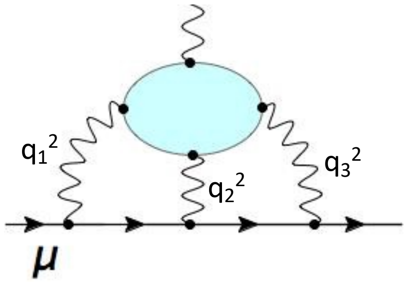
			0.37 ppm
QED 	+…	$116\,584\,718.9\,(1) \times 10^{-11}$	0.001 ppm
Weak 	+…	$153.6\,(1.0) \times 10^{-11}$	0.01 ppm
Hadronic…			
…Vacuum Polarization (HVP) α^2 	+…	$6845\,(40) \times 10^{-11}$ [0.6%]	0.34 ppm
…Light-by-Light (HLbL) α^3 	+…	$92\,(18) \times 10^{-11}$ [20%]	0.15 ppm



➤ SM uncertainty dominated by hadronic contributions, now with δ HVP > δ HLbL

$a_\mu^{hadronic}$: non-perturbative, the limiting factor of the SM prediction





- **Q:** What's in the hadronic (**Vacuum Polarisation** & **Light-by-Light scattering**) blobs?

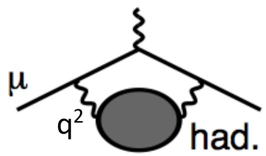
  A: Anything `**hadronic**' the virtual photons couple to, i.e. **quarks + gluons** + **photons**

  But: low q^2 photons dominate loop integral(s) ➠ cannot calculate blobs with perturbation theory

- **Two very different** (model independent) **strategies:**

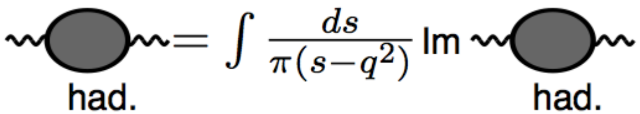
  1. use wealth of hadronic data, `**data-driven dispersive methods**':
     - data combination from many experiments, **radiative corrections** required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `**lattice QCD**':
     - finite size, finite lattice spacing, artifacts from lattice actions, **QCD + QED** needed
     - numerical Monte Carlo methods require large computer resources

a_μ^{HVP} : Basic principles of **dispersive** data-driven method



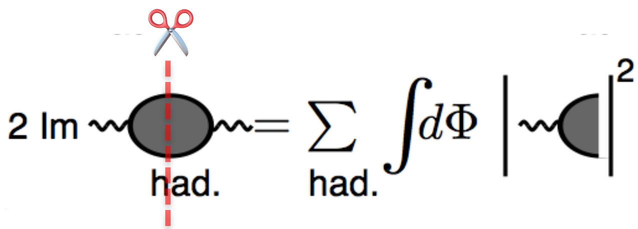
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➟ analyticity ➟ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➟ Optical Theorem:

imaginary part (`cut diagram') =

sum over |cut diagram|^2, i.e.

∝ sum over all total hadronic cross sections

$$a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s} \hat{K}(s) \sigma_{\mathrm{had}}(s)$$

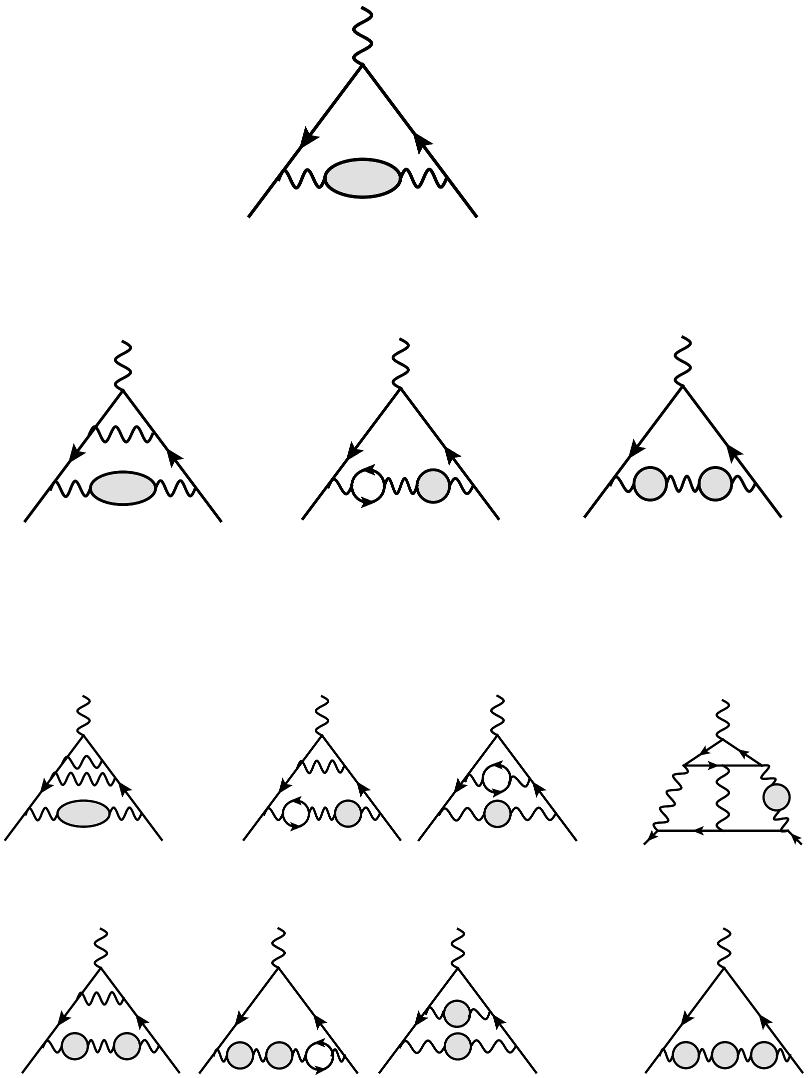
- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$ ♣

- Total hadronic cross section $\boldsymbol{\sigma_{had}}$ from > 100 data sets for **e^+e^- ➞ hadrons** in > 35 final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of input data
- pQCD only at large s, **no modelling** of $\sigma_{had}(s)$, direct data integration

a_μ^{HVP} : Higher orders & power counting; WP20 values in $\mathbf{10^{-11}}$



➤ All hadronic blobs also contain photons,
i.e. real + virtual corrections in $\sigma_{had}(s)$

- **LO: 6931(40)**

- **NLO: - 98.3(7)**

  from three classes of graphs:
  - 207.7(7) + 105.9(4) + 3.4(1) [KNT19]
  (photonic, extra e-loop, 2 had-loops)

- **NNLO: 12.4(1)** [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

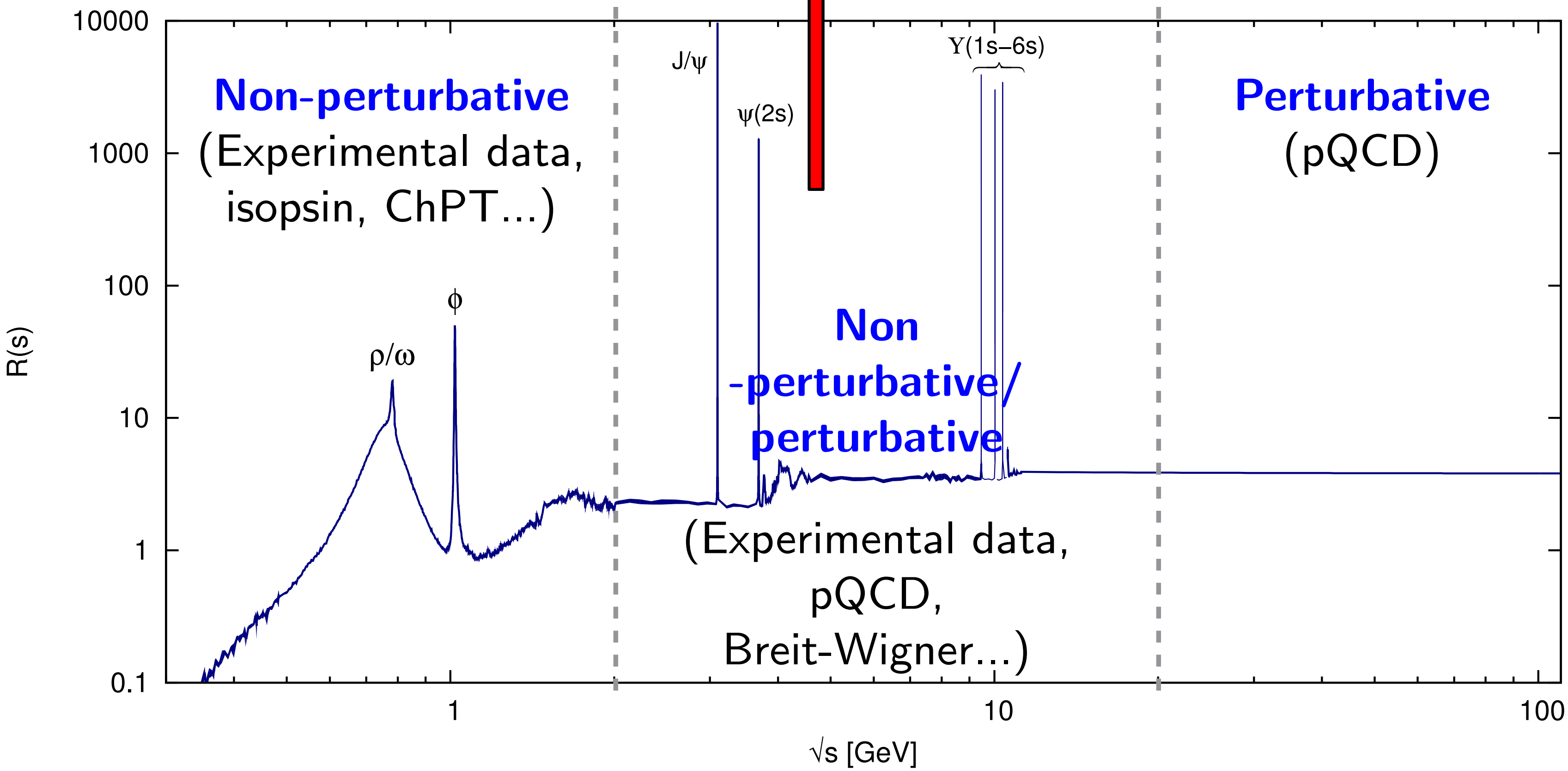
  from five classes of graphs:
  8.0 - 4.1 + 9.1 - 0.6 + 0.005

➥ good convergence,
iterations of hadronic blobs _very_ small

➠ `double-bubbles' very small

HVP disp.: cross section (in terms of R-ratio) input

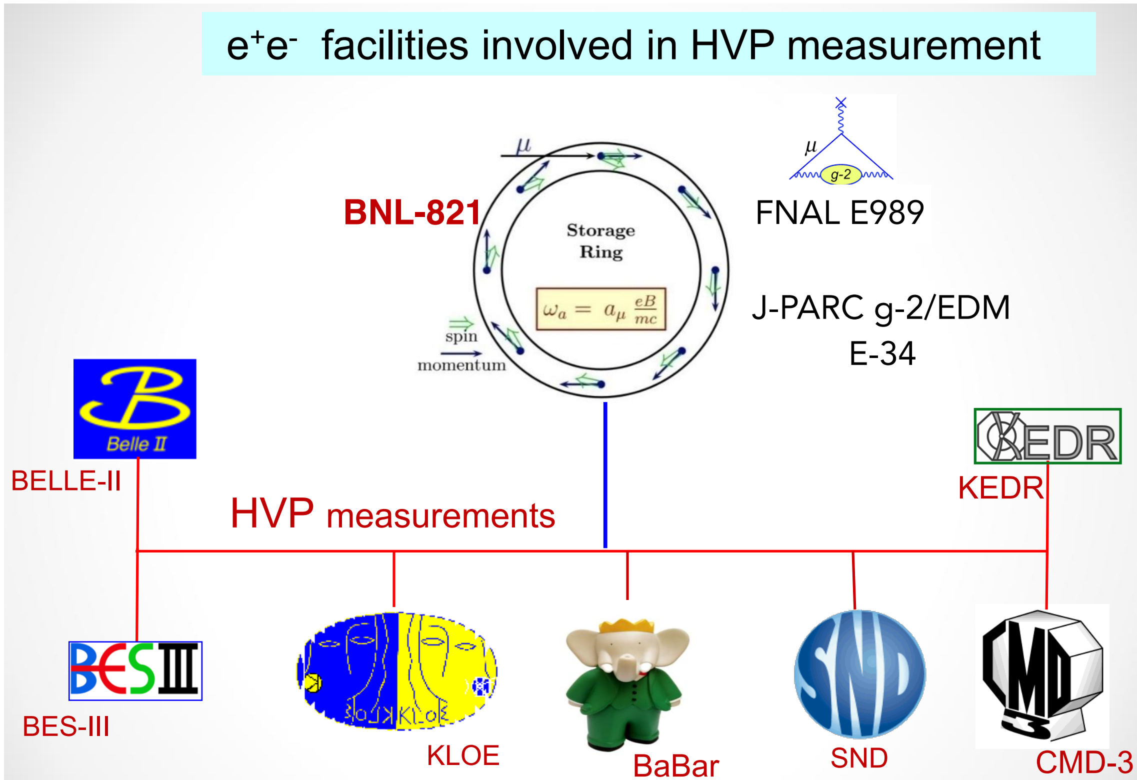
$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\mathrm{had},\gamma}(s)}{4\pi\alpha^2/3s}$$

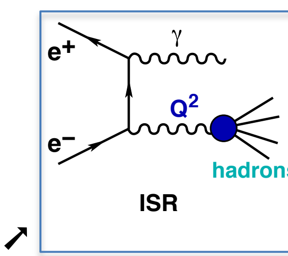


Must build full hadronic cross section/R-ratio…

HVP: Recent (of >25 years) experiments providing input $\sigma_{had}(s)$ data

S. Serednyakov (for SND) @ HVP KEK workshop





- Different methods: \`**Direct Scan**' (tunable e^+e^- beams) &
  \`**Radiative Return**' (Initial State Radiation scan at fixed cm energy) ↗
- Over last decades detailed studies of **radiative corrections** & **Monte Carlo Generators** for $\sigma_{had}(s)$
  - ➤ **RadioMonteCarLow** Working Group report: *Eur. Phys. J.* C66 (2010) 585-686
  - ➤ full NLO radiative corrections in ISR MC ***Phokhara***: Campanario et al, PRD 100(2019)7,076004

HVP dispersive: cross section compilation

How to get the most precise σ^0_{had}? Use of $e^+e^- \rightarrow$ hadrons $(+\gamma)$ data:

- **Low energies: sum ~35 exclusive channels**, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …, [now very limited use of iso-spin relations for missing channels]

- **Above √s ~1.8 GeV:** use of **inclusive data or pQCD** (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of **data combination** (locally in √s, with **error inflation if tensions**):
  - many experiments, different energy ranges and bins,
  - **statistical + systematic errors** from many different sources, use of **correlations**

  ➤ Significant differences between **DHMZ** and **KNT** in use of correlated errors:
  - KNT allow non-local correlations to influence mean values,
  - DHMZ restrict this but retain correlations for errors, also estimate cross channel corrs.

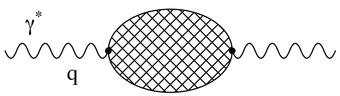
- **σ^0_{had}** means the 'bare' cross section, i.e. excluding 'running coupling' (**VP**) effects, but including Final State (γ) Radiation:

  ☛ data need radiative corrections, compilations estimate additional uncertainty,

  e.g. in KNT: $\delta a_\mu^{\text{had, VP}} = \mathbf{2.1 \times 10^{-11}}$, and $\delta a_\mu^{\text{had, FSR}} = \mathbf{7.0 \times 10^{-11}}$

Rad. Corrs.: HVP for running α(q²). Undressing

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi \; =$ 

Full photon propagator $\sim \; 1 \; + \; \Pi \; + \; \Pi \cdot \Pi \; + \; \Pi \cdot \Pi \cdot \Pi \; + \; \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) \; = \; \frac{\alpha}{1 - \mathrm{Re}\Pi(q^2)} \; = \; \alpha \, / \, \left(1 - \Delta\alpha_{\mathrm{lep}}(q^2) - \Delta\alpha_{\mathrm{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) \; = \; -\frac{q^2}{4\pi^2\alpha} \, \mathrm{P} \int_{m_\pi^2}^{\infty} \frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s - q^2} \, , \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1 - \Pi|^2}$$

[$\to$ σ^0 requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the $|$full photon propagator$|^2$, i.e. $|$infinite sum$|^2$.

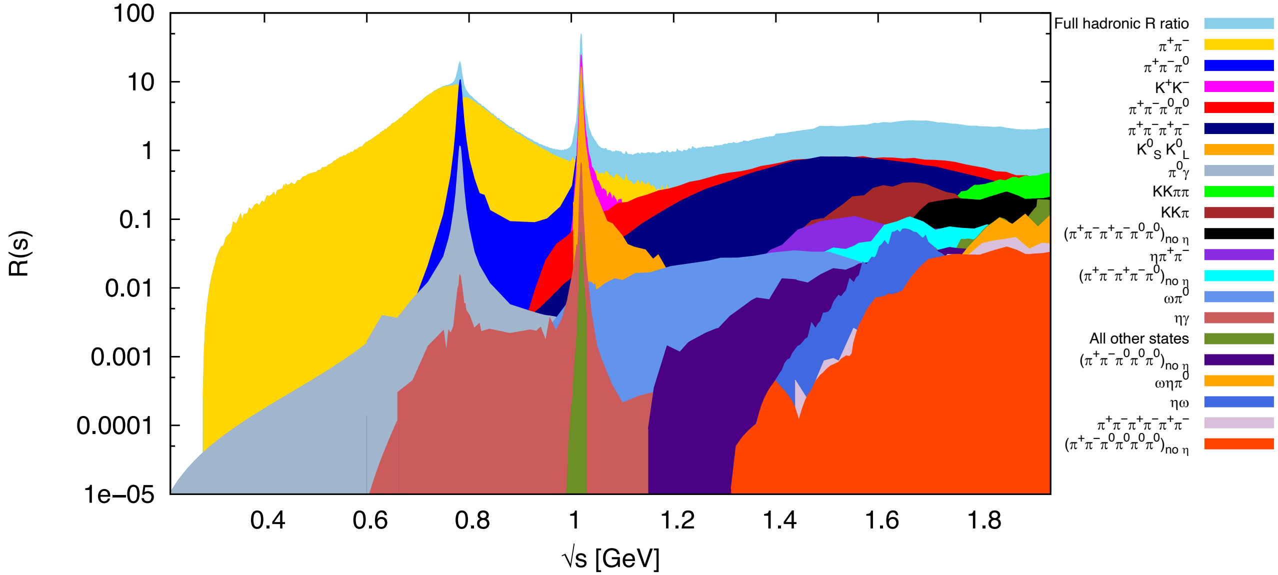
$\to$ To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Rad. Corrs.: Final State γ Radiation

- Real + virtual , must be included in σ^0_{had} as part of the (hadronic) dynamics
- In measured cross sections, virtual and soft/collinear photons are always included,
- but some events with hard real radiation are cut-off by experimental analyses (through event selection/classification, cuts, acceptances):
  -- limited phase space for hard radiation at low energies in scan mode
  -- no problem if γ missed but the event counted, but
  -- possibly important effect in radiative return (ISR) mode, depending on energy

- Experiments account for this and add (back missed) FSR in their data analyses
  -- using **MC generators** with **corrections based on scalar QED** for **π**s and **K**s
  (checked to work ok at low energies when hadronic substructure hardly resolved)
  -- for analyses based on Radiative Return (in particular for the 2π channel),
  ISR and FSR are an integral part of the MCs used (*EVA*, *Phokhara*)
  -- **possible limitations for accuracy discussed at Strong2020 WorkStop/ThinkStart,**
  work planned for higher order corrections & MC implementation

HVP disp: Landscape of $\sigma_{had}(s)$ data. Most important $\pi^+\pi^-$ channel



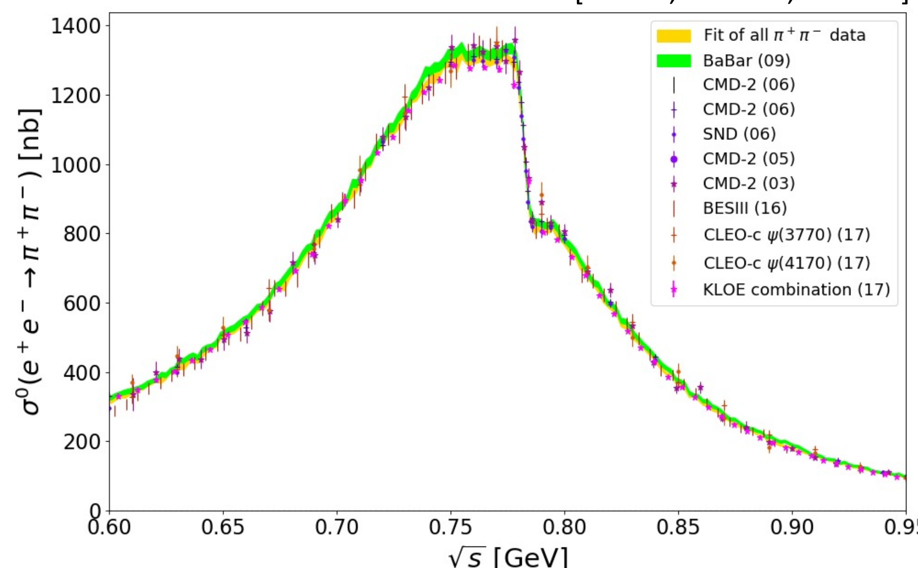
[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of 2**π**

$\pi^+\pi^-$:

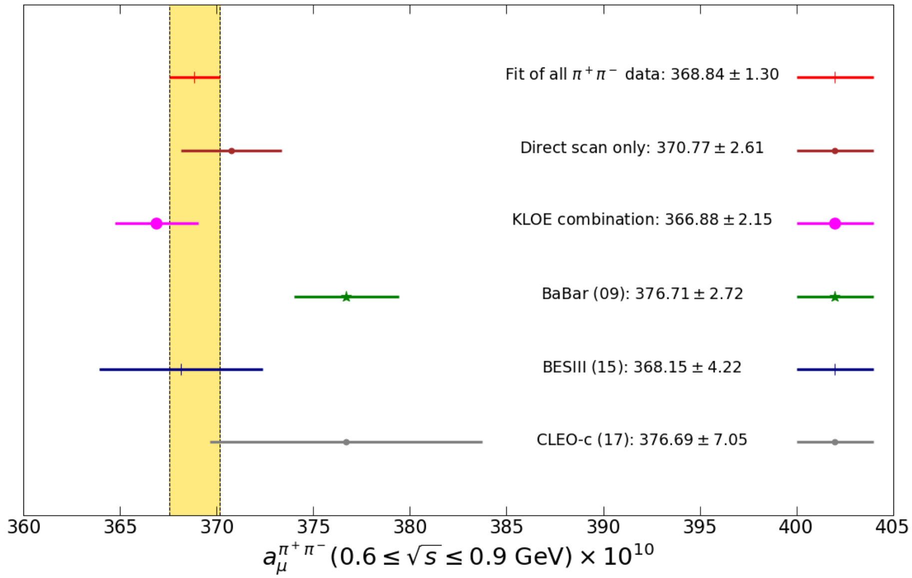
- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from **6 independent experiments** with different systematics and different radiative corrections
- Data sets from Radiative Return dominate, **until now…**

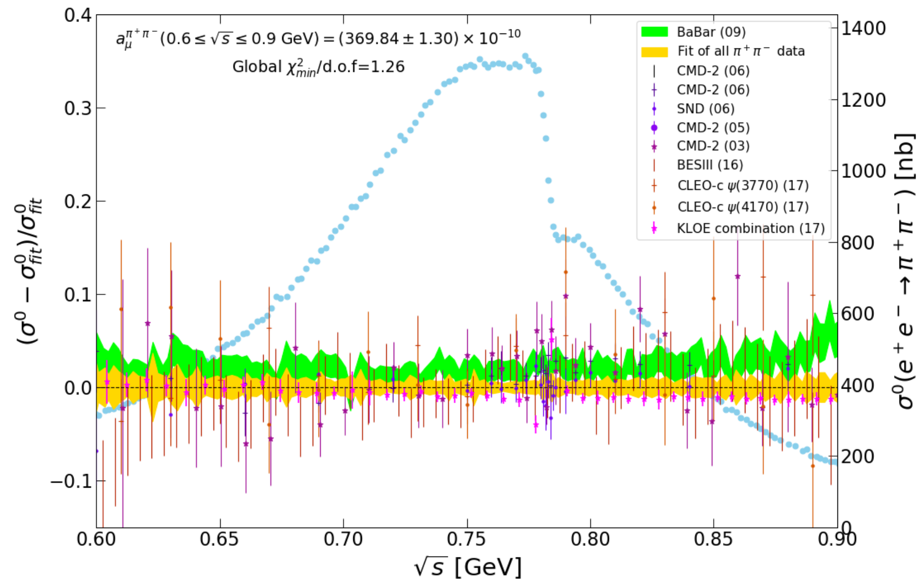
[KNT19, PRD101, 014029]



a_μ^{HVP} : $\pi^+\pi^-$ channel **KLOE vs. Babar puzzle, enlarged WP error**

[Plots from KNT19]

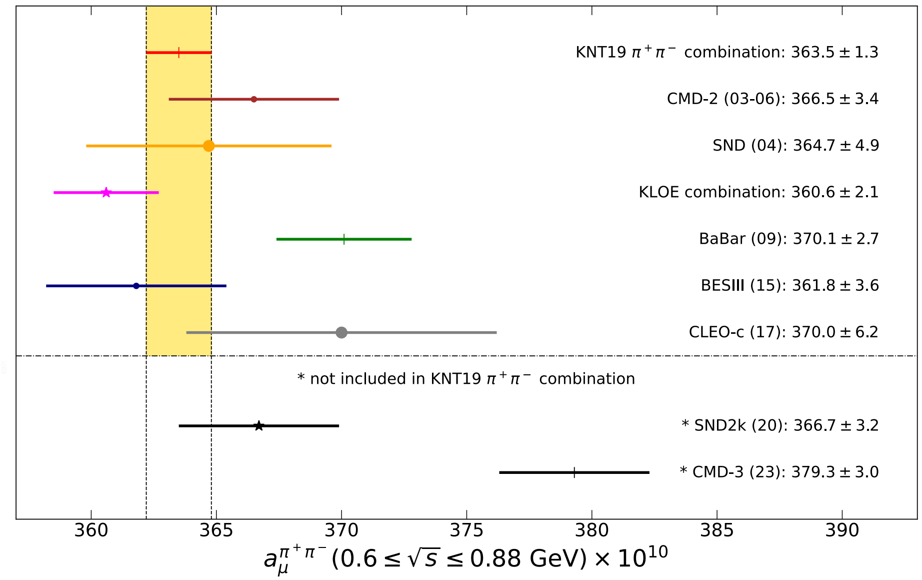


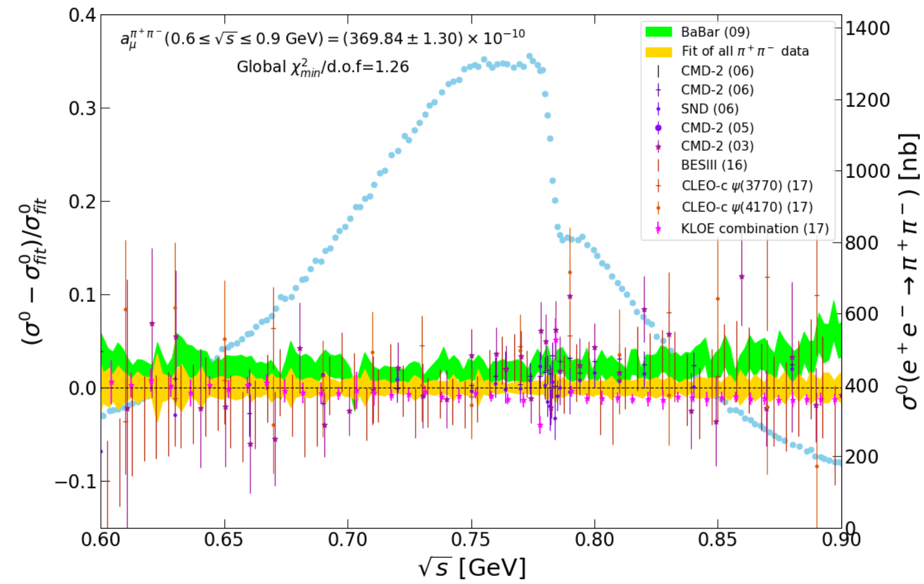


- **Tension** between different sets, especially between the most precise 4 sets from **BaBar** and **KLOE**
- Inflation of error with **local** χ^2_{min} accounts for tensions, leading to a ~**14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)
- Differences in data and methods accounted for in **WP merging procedure,** leading to enlarged error for a_μ^{HVP}

a_μ^{HVP} : $\pi^+\pi^-$ channel **KLOE vs. Babar puzzle, enlarged WP error**

[Plot from KNT19 (updated by Alex Keshavarzi)]



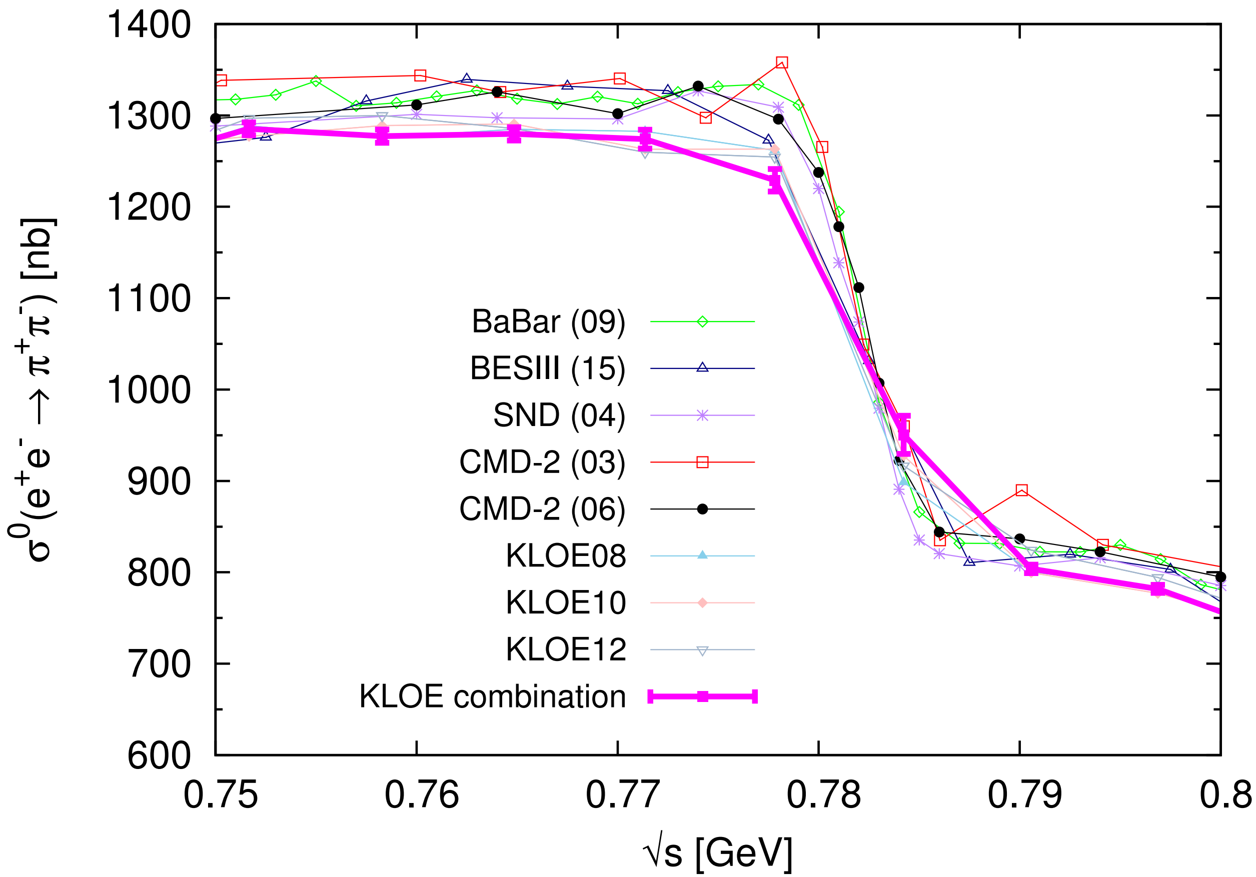


- **Tension** between different sets, especially between the most precise 4 sets from **BaBar** and **KLOE**
- Inflation of error with **local** χ^2_{min} accounts for tensions, leading to a **~14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)
- Differences in data and methods accounted for in **WP merging procedure,** leading to enlarged error for a_μ^{HVP}. **Procedure not well suited to cover CMD-3**

HVP: $\pi^+\pi^-$ channel

- **Tension** between data sets from KLOE, BaBar, CMD-2, SND and BESIII in the **ρ-ω** interference region
- Note that some differences, possibly due to binning effects, are washed out in the dispersion integral for $a_\mu^{2\pi}$

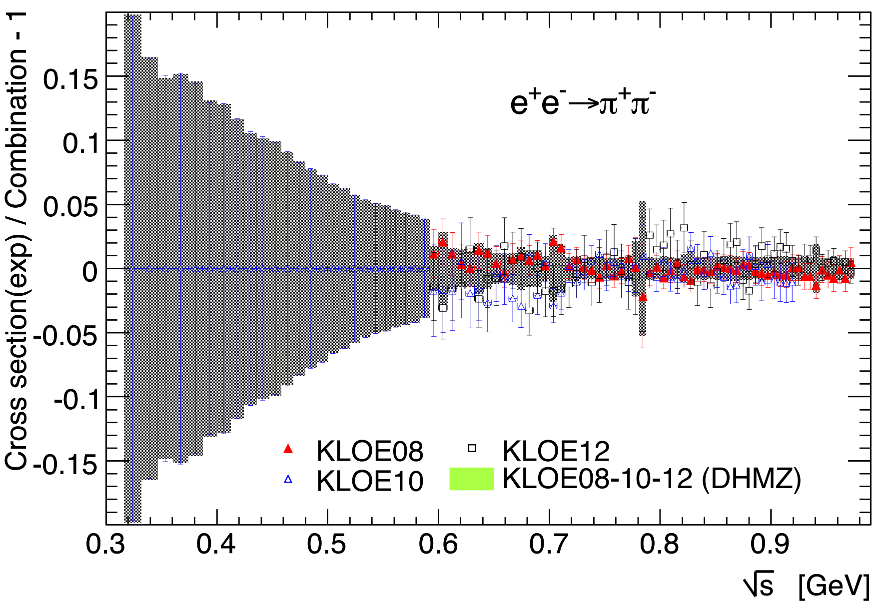
Figure from KLOE (+KT) combination paper JHEP 03(2018)173

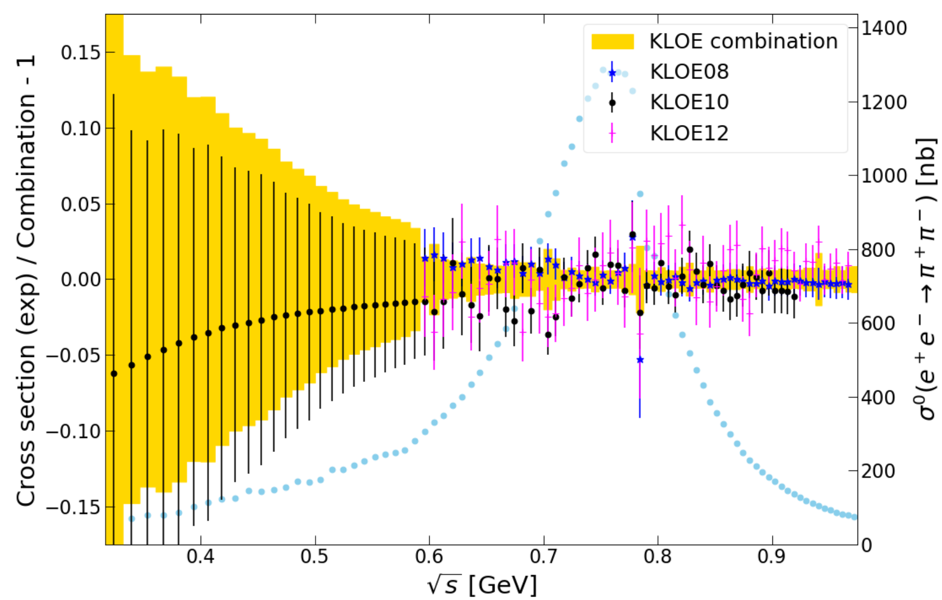


HVP: $\pi^+\pi^-$ channel

- Combination of same three KLOE data sets by DHMZ (left) and KNT (right), leading to
- different results, depending on use of **long-range correlations** through systematic errors;

  -- DHMZ: restricted to error estimate, but not used to determine combination mean values
  -- KNT: full use of correlated errors in fit, allowing change of mean values within errors

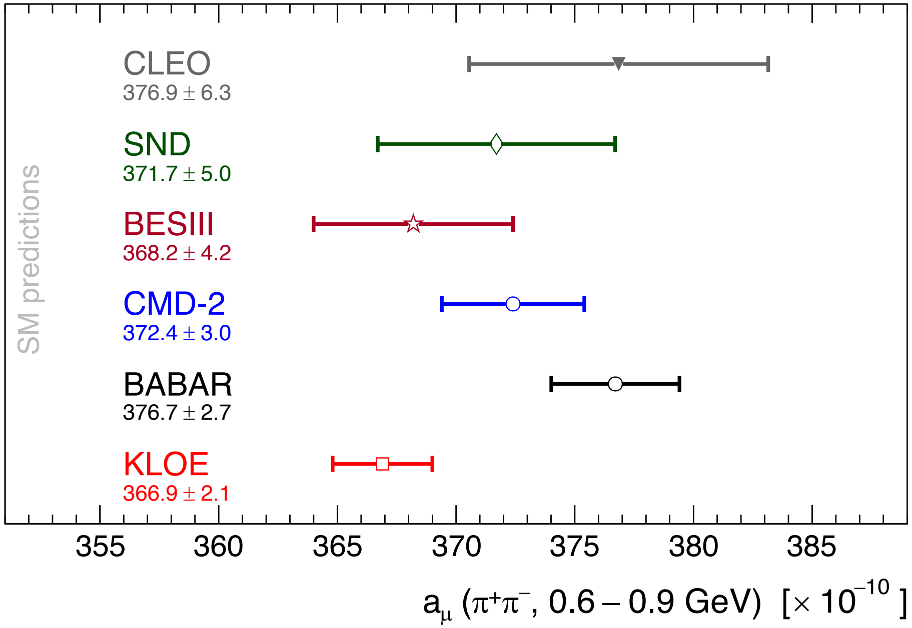




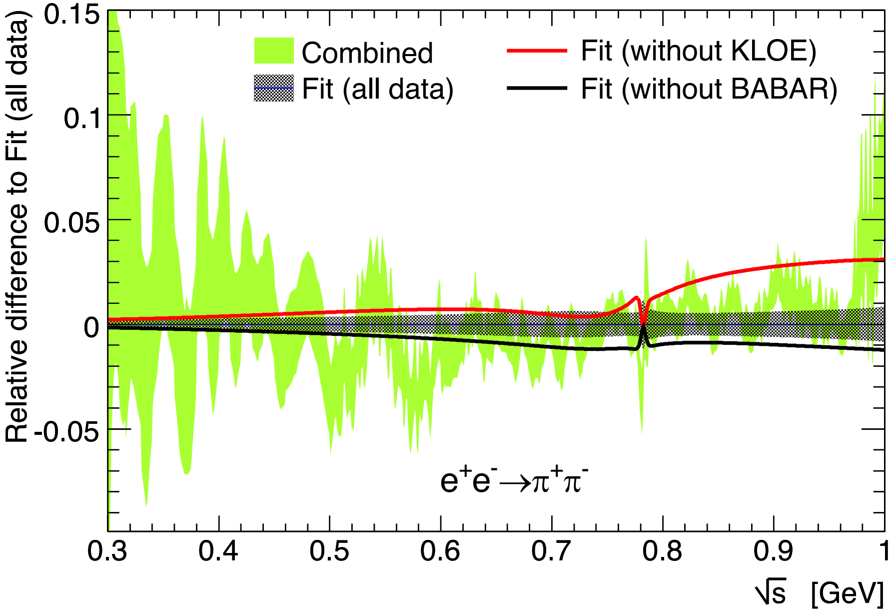
HVP: $\pi^+\pi^-$ channel [DHMZ, *Eur. Phys. J.* C 80(2020)3, 241]

- In addition they employ a fit, based on **analyticity + unitarity + crossing symmetery,** similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraints/lower errors at low energies
- For 2π, based on difference between result for $a_\mu^{\pi\pi}$ w/out KLOE and BaBar, sizeable additional systematic error is applied and mean value adjusted

arXiv:1908.00921 Figure 5:

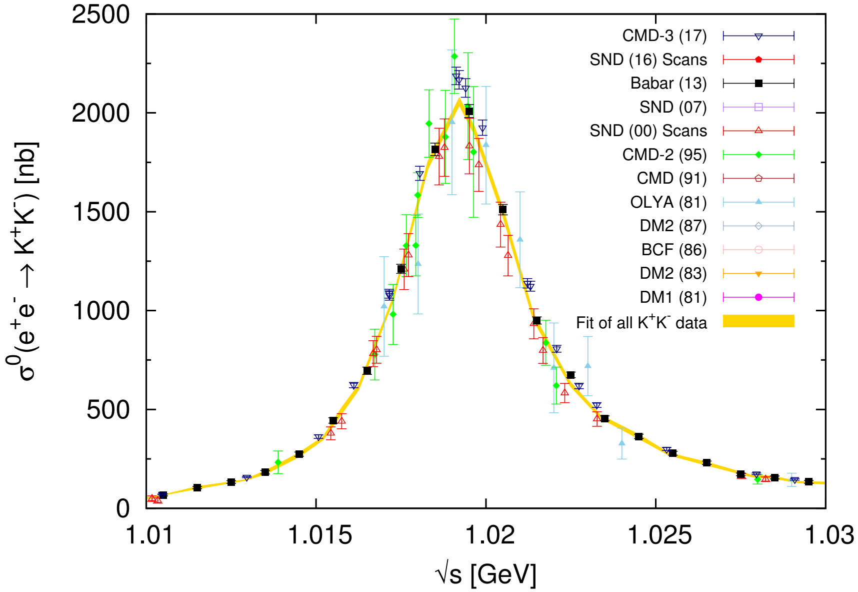


arXiv:1908.00921 Figure 6:



HVP: Kaon channels [KNT18, PRD97, 114025]

K^+K^-



New data:

BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

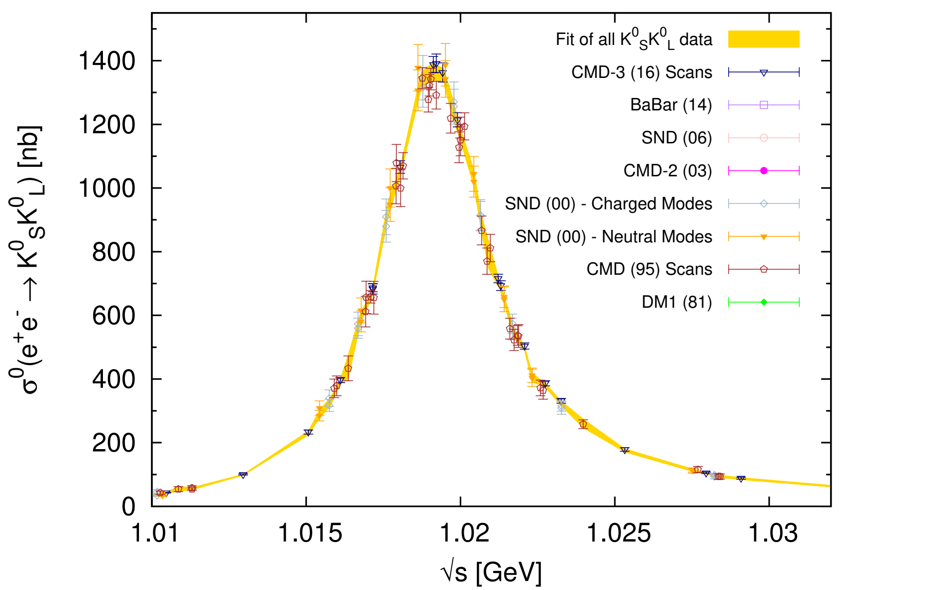
Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\rm tot}$$

HLMNT11: $22.15 \pm 0.46_{\rm tot}$

Large increase in mean value

$K_S^0 K_L^0$



New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

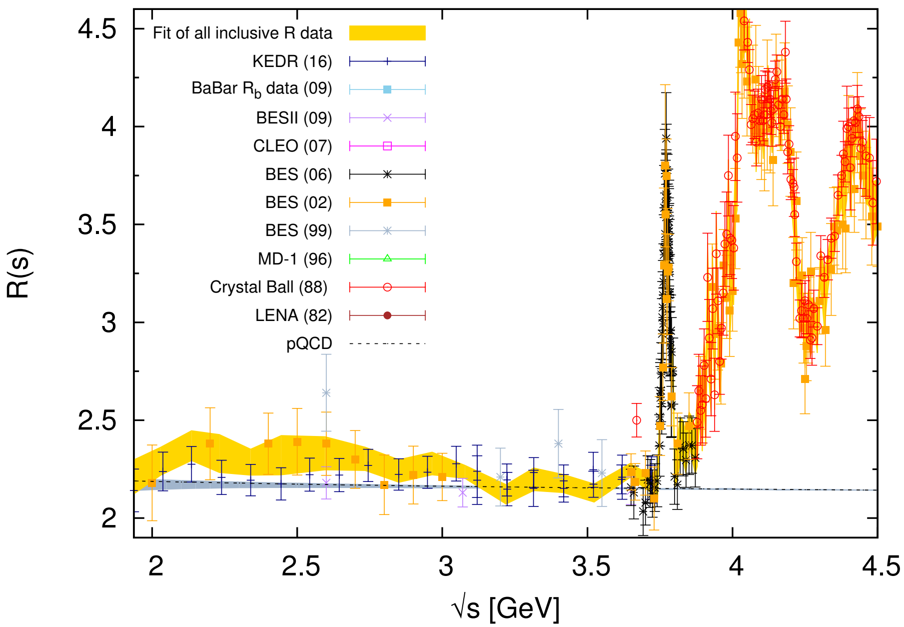
$$a_\mu^{K_S^0 K_L^0} = 13.04 \pm 0.19_{\rm tot}$$

HLMNT11: $13.33 \pm 0.16_{\rm tot}$

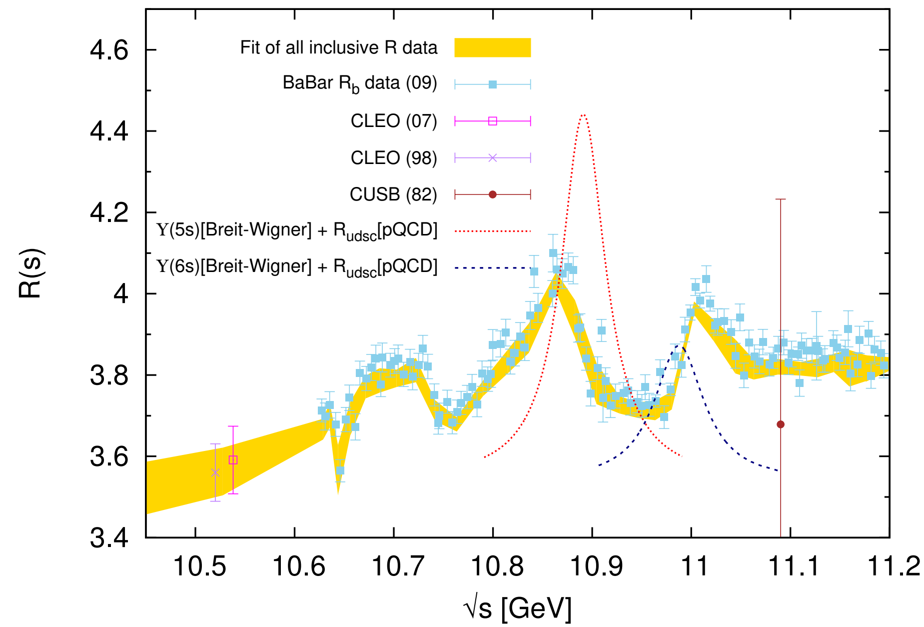
Large changes due to new precise measurements on ϕ

HVP: σ_{had} inclusive region [KNT18]

$\Rightarrow$ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

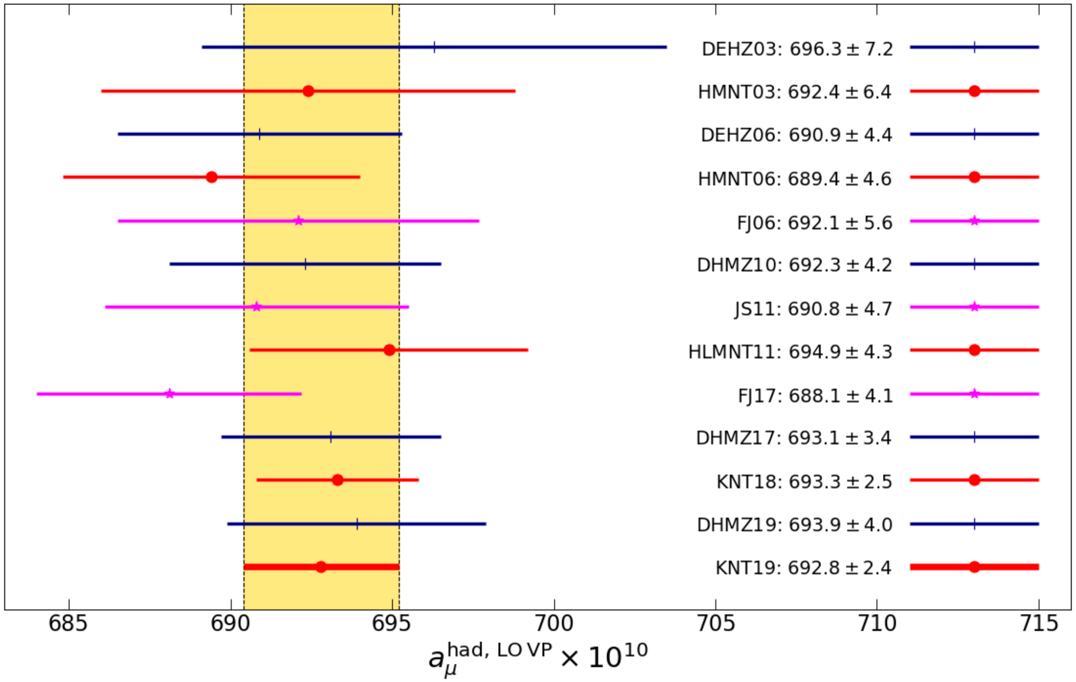
HVP: White Paper comparison

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]

a_μ^{HVP} : > 20 years of data based predictions, `pies'



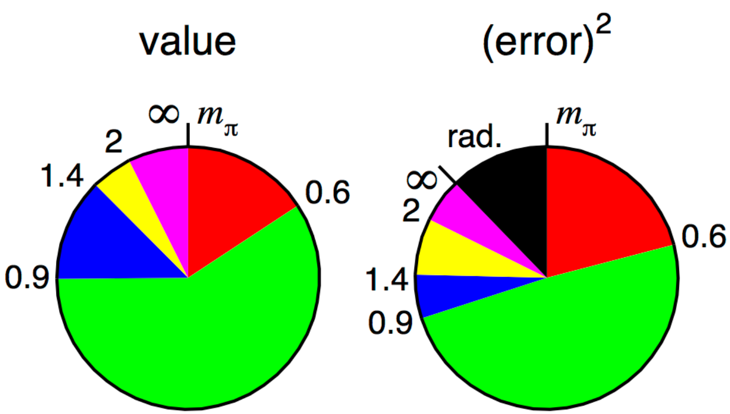
- **Stability** and **consolidation** over two decades thanks to more and better data input and improved compilation procedures

- Compare with **merged** DHMZ & KNT WP20 value:

$a_\mu^{had,\,LO\,VP}$(WP20) = 693.1(4.0)×10^{-10}

Pie diagrams for KNT compilation:

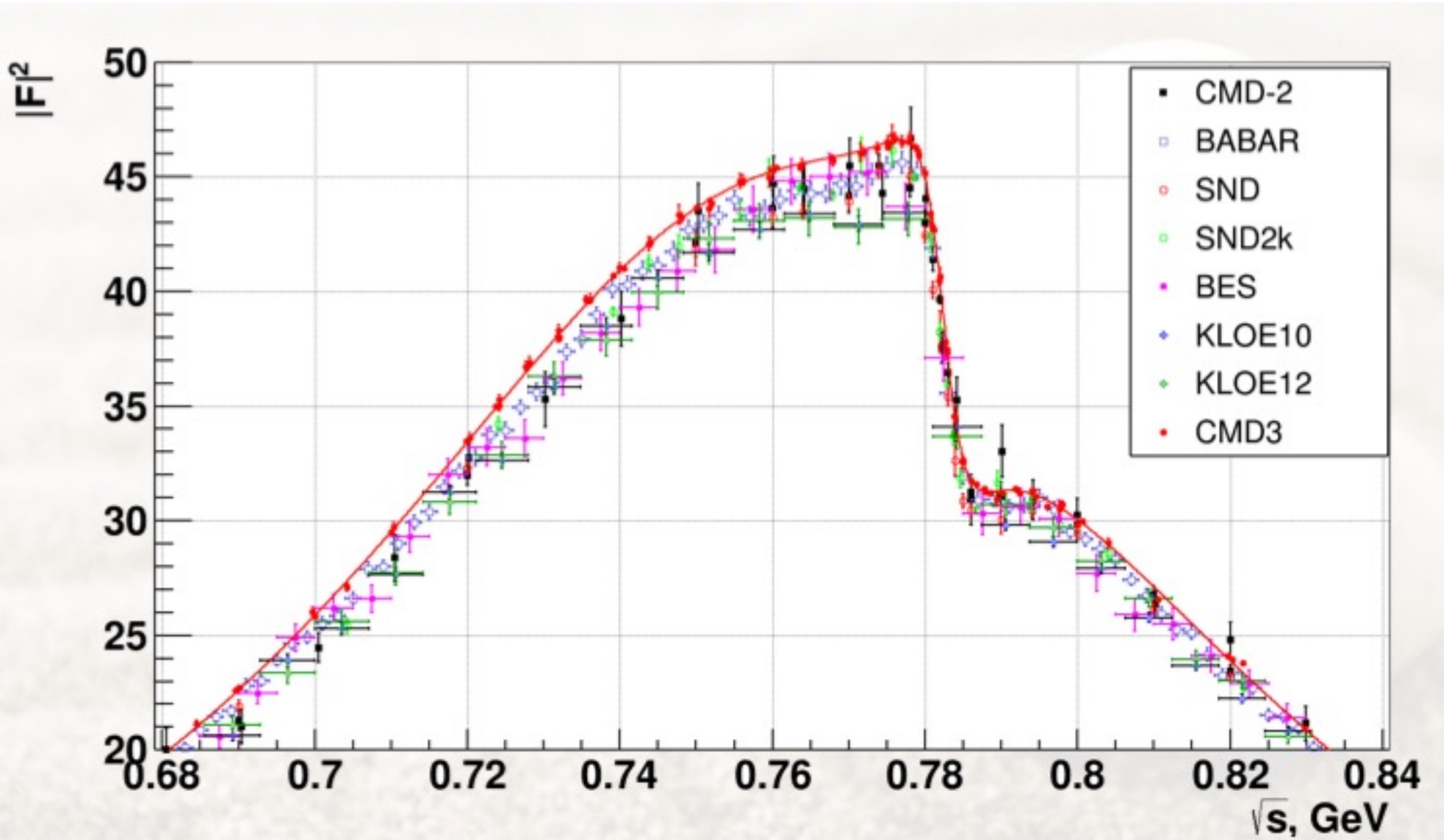
- error still dominated by the two pion channel
- significant contribution to error from additional **uncertainty from radiative corrections**
- **Is all this invalidated by the recent CMD-3 data?**



New CMD-3 $\pi^+\pi^-$ data vs. other experiments

Slides from Fedor Ignatov's TI talk 27.3.2023

arXiv:2302.08834

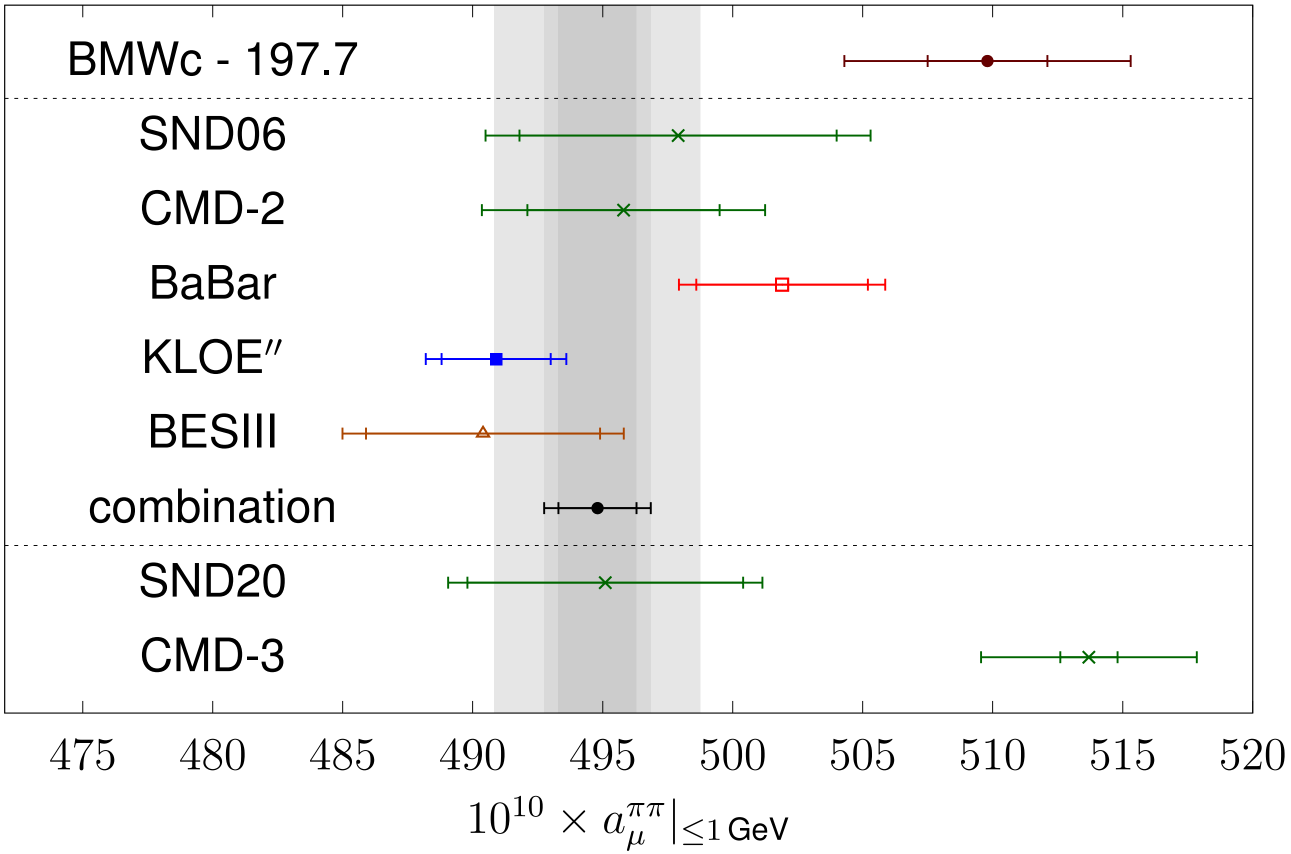


Theory Initiative: Sep. 2023 workshop at Bern

Peter Stoffer: **studies of Colangelo et al. with analyticity&unitarity based fits:**
(no combination w. CMD-3 yet)

More tensions: CMD-3

→ F. Ignatov et al. (CMD-3), 2302.08834 [hep-ex]



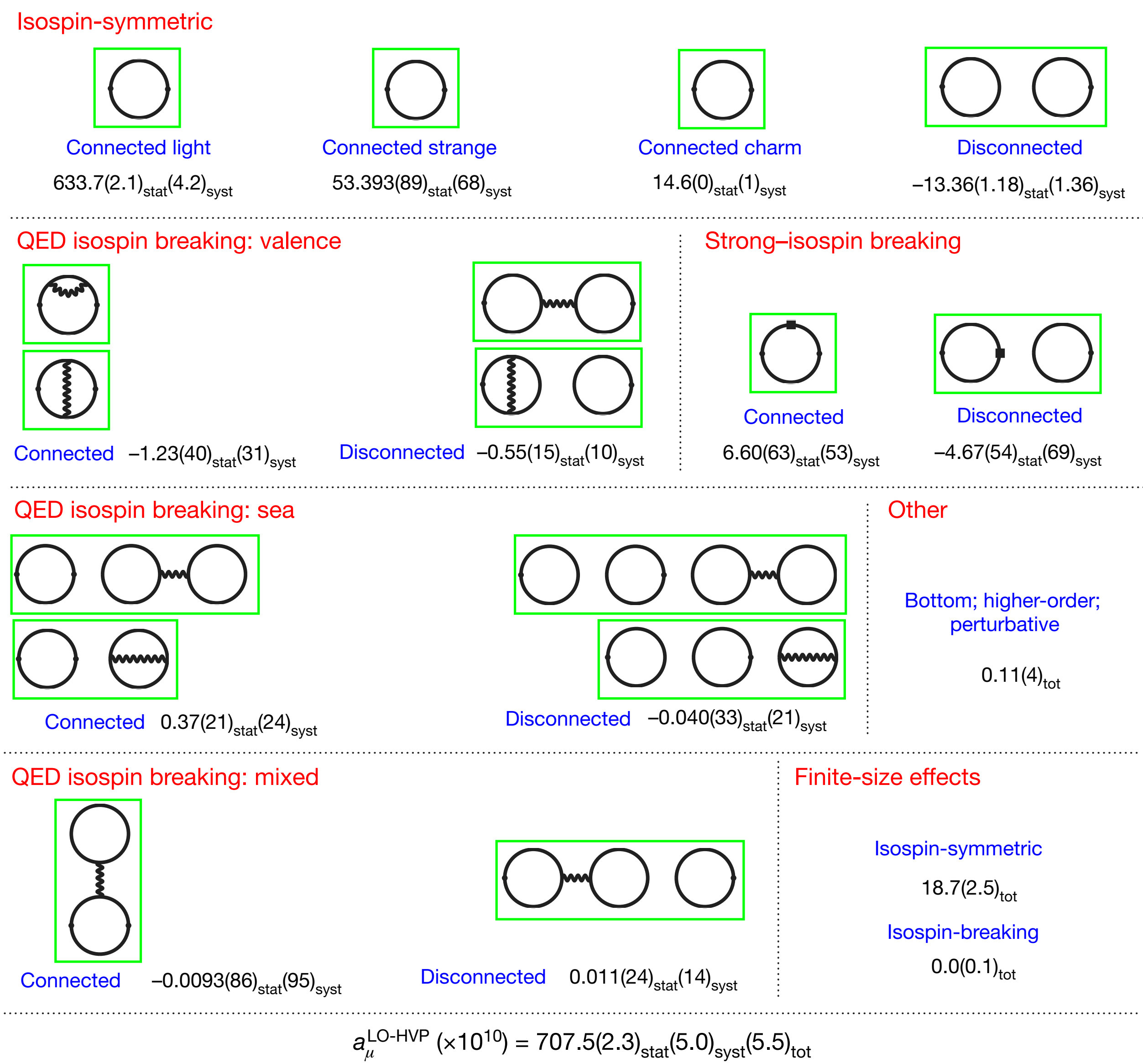
Theory Initiative: Sep. 2023 workshop at Bern

Michel Davier's summary report of the **`49 Questions to CMD-3'** (all answered by Fedor):

Conclusions

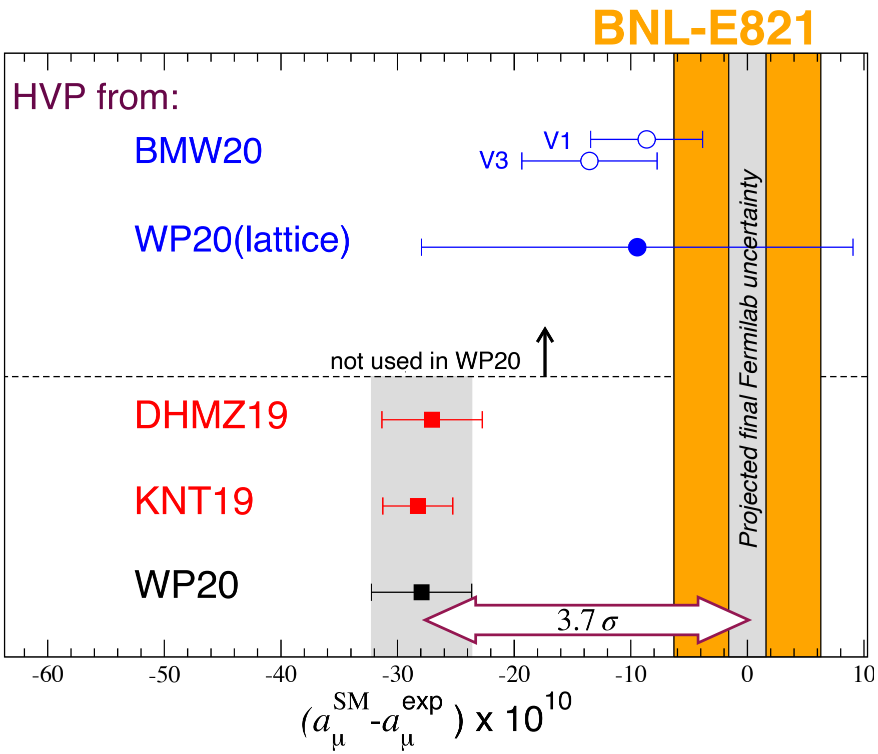
- Difficult exercise: sophisticated analyses are not easy to penetrate without access to the data
- However we got documented answers on detailed questions covering the important aspects of the analysis
- It is fair to say that no major issue significantly impacting the results has been identified
- The strength of the analysis lies in (1) the large statistics accumulated giving the possibility to perform systematic tests with high precision, (2) improved performance of the CMD-3 detector, and (3) the fact that two independent methods were used for channel separation
- Still several points remained unclear to us and /or not enough convincing with the information available
- Possible effects on the results from these minor issues need to be quantified with respect to the claimed accuracy
- Need guidance from CMD-2/3 on how to handle their data

a_μ^{HVP} : Lattice result from **BMW** [Borsanyi et al., Nature 2021]

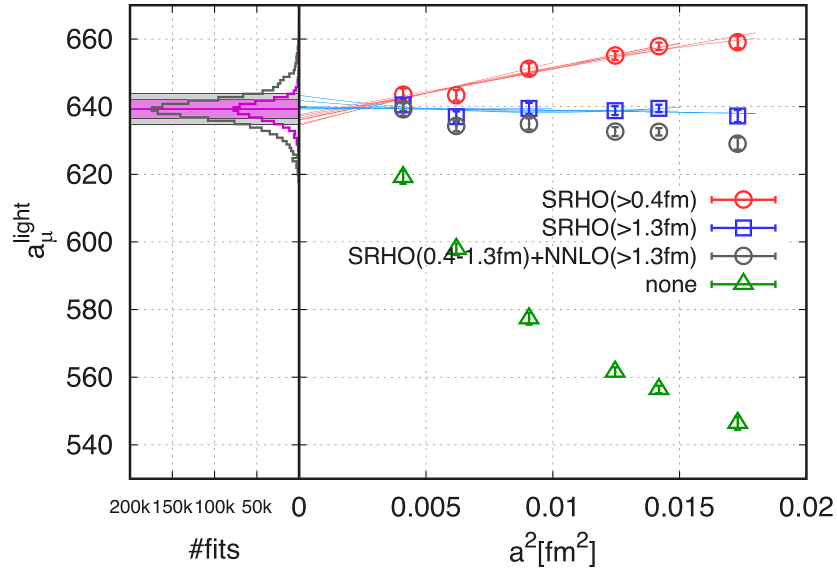


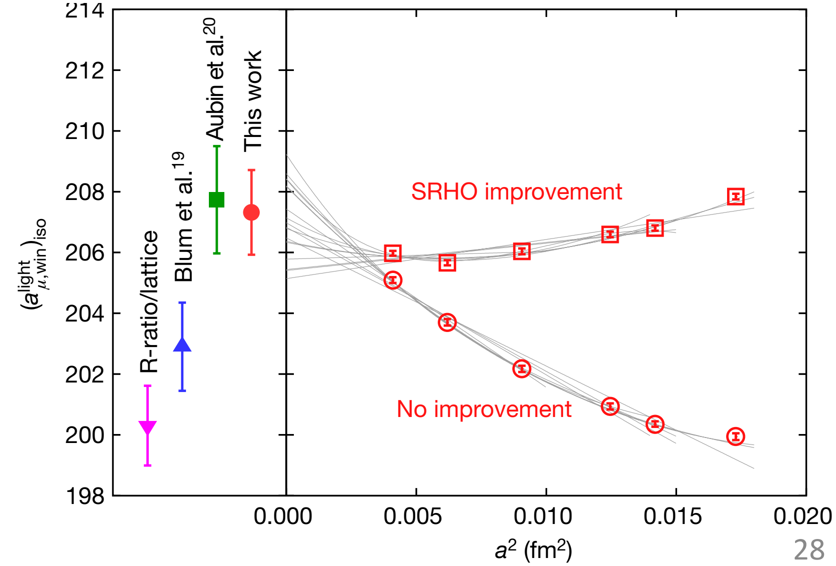
- First lattice prediction with errors matching the data-driven approach
- Current-current correlators, summed over all distances and integrated over time (TMR)
- Using a L~6fm lattice (11fm for finite size corrections)
- Physical quark masses
- Strong + QED isospin breaking corrections

a_μ^{HVP} : Tension between **data-driven & BMW**. Systematics



BMW20 [Borsanyi et al, arXiv:2002.12347, 2021 Nature]





BMW20: large systematics from **continuum limit,** large taste-breaking corrections (`SRHO')

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central `**window**' compared to other lattice and data-driven results (**3.7σ** tension)

a_μ^{HVP}: **Window method** for more detailed comparison

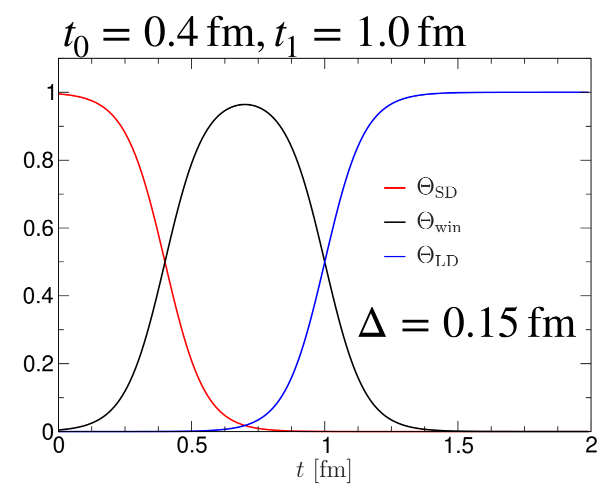
$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)$$

- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $t : 0 \rightarrow t_0$

Intermediate (W) $t : t_0 \rightarrow t_1$

Long Distance (LD) $t : t_1 \rightarrow \infty$

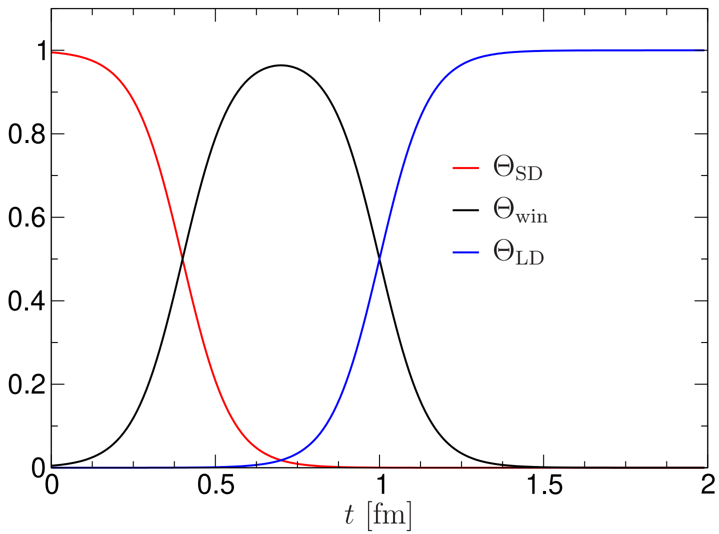


- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

a_μ^{HVP} : **Window method** for more detailed comparison

Correspondence to kernels for comparison with (time-like) dispersive approach:



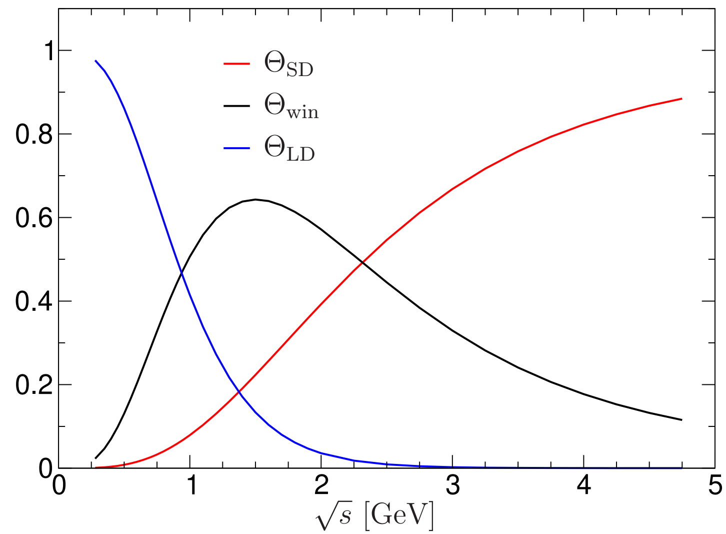
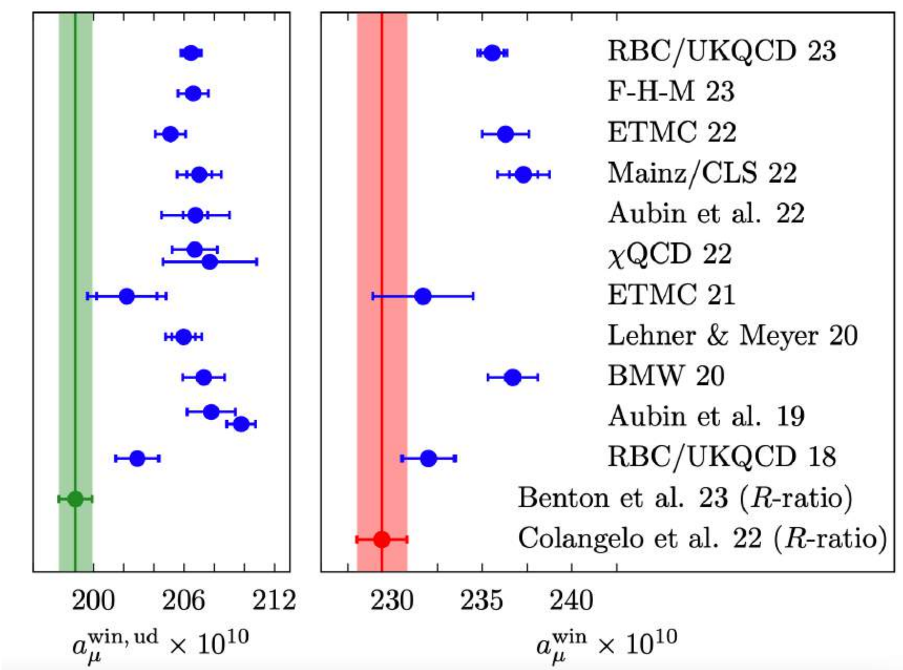


Fig.: G. Colangelo, PWA12/ATHOS7 2021

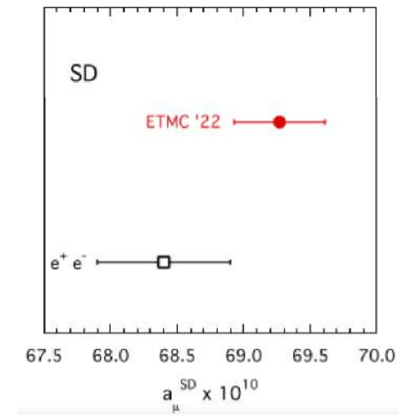
a_μ^{HVP} : **Window method** for more detailed comparison

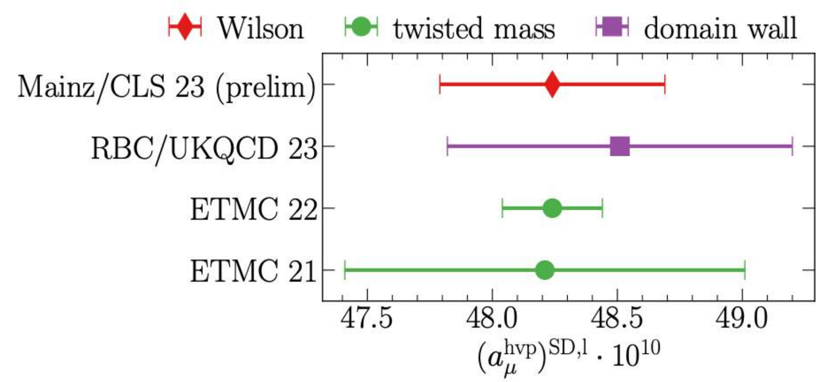
Current lattice predictions for `middle' and Short Distance Euclidean Windows:

Intermediate window (not noisy on Lattice)



Short distance window (noisy on Lattice, small contribution)





No lattice evaluation for long distance window yet (noisy on lattice, largest contribution)

Muon g-2 Theory Initiative working tirelessly to better results and scrutinise differences…

➤ `Consolidation of discrepancy' with data-driven results in middle window

Theory Initiative: Sep. 2023 workshop at Bern

Aida El-Khadra: TI outlook and plans:

WP update: proposed timeline

Goal

Obtain the best possible prediction for a_μ **before** the Fermilab g-2 experiment releases their final measurement (based on runs 4,5,6) in 2025.

Considerations

Writing a WP is a major undertaking, we should make sure it's worth the effort.
➠ Timing of WP update informed by availability of new results & information

Summarize the status of SM predictions
➠ Include everything in update to enable detailed comparisons between the different approaches (e.g. lattice/dispersive) for HVP & HLbL and related quantities

➠ Aim WP update for late 2024

Pathways to solving the (HVP) puzzles

- No easy way out! Signs for Beyond the Standard Model physics?

- BSM at high scales? Many explanations for `**4.2σ**' **puzzle**, few seem natural, NP smoking guns in the flavour sector weakened

- BSM `faking' low $\boldsymbol{\sigma_{had}}$? Possible but not probable
  [DiLuzio, Masiero, Paradisi, Passera, *Phys.Lett.B* 829 (2022) 137037]

  .. a new Z' [Coyle, Wagner, 2305.02354]

  … or even new hadronic states (like sexa-quarks [Farrar, 2206.13460]) ?

- Situation now very complicated due to emerged **lattice & CMD-3 puzzles**

- **More & more precise data are needed (and coming) to clarify data puzzle:**
  BaBar, CMD-3, SND, BES III, Belle II, and **KLOE**

- To avoid any possible bias, **blinded analyses** are now the standard, for both experiments (g-2 and $\boldsymbol{\sigma_{had}}$) and lattice, and also the next KNT+W compilation

- The third way: **MUonE**

KLOE 2π, RC & MC activities have started

- Challenges and opportunities to get a clearer understanding of the puzzles from data,

  to re-establish a stable SM prediction of g-2 [and the running QED coupling, $\alpha(M_Z^2)$]

- New Liverpool+ effort to analyse the full statistics KLOE **2π** data (**integrated *L* ~ 1.7 fb^{-1}**):

  - Leverhulme International Professor G. Venanzoni has created sizeable team of exp+Th+MC in Liverpool and with external collaborators

- Goal: sub-percent accuracy for $e^+e^- \to \pi^+\pi^-$,
  and improvement of a factor of ~2 on the total uncertainty => **$\Delta a_\mu^{HLO} \lesssim 0.4\%$**

- This will require significant involvement from theoretical groups

  - improvement of MC(s) to better describe **ISR and FSR** (PHOKHARA, …)

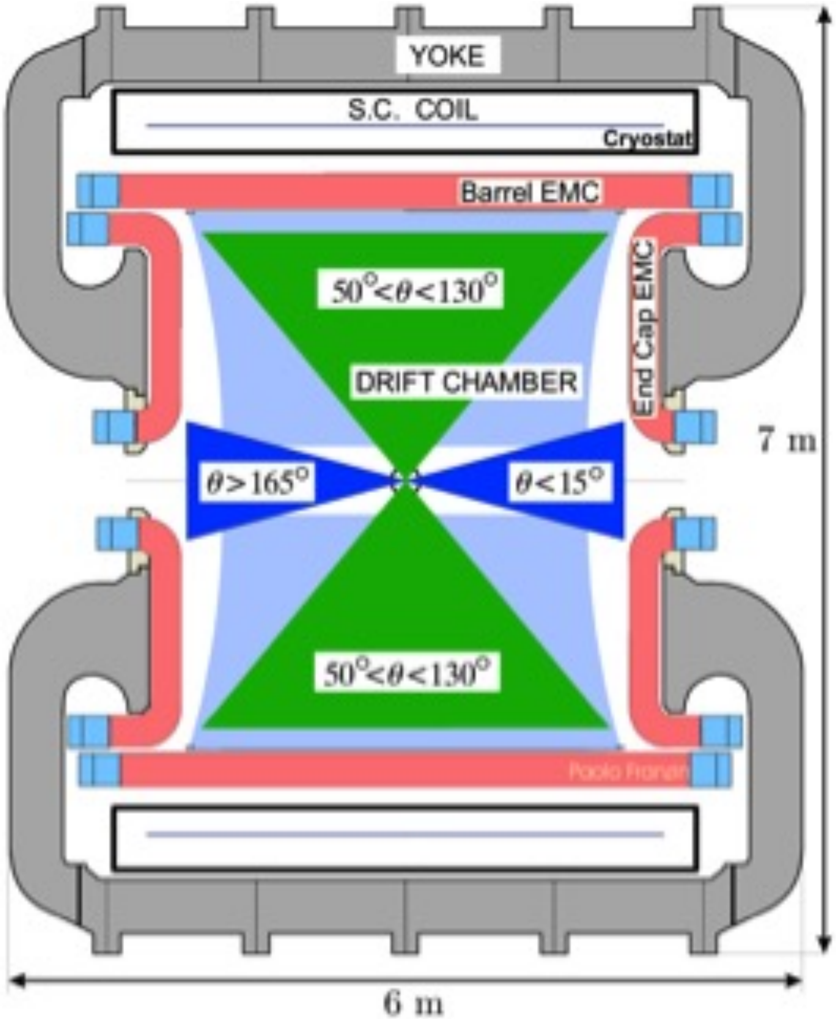
  - main aim is NNLO for ISR and improvement of/consistent FF treatment for FSR

  - other MC groups have agreed to also concentrate on $e^+e^- \to \pi^+\pi^-$, $\mu^+\mu^-$, e^+e^-

    (Babayaga, Sherpa, McMule, KKMC)

  - ongoing activity: 5th WorkStop/ThinkStart: Radiative corrections and MC tools for Strong 2020

KLOE 2π analyses



Large Angle:

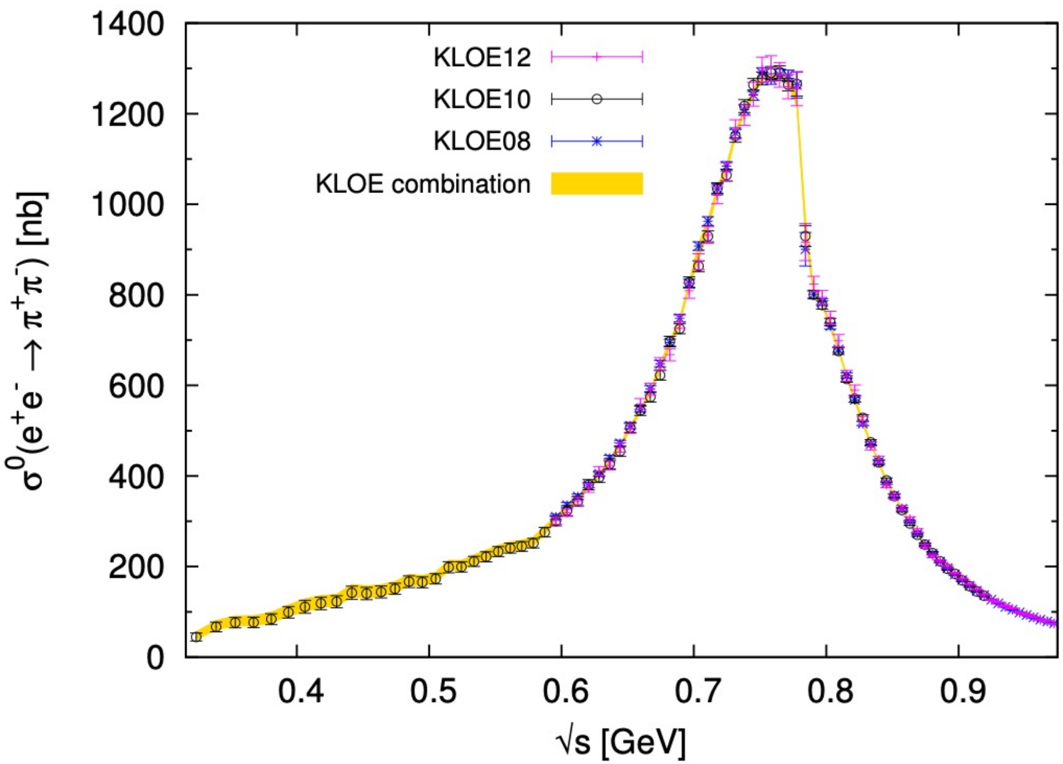
2 pion (muon) tracks at 50° < $\vartheta_{\pi,\mu}$ < 130°

Small angle photon selection:

ϑ_{miss} < 15°; ϑ_{miss} > 165°

- high statistics for ISR events
- low FSR contribution
- easy to suppress $\varphi \to \pi^+\pi^-\pi^0$ background
- photon momentum from kinematics: $\vec{p}_\gamma = \vec{p}_{miss} = -(\vec{p}_+ + \vec{p}_-)$
- threshold region not accessible

KLOE 2π results



KLOE05

Small Angle analysis of 140 pb^{-1} @ m_φ
KLOE Coll. Phys. Lett. B 606 (2005)

KLOE08

Small Angle analysis of 240 pb^{-1} @ m_φ
KLOE Coll. Phys. Lett. B 670 (2009)

KLOE10

Large angle analysis of 250 pb^{-1} @ 1 GeV
KLOE Coll. Phys. Lett. B 700 (2011)

KLOE12

KLOE08 with normalisation to $e^+e^- \to \mu^+\mu^-$
KLOE Coll. Phys. Lett. B 720 (2013)

Combination of three sets *JHEP 1803 (2018) 173:*

$$a_\mu^{\pi\pi}\,[0.1 < s < 0.95\ \text{GeV}^2] = (489.8 \pm 1.7_{\text{stat}} \pm 4.8_{\text{sys}}) \times 10^{-10}$$

KLOE 2π uncertainties

We aim to improve:

Syst. errors (%)	$\Delta^{\pi\pi} a_\mu$ abs [4]	$\Delta^{\pi\pi} a_\mu$ ratio
Background Filter (FILFO)	negligible	negligible
Background subtraction	0.3	0.6
Trackmass	0.2	0.2
Particle ID	negligible	negligible
Tracking	0.3	0.1
Trigger	0.1	0.1
Unfolding	negligible	negligible
Acceptance ($\theta_{\pi\pi}$)	0.2	negligible
Acceptance (θ_π)	negligible	negligible
Software Trigger (L3)	0.1	0.1
Luminosity	0.3 ($0.1_{th} \oplus 0.3_{exp}$)	-
$\sqrt{s}$ dep. of H	0.2	-
Total exp systematics	0.6	0.7
Vacuum Polarization	0.1	-
FSR treatment	0.3	0.2
Rad. function H	0.5	-
Total theory systematics	0.6	0.2
Total systematic error	0.9	0.7

possible corrs. to naïve ISR-FSR factorization for radiator function

WorkStop/ThinkStart: WorkStop Nr 5 (Adrian Signer's intro)

- idea: make a next step in

Radiative corrections and Monte Carlo tools for
low-energy hadronic cross sections in $e^+ e^-$ collisions

Eur. Phys. J. C (2010) 66: 585–686
DOI 10.1140/epjc/s10052-010-1251-4

THE EUROPEAN PHYSICAL JOURNAL C

Review

- inspired by [0912.0749]

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

S. Actis[38], A. Arbuzov[9,e], G. Balossini[32,33], P. Beltrame[13], C. Bignamini[32,33], R. Bonciani[15], C.M. Carloni Calame[35], V. Cherepanov[25,26], M. Czakon[1], H. Czyż[19,a,f,i], A. Denig[22], S. Eidelman[25,26,g], G.V. Fedotovich[25,26,e], A. Ferroglia[23], J. Gluza[19], A. Grzelińska[8], M. Gunia[19], A. Hafner[22], F. Ignatov[25], S. Jadach[8], F. Jegerlehner[3,19,41], A. Kalinowski[29], W. Kluge[17], A. Korchin[20], J.H. Kühn[18], E.A. Kuraev[9], P. Lukin[25], P. Mastrolia[14], G. Montagna[32,33,b,d], S.E. Müller[22,f], F. Nguyen[34,d], O. Nicrosini[33], D. Nomura[36,h], G. Pakhlova[24], G. Pancheri[11], M. Passera[28], A. Penin[10], F. Piccinini[33], W. Płaczek[7], T. Przedzinski[6], E. Remiddi[4,5], T. Riemann[41], G. Rodrigo[37], P. Roig[27], O. Shekhovtsova[11], C.P. Shen[16], A.L. Sibidanov[25], T. Teubner[21,h], L. Trentadue[30,31], G. Venanzoni[11,c,i], J.J. van der Bij[12], P. Wang[2], B.F.L. Ward[39], Z. Was[8,g], M. Worek[40,19], C.Z. Yuan[2]

- consolidate and implement the progress since 2010

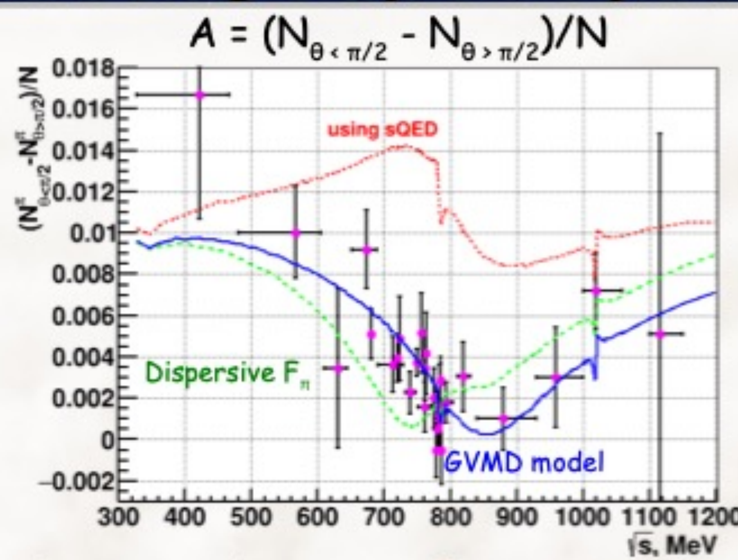
→ the motivation for this is clear from the theory perspective

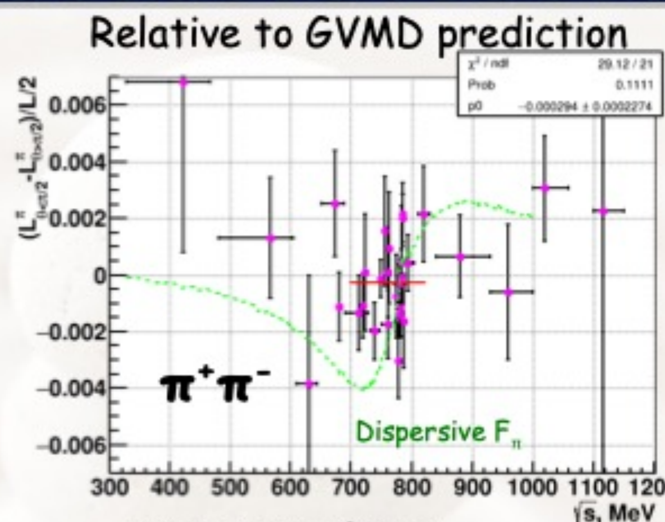
Fedor Ignatov's talk on MC generators:

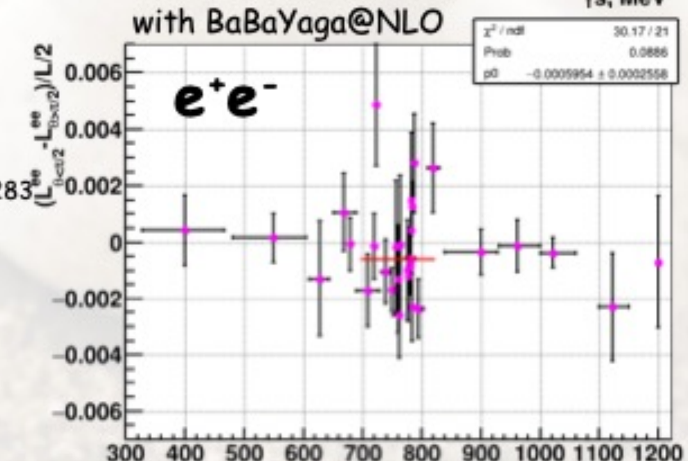
☞ **Need to study FF models**

Charge asymmetry in e+e- -> π+π-

Precalculated amplitude of box diagram above sQED was added to MCGPJ







Conventional sQED approach gives ~ 1% inconsistency

The theoretical model within GVMD was introduced, describes well the CMD-3 data R.Lee et al., Phys.Lett.B 833 (2022) 137283 was confirmed by calculation in dispersive formalism M.Hoferichter et al., JHEP 08 (2022) 295

Average at $\sqrt{s}$ = 0.7-0.82 GeV:

$\pi^+\pi^-$: $\langle \delta A \rangle$ = -0.029 ± 0.023 %

e^+e^-: $\langle \delta A \rangle$ = -0.060 ± 0.026 %

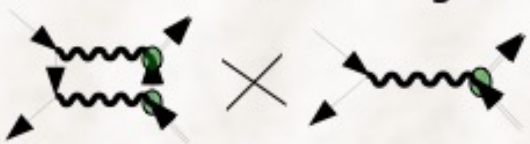
Fedor Ignatov's talk on MC generators:

☞ **Need to study FF models**

How it can affect pion form factor measurements?

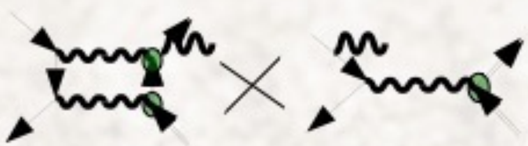
Usually event selections in analyses are charge/angle symmetric

Main effect at lowest order comes from:
Interference of box vs born diagrams



=> only charge-odd contribution
effect is integrated out
in full cross-section

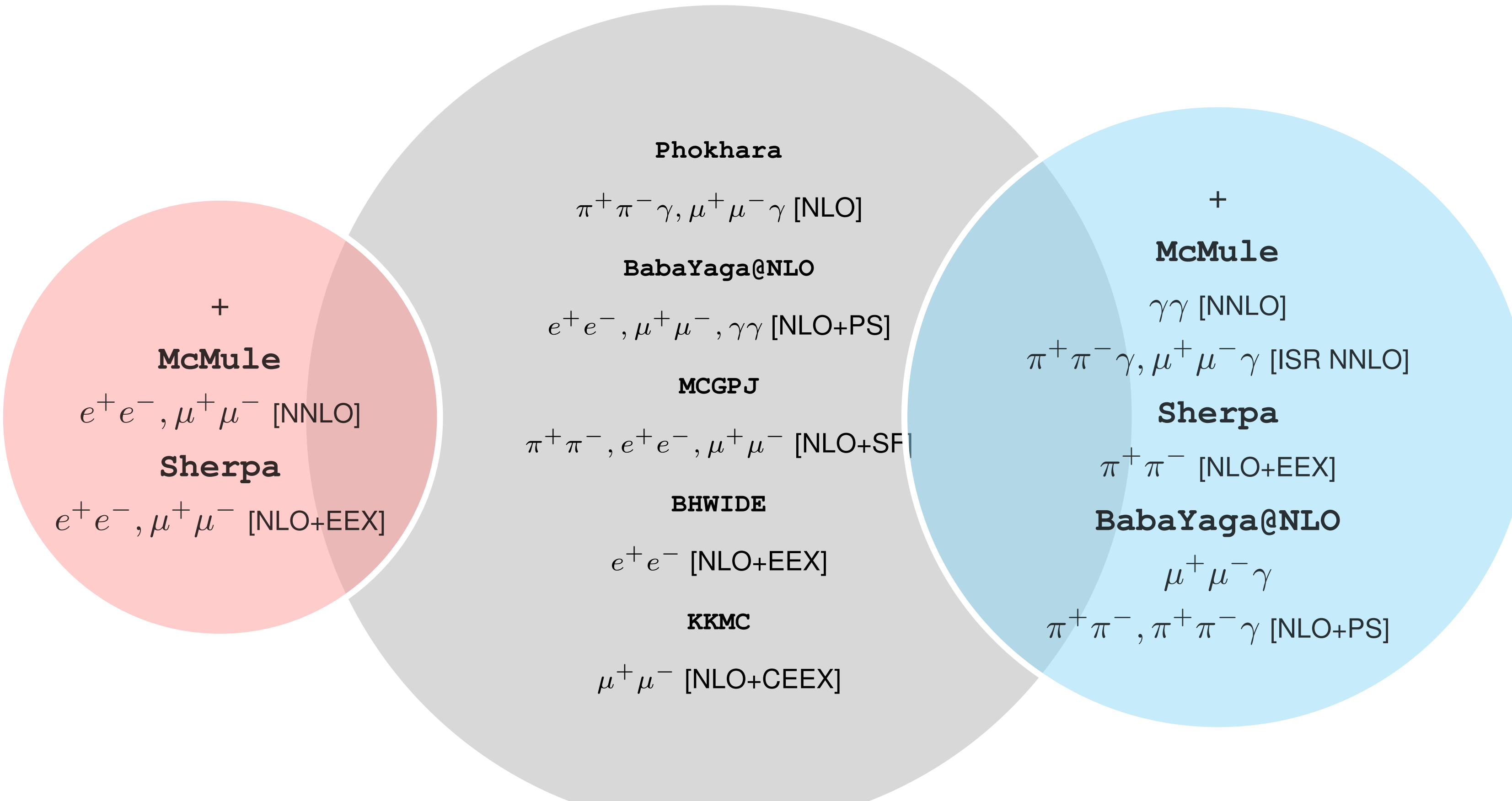
Interference of ISR & box vs FSR (or v.v.)



=> charge-even
can affect integrated cross-section

Carlo Carloni Calame & Marek Schoenherr:

Workstop/Thinkstart outcome for WP4



-- (C)EEX: (Coherent) Exclusive Exponentiation, based on YFS exponentiation, coherent is on amplitude level
-- Sherpa also working to include photon splitting in exponentiation, see Lois Flower's talk

Outlook / Conclusions

- The still **unresolved muon g-2 discrepancy** has triggered a lot of experimental & theory activities, including experiments, the Muon g-2 Theory Initiative & **lattice**

- **Much progress** has been made for **HLbL** (disp. & lattice), previously the bottleneck

- For **HVP dispersive**, the **TI published a conservative consensus (WP20)**

  -- no WP update since 2020 yet, **current discrepancies not understood**

  ➤ the resolution of the **puzzles** in the crucial **2π** channel requires further new data

  -- expected/**puzzling new σ_{had} data for 2π** and other channels from

  BaBar, **CMD-3**, **SND**, **BES III**, **Belle II**, and **KLOE** (Liverpool analysis has started)

  ➤ **if** new precise data **agree**, the **$a_\mu^{2\pi}$ puzzle** may go away and the error down

  -- but **further theory effort** (**NNLO$^+$ rad. corrs. & MCs**) **will be crucial**

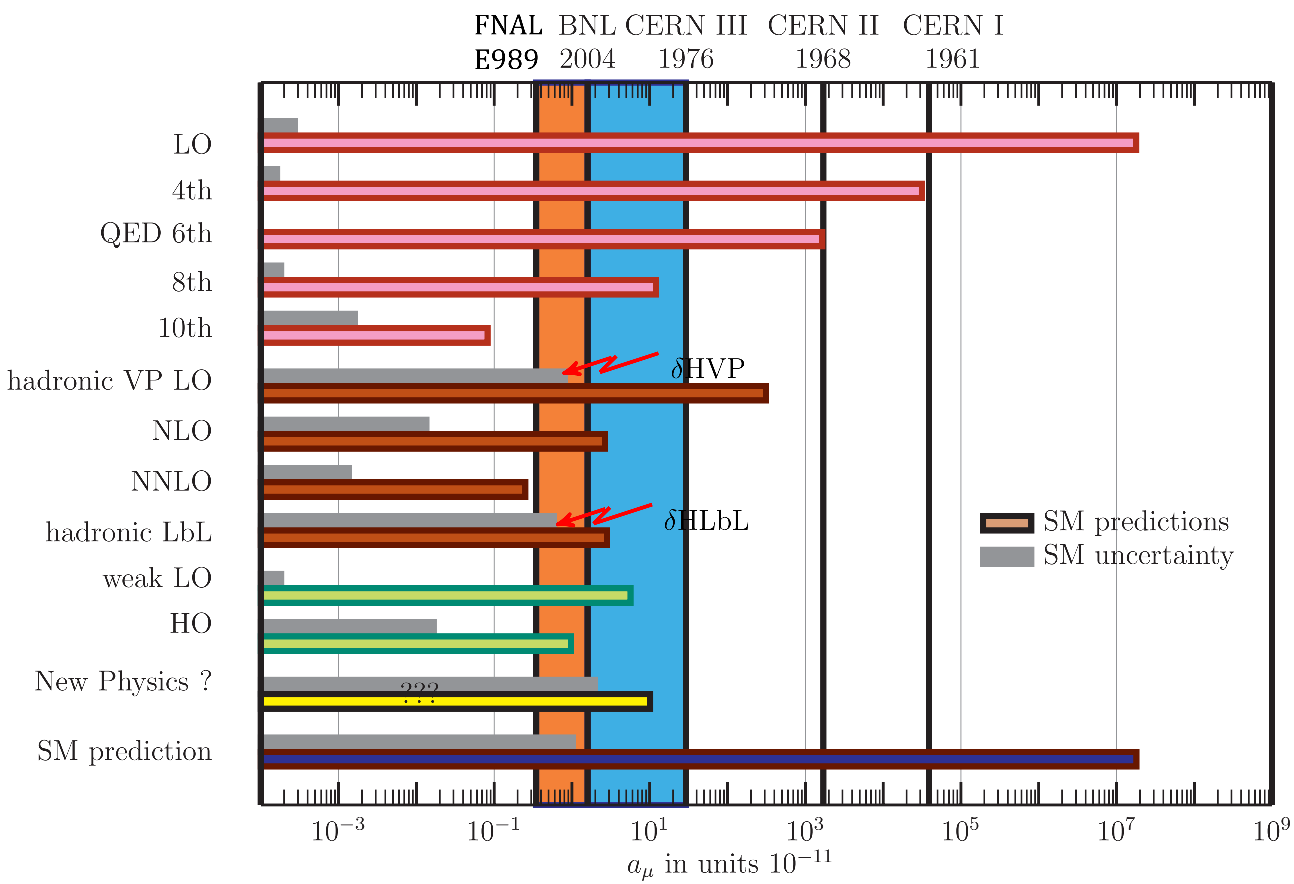
  ➤ this may solve the **lattice puzzle** too. **Longer term, 3rd way: MUonE**

✤ ***There is a lot to do in Exp, Theory, RCs & MCs beyond/before the HL LHC …***

Extras

Why HVP: g-2 exp vs theory - sensitivity chart

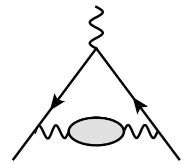
Plot from Fred Jegerlehner



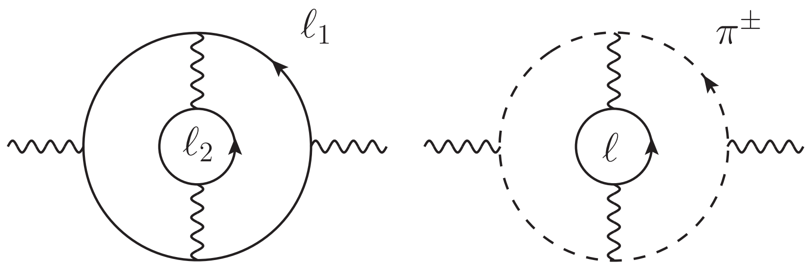
- ➤ Need to control the hadronic contributions

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$$

a_μ^{HVP} : short detour into double-bubbles

- What if the blob in  is a `double-bubble' ?

- Purely leptonic graphs (left diagram below) are part of four-loop QED corrections



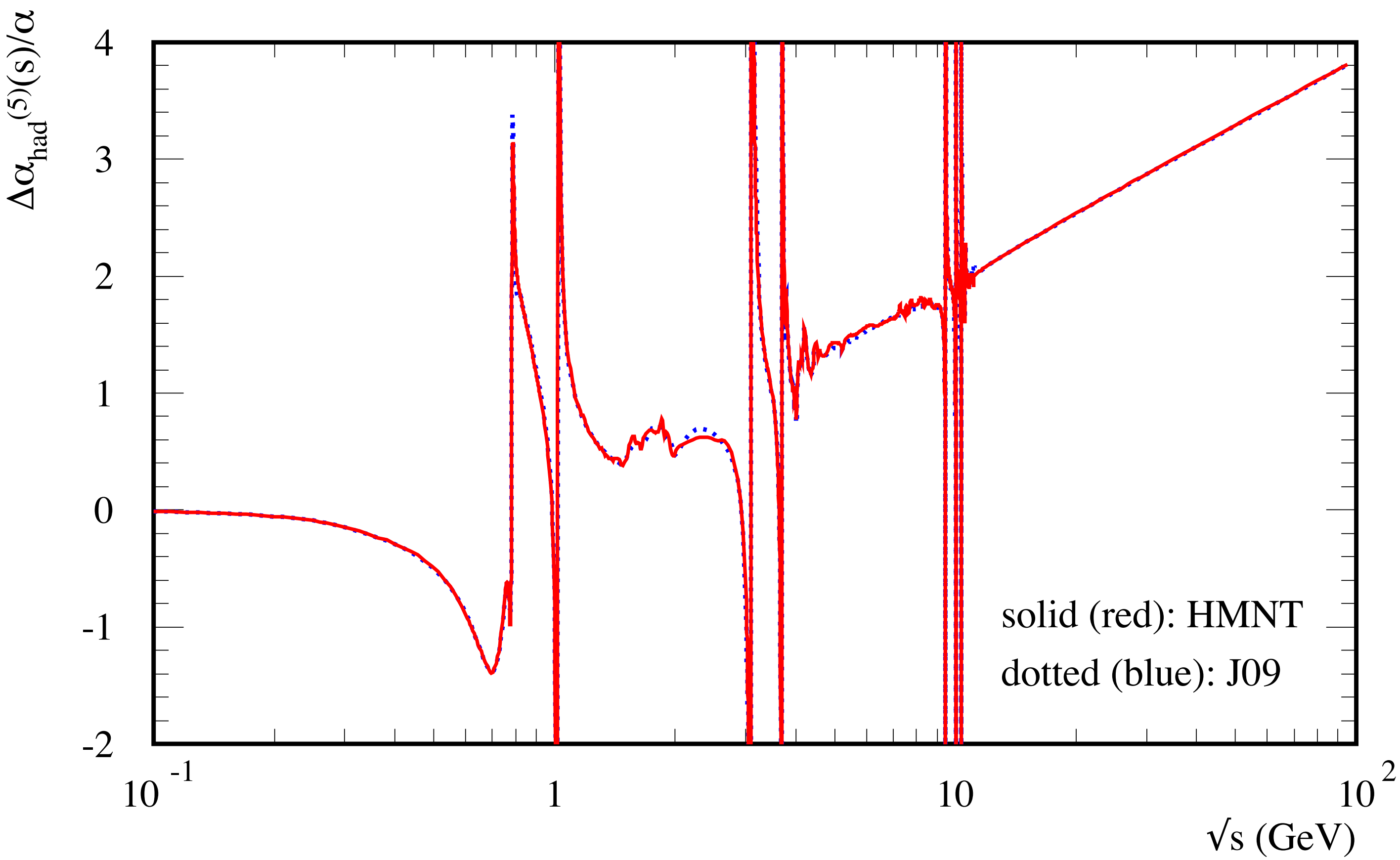
- But possibly enhanced contributions from mixed hadronic-leptonic double bubble graphs (right diagram above) are not included in the hadronic **NNLO** HVP corrections quoted above

- Our recent work has estimated these remaining NNLO contributions to $\mathbf{a_\mu}$ to be **below 1×10^{-11}** and hence not critical at the level of the experimental accuracy

M Hoferichter + TT, *Phys. Rev. Lett.* 128 (2022) 11, 112002

Rad. Corrs.: HVP for running $\alpha(q^2)$. Undressing

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



For demonstration only, results >10 years old!

Different groups use their own HVP routines:

- Fred Jegerlehner,
- DHMZ,
- KNT,
- Novosibirsk (Fedor Ignatov)

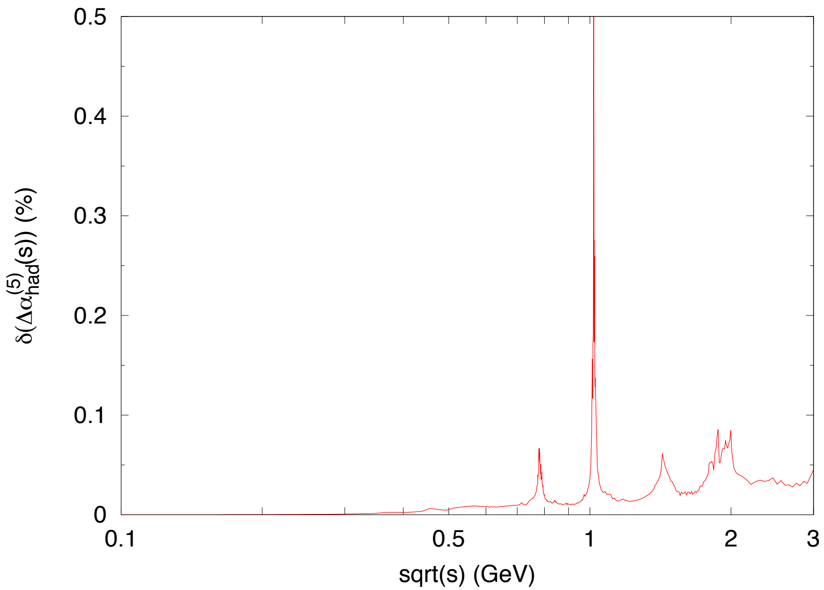
→ with new version big differences (with 2003 version) gone

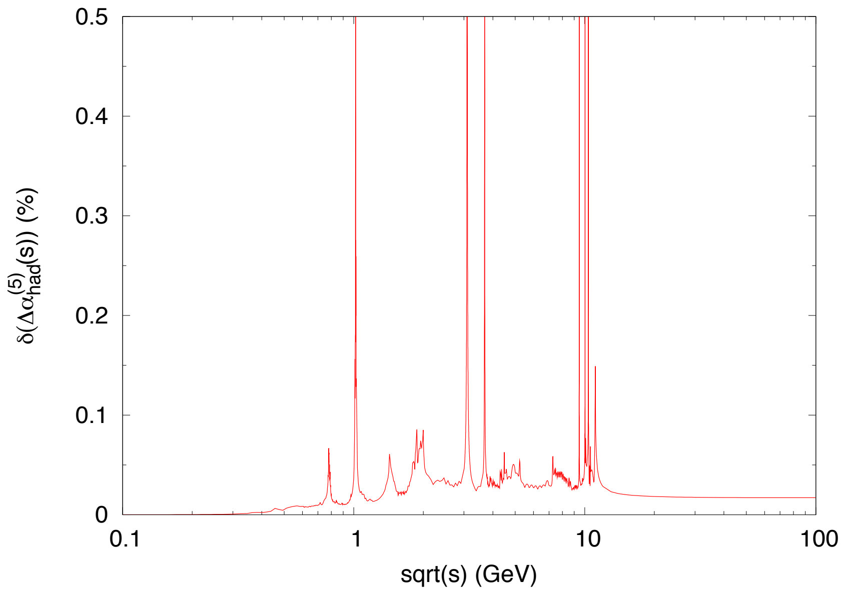
– smaller differences remain and reflect different choices, smoothing etc.

Rad Corrs: HVP for running α(q²). Accuracy

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

Error of VP in the timelike regime at low and higher energies (HLMNT compilation):





$\longrightarrow$ Below one per-mille (and typically $\sim 5 \cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

Rad Corrs: ISR. Scan vs ISR method. Phokhara

- ISR is always there, also for `**direct scan**' measurements, well understood theoretically and routinely taken into account in the experimental analyses
  (deconvolution of measured hadrons (+γ) cross section to get the cross section w/out ISR)

- In `**Radiative Return**' analyses, ISR emission defines already the lowest order process, hence higher orders, including FSR, are crucial

- The origin of additional photons can not be determined on an event-by-even basis

- Making use of high luminosities at meson factories, large event numbers can still be achieved with the ISR method, despite the parametric $\boldsymbol{\alpha/\pi}$ suppression

- Different variants: w. or w/out γ detection (large/small angle), luminosity from Bhabha or $\boldsymbol{\mu^+\mu^-}$

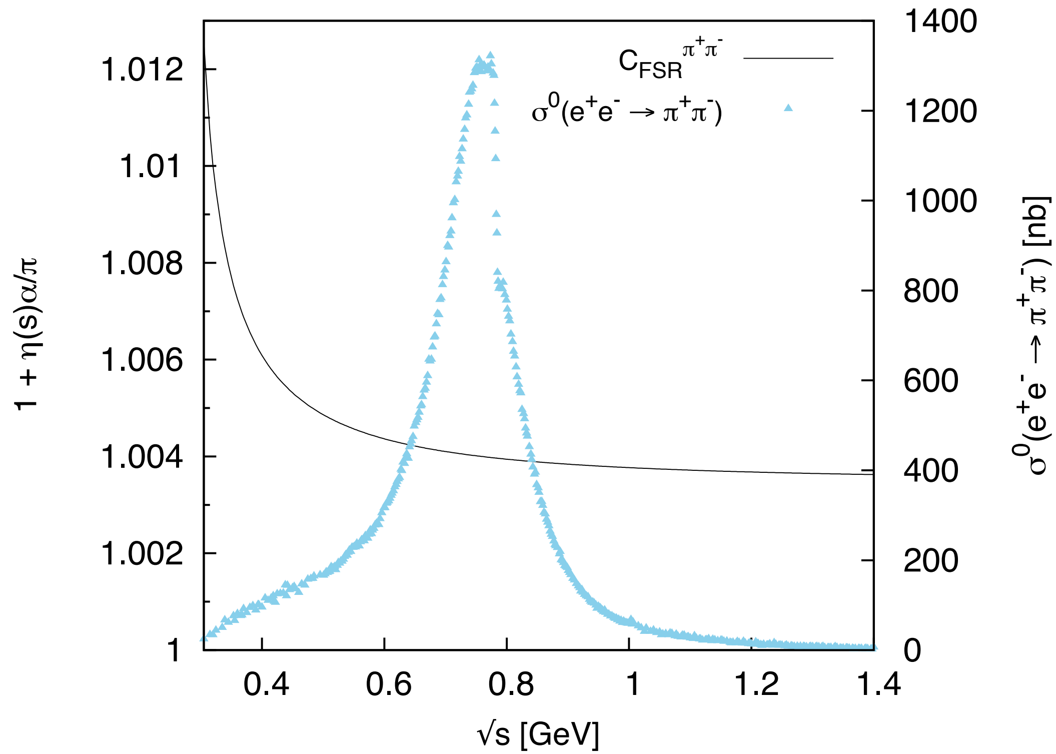
- Crucial Monte Carlo generator: ***Phokhara***
  - now with complete NLO corrections for $e^+e^- \rightarrow \boldsymbol{\mu^+\mu^-\gamma}, \boldsymbol{\pi^+\pi^-\gamma}$
  - but was not available for the earlier KLOE & BaBar analyses
  - further studies needed to clarify the role of these (and other) higher order corrections for the data obtained via ISR studies

Rad. Corrs.: inclusive Final State γ Radiation in sQED

- \`Schwinger' formula for inclusive (r+v) FSR: $$\sigma^0_{\text{had},(\gamma)}(s) = \sigma^0_{\text{had}}(s)\left(1+\eta(s)\frac{\alpha}{\pi}\right)$$

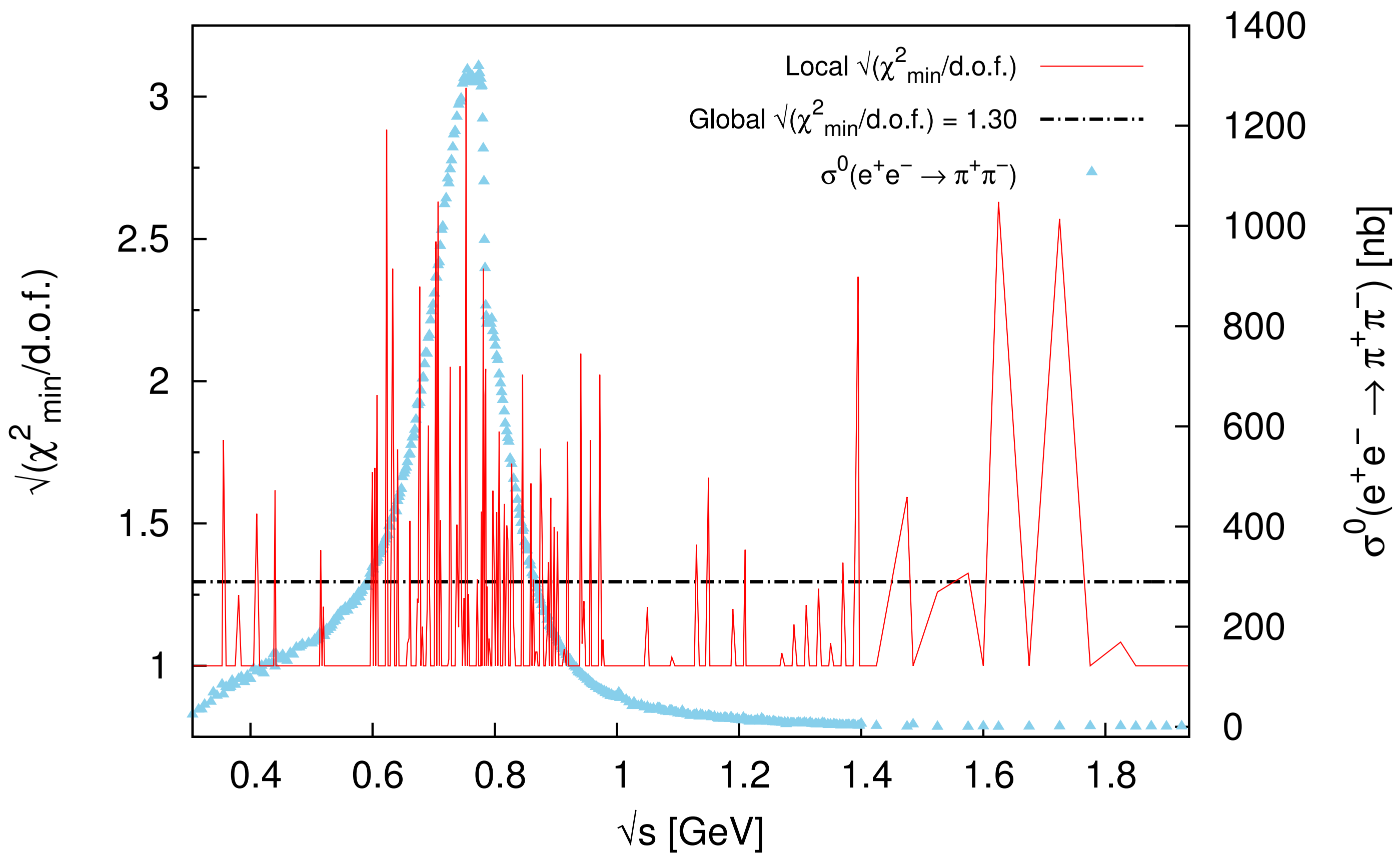
  [\`hard' real radiation (above a cutoff) is finite and easy to calculate as part of $\eta(s)$]

- <u>Example 2π:</u> inclusive correction compared to cross section in the ρ peak region



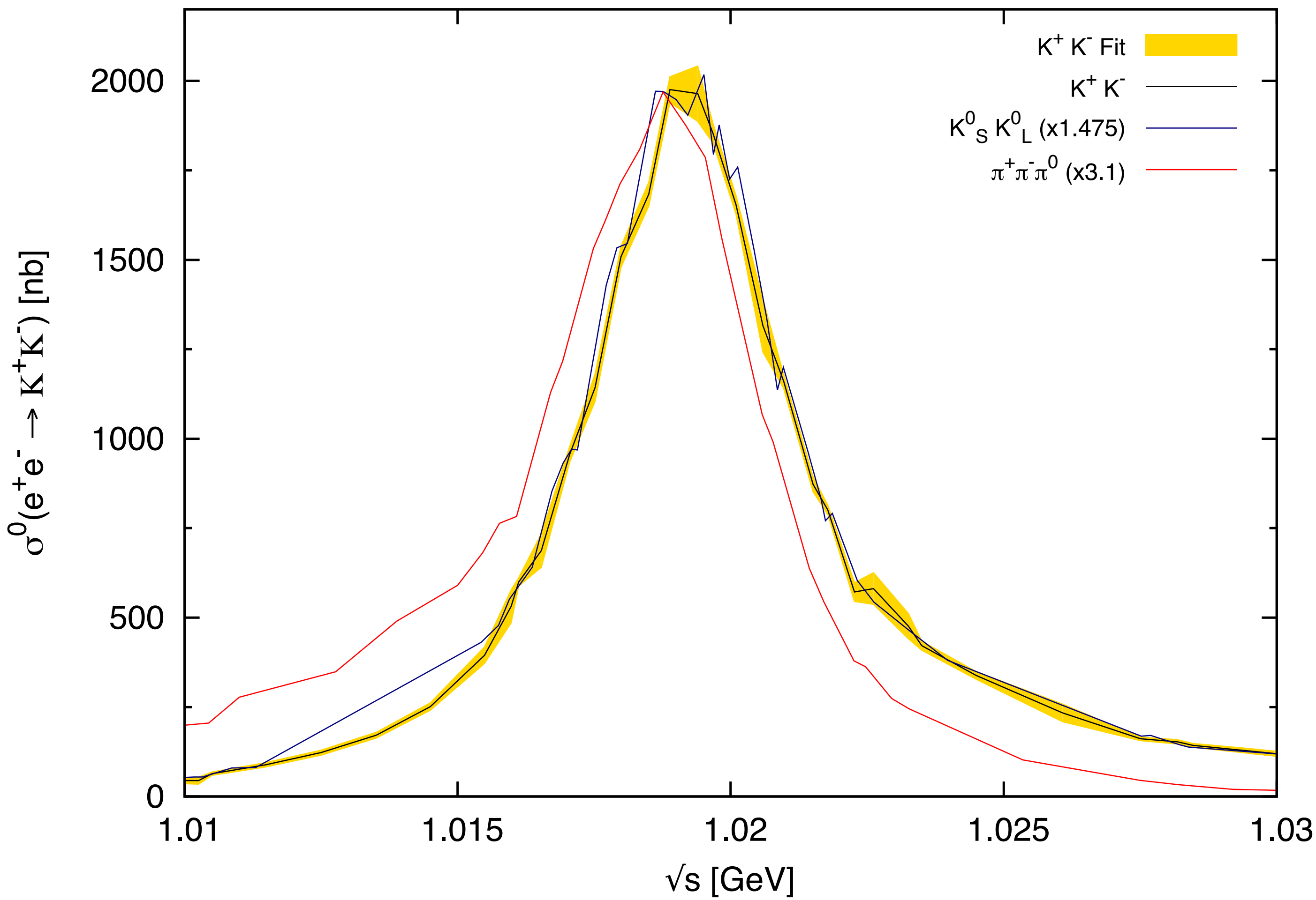
HVP: $\pi^+\pi^-$ channel. Error inflation in KNT

- Inflation of error with **local χ^2_{min}** accounts for tensions, leading to a ~14% error inflation, with overlay of 2**π** cross section fit (blue markers) and global χ^2_{min} (dash-dotted line)



HVP: Φ in different final states K^+K^-, $K_S^0K_L^0$, $\pi^+\pi^-\pi^0$

➤ Direct data integration automatically accounts for all hadronic dynamics, no resonance fits/parametrisations or estimates of mixing effects needed.



For demo. only, does not include latest data

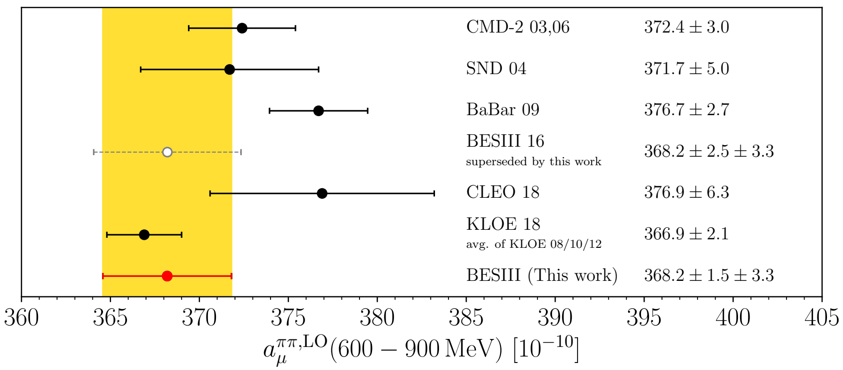
HVP: New/updated data sets since KNT19

- **pi+pi-pi0**, BESIII (2019), arXiv:1912.11208
- **pi+pi-** [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021) 135982 (erratum)
- **K+K-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- **etapi0gamma** (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-**, SND (2020), JHEP 01 (2021) 113
- **etaomega** → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- **pi+pi-pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- **pi+pi-2pi0omega**, BaBar (2021), Phys. Rev. D 103, 092001
- **etaetagamma**, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- **etaomega**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **omegaetapi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **pi+pi-4pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0pi0eta**, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- **pi+pi-3pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **2pi+2pi-3pi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **omega3pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi+pi-eta**, BaBar (2021), Phys. Rev. D 103, 092001
- **inclusive**, BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- ...

HVP: New/updated data sets since KNT19

- No new full KNT update at this stage yet, *preliminary estimates* show no big surprises
- KNT analysis framework *blinded* in autumn 2022 (see Alex's talk at TI meeting in Edinburgh)

- **pi+pi-,** inclusion of BESIII (2020 erratum) & SND (2020):



Measurement	$a_\mu(\pi\pi) \times 10^{10}$
This work	$409.79 \pm 1.44 \pm 3.87$
SND06	$406.47 \pm 1.74 \pm 5.28$
BaBar	$413.58 \pm 2.04 \pm 2.29$
KLOE	$403.39 \pm 0.72 \pm 2.50$

(not yet full statistics, systematics?)

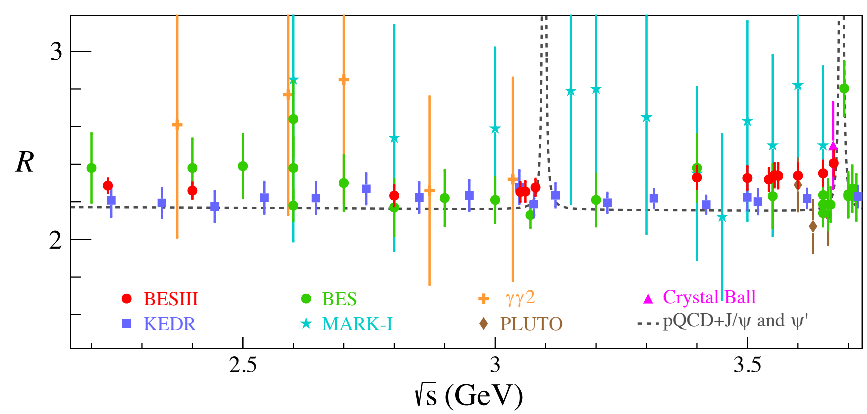
$a_\mu^{2\pi}$ [0.305 … 1.937 GeV] (KNT19) = (503.46 ± 1.91) × 10^{-10} ➡ (503.88 ± 1.79) × 10^{-10} (prel.)

- **inclusive**, inclusion of BESIII (2021):

  $a_\mu^{\mathrm{incl.}}$ [1.937 … 11.2 GeV] (KNT19) =

  (43.55 ± 0.67) × 10^{-10} ➡

  (43.16 ± 0.59) × 10^{-10} (prel.)



HVP: White Paper merging procedure

Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or similar experimental input
- Includes correlations and their different treatment as much as possible
- Allows to give one recommended (merged) result, which is conservative w.r.t. the underlying (and possibly underestimated) systematic uncertainties
- Note: Merging leads to a bigger error estimate compared to individual evaluations; error `corridor' defined by embracing choices goes far beyond χ^2_{min} inflation

➠ $a_\mu^{HVP,\,LO} = 693.1\,(4.0) \times 10^{-10}$ is the result used in the WP `SM2020' value

- This result does not include lattice, but in 2020 was compatible with published full results, apart from the BMW prediction:

  $a_\mu^{HVP,\,LO}$ (BMW) = 707.5 (5.5) × 10^{-10} [Nature 2021] ⇝ **1.5/2.1 σ** tension w. exp/WP20

Many efforts are ongoing to understand this new puzzle!

Channel	Energy range [GeV]	$a_\mu^{\text{had,LO VP}} \times 10^{10}$	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	New data
		Chiral perturbation theory (ChPT) threshold contributions		
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	...
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	...
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	...
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	...
		Data based channels ($\sqrt{s} \le 1.937$ GeV)		
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	...
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\text{no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	...
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\text{no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	...
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	...
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	...
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to \text{npp})K\bar{K}_{\text{no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
		Estimated contributions ($\sqrt{s} \le 1.937$ GeV)		
$(\pi^+\pi^-3\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	...
$(\pi^+\pi^-4\pi^0)_{\text{no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	...
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	...
$\omega(\to \text{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	...
$\omega(\to \text{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	...
$\omega(\to \text{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	...
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	...
		Other contributions ($\sqrt{s} > 1.937$ GeV)		
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	...	6.26 ± 0.19	7.07 ± 0.22	...
ψ'	...	1.58 ± 0.04	2.51 ± 0.06	...
$\Upsilon(1S-4S)$	...	0.09 ± 0.00	1.06 ± 0.02	...
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	...
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	...

Table from KNT18,
PRD 97(2018)114025

Update: KNT19
LO+NLO HVP for
$a_{e,\mu,\tau}$ & hyperfine splitting of muonium

PRD101(2020)014029

Breakdown of HVP contributions in ~35 hadronic channels

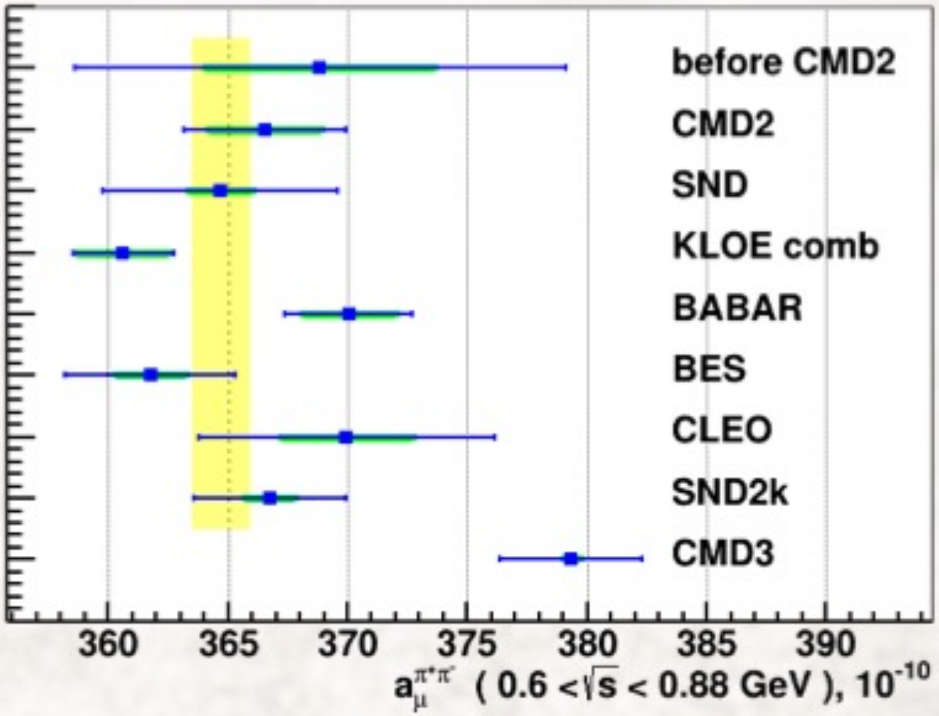
From 2-11 GeV, use of inclusive data, pQCD only beyond 11 GeV

New CMD-3 $\pi^+\pi^-$ puzzle for a_μ^{HVP}

Slides from Fedor Ignatov's TI talk 27.3.2023

arXiv:2302.08834

$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty} \frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$



$0.6 < \sqrt{s} < 0.88$ GeV

	$a_\mu^{\pi\pi,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

RHO2013	380.06 ± 0.61 ± 3.64
RHO2018	379.30 ± 0.33 ± 2.62 $\times10^{-10}$
Sum	379.35 ± 0.30 ± 2.95

a_μ (SM): White Paper https://doi.org/10.1016/j.physrep.2020.07.006

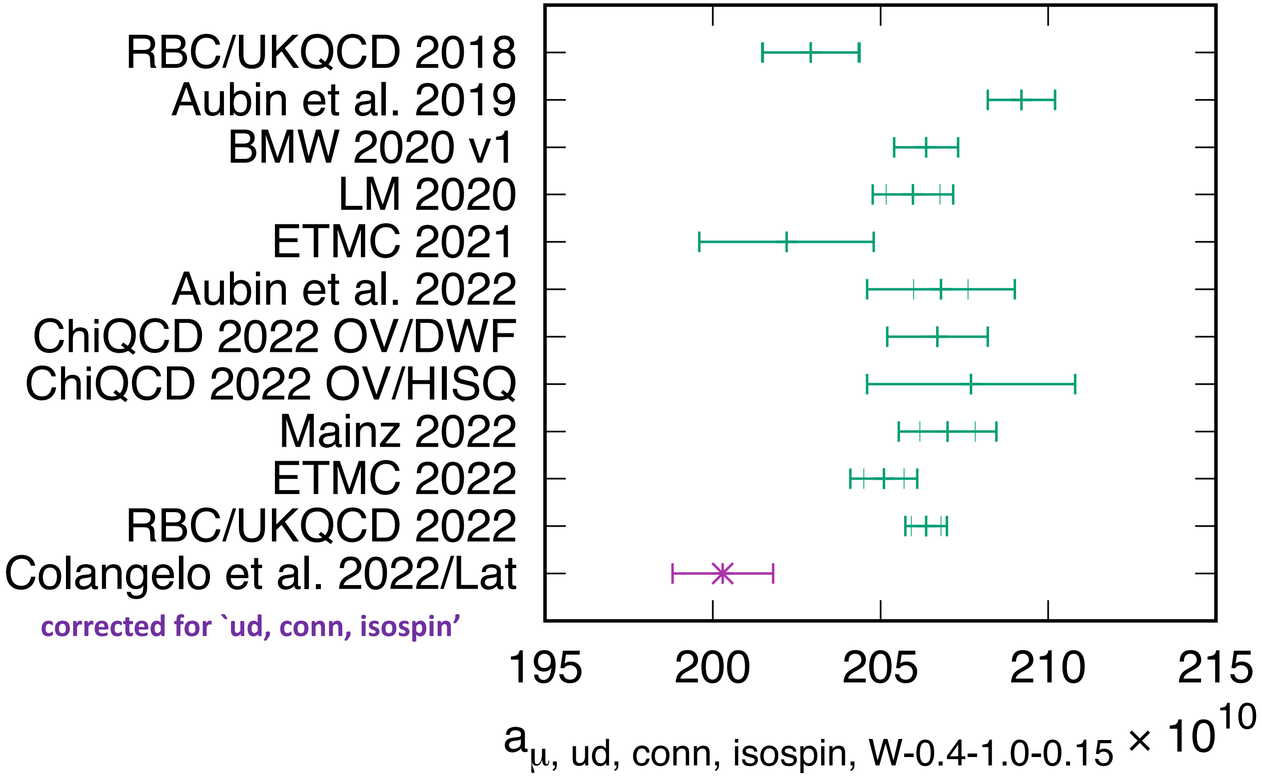
White Paper [T. Aoyama et al, arXiv:2006.04822], 132 authors, 82 institutions, 21 countries

Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	279(76)	

w.r.t. BNL only

a_μ^{HVP} : `Window Fever'

Plot from C Lehner's talk at the TI Edinburgh workshop 5-9.9.'22



Another ~4σ puzzle:

- Lattice QCD `easiest' in the middle window
- Comparison not direct, but heavier quark and iso-spin breaking contributions unlikely to change much
- So why is there such a large disagreement w. the data?

- **3.9σ tension** betw. RBC/UKQCD 2022 and data-driven

  [Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner (22)]
- also new FNAL/HPQCD/MILC result: 206.1(1.0) [arXiv:2301.08274]
- **Agreement** of different lattice results, check of **universality** betw. lattice methods

a_μ^{HVP} : **Window Fever,** where to go from here

- Shorter term: <u>further window studies</u>, with short- and long-distance windows needed to better understand the emerged discrepancy

- Longer term: <u>full $\mathbf{a_\mu}$ with high precision</u> from other lattice collaborations

- For now there is a big puzzle

- **Could σ_{had} (w/out CMD-3) be so wrong?** → future indep. check via **MUonE** @CERN

  - If cross sections would shift up at energies above ~1-2 GeV, this would change **$\Delta\alpha(M_Z^2)$** and the SM **EW precision fits** would be in trouble

    [Crivellin, Hoferichter, Manzari, Montull ('20) / Keshavarzi, Marciano, Passera, Sirlin ('20) / Malaescu, Schott ('20)]

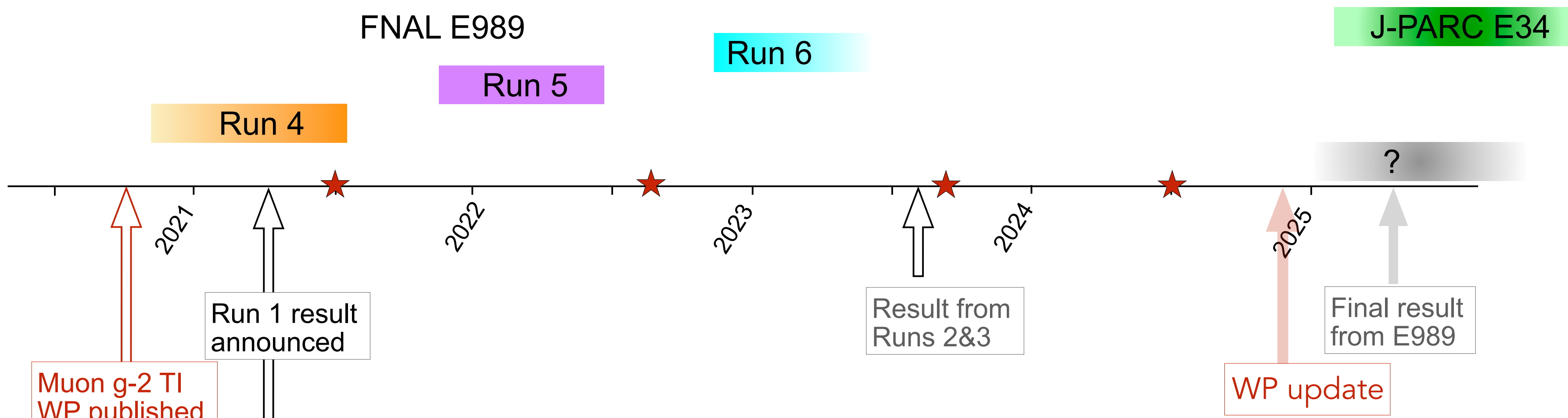
  - Most important $\pi^+\pi^-$ channel constrained by analyticity and unitarity, **but CMD-3**

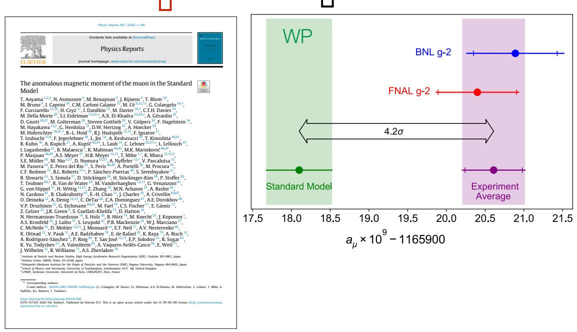
  - First detailed comparisons of lattice with data-driven window evaluations show that to reconcile data-driven with lattice ~40% of the shift must come from above 1 GeV for any reasonable cross section shifts (so not only $\pi^+\pi^-$ would need change)

    [Colangelo at LatticeNET workshop in Benasque 11-17.9.'22]

Theory Initiative: Sep. 2023 workshop at Bern

Aida El-Khadra: TI outlook and plans:





Theory Initiative:

☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT

☆ 2nd CMD-3 discussion meeting

☆ 8/9/2023: Status of Muon g-2 Theory in SM

★ TI workshops:

Jun 2021 @ KEK (virtual)
Sep 2022 @ Higgscentre
Sep 2023 @ Bern
Sep 2024 @ KEK or KMI

WorkStop/ThinkStart: history & papers (Adrian Signer's intro)

UZH 13–16 Sep 2016 [1705.01827]

Eur. Phys. J. C (2017) 77:471
DOI 10.1140/epjc/s10052-017-5023-2

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

To *d*, or not to *d*: recent developments and comparisons of regularization schemes

C. Gnendiger[1,a], A. Signer[1,2], D. Stöckinger[3], A. Broggio[4], A. L. Cherchiglia[5], F. Driencourt-Mangin[6], A. R. Fazio[7], B. Hiller[8], P. Mastrolia[9,10], T. Peraro[11], R. Pittau[12], G. M. Pruna[1], G. Rodrigo[6], M. Sampaio[13], G. Sborlini[6,14,15], W. J. Torres Bobadilla[6,9,10], F. Tramontano[16,17], Y. Ulrich[1,2], A. Visconti[1,2]

UZH 4–7 Feb 2019 [2004.13663]

Eur. Phys. J. C (2020) 80:591
https://doi.org/10.1140/epjc/s10052-020-8138-9

THE EUROPEAN PHYSICAL JOURNAL C

Review

Theory for muon-electron scattering @ 10 ppm

A report of the MUonE theory initiative

P. Banerjee[1], C. M. Carloni Calame[2], M. Chiesa[3], S. Di Vita[4], T. Engel[1,5], M. Fael[6], S. Laporta[7,8], P. Mastrolia[7,8], G. Montagna[2,9], O. Nicrosini[2], G. Ossola[10], M. Passera[8], F. Piccinini[2], A. Primo[5], J. Ronca[11], A. Signer[1,5,a], W. J. Torres Bobadilla[11], L. Trentadue[12,13], Y. Ulrich[1,5], G. Venanzoni[14]

Florence 4–6 Nov 2019 [2012.02567]

Eur. Phys. J. C (2021) 81:250
https://doi.org/10.1140/epjc/s10052-021-08996-y

THE EUROPEAN PHYSICAL JOURNAL C

Review

May the four be with you: novel IR-subtraction methods to tackle NNLO calculations

W. J. Torres Bobadilla[1,2,a], G. F. R. Sborlini[3], P. Banerjee[4], S. Catani[5], A. L. Cherchiglia[6], L. Cieri[5], P. K. Dhani[5,7], F. Driencourt-Mangin[2], T. Engel[4,8], G. Ferrera[9], C. Gnendiger[4], R. J. Hernández-Pinto[10], B. Hiller[11], G. Pelliccioli[12], J. Pires[13], R. Pittau[14], M. Rocco[15], G. Rodrigo[2], M. Sampaio[6], A. Signer[4,8], C. Signorile-Signorile[16,17], D. Stöckinger[18], F. Tramontano[19], Y. Ulrich[4,8,20]

Durham 3–5 Aug 2022

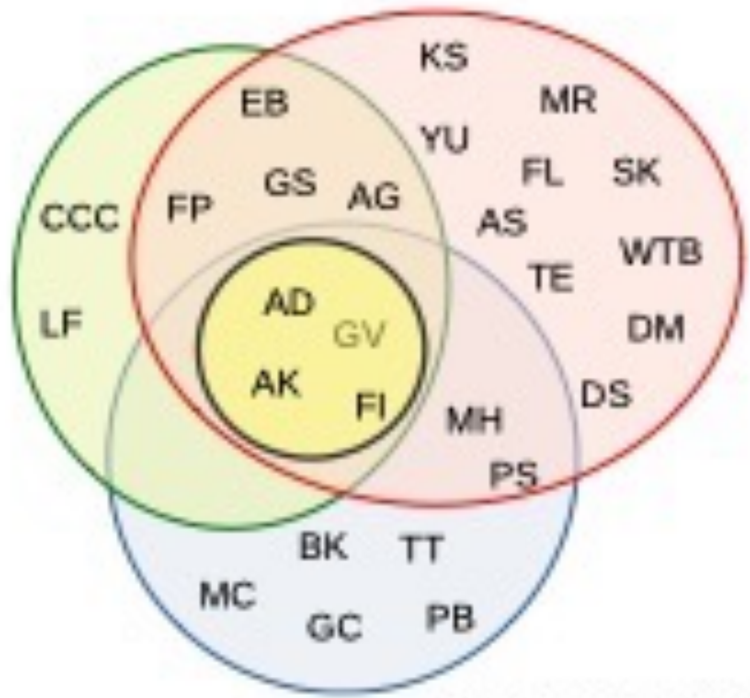
N^{3}LO kick-off WorkStop/ThinkStart

https://conference.ippp.dur.ac.uk/event/1104/

WorkStop/ThinkStart: WorkStop Nr 5 (Adrian Signer's intro)

Team: P. Beltrame, E. Budassi, C. Carloni Calame, G. Colangelo, M. Cottini, A. Driutti, T. Engel, L. Flower, A. Gurgone, M. Hoferichter, F. Ignatov, S. Kollatzsch, B. Kubis, A. Kupsc, F. Lange, D. Moreno, F. Piccinini, M. Rocco, K. Schönwald, A. Signer, G. Stagnitto, D. Stöckinger, P. Stoffer, T. Teubner, W. Torres Bobadilla, Y. Ulrich, G. Venanzoni

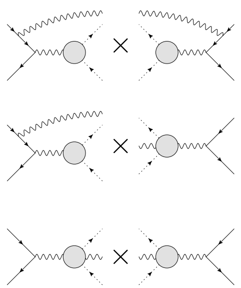
WP1:	QED for leptons at NNLO
WP2:	Form factor contributions at N^3LO
WP3:	Processes with hadrons
WP4:	Parton showers
WP5:	Experimental input



Zurich ThinkStart: diagram classification ISR (P. Stoffer's WP3 summary)

[From: 5th WorkStop/ThinkStart: Radiative corrections and MC tools for Strong 2020, Zurich, 5-9 June 2023]

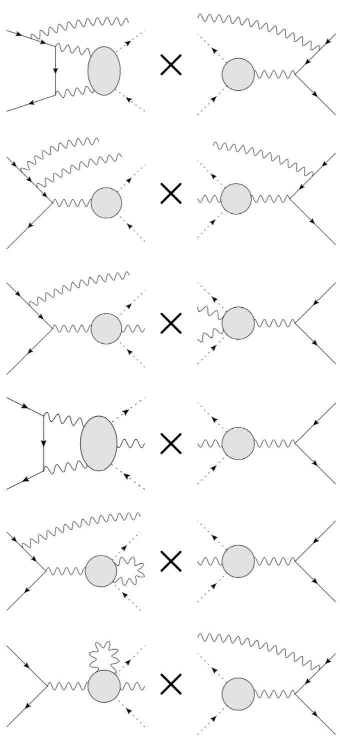
ISR experiments: LO



pure QED

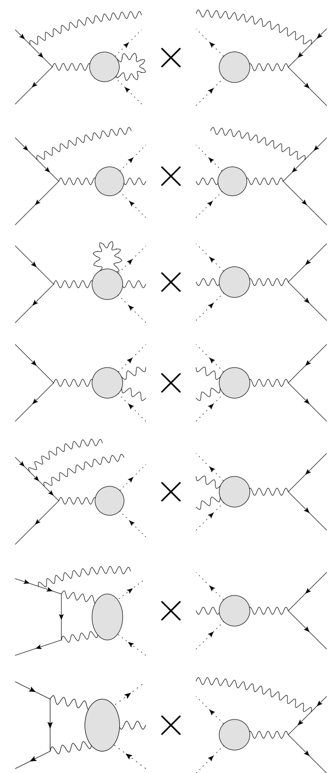
contributing only to charge asymmetry

suppressed by cuts?
PHOKHARA: sQED + resonance approximations
dispersive approach by Colangelo et al.



contributes only to asymmetry

ISR experiments: NLO (omitting pure QED corrections to LO)



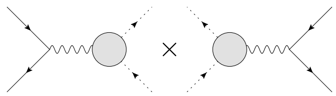
PHOKHARA: sQED + resonance approximations
dispersive approach by Colangelo et al.

contained in PHOKHARA
pure FSR: sufficiently suppressed by experimental cuts?

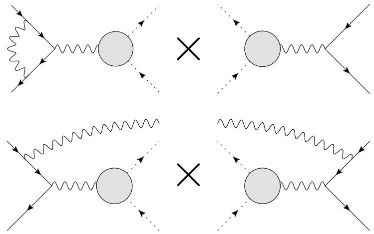
???
PHOKHARA: sQED, multiplied by form factors *outside* loop
ISR–FSR interference
potential red flag identified during WorkStop

Zurich ThinkStart: diagram classification scan (P. Stoffer's WP3 summary)

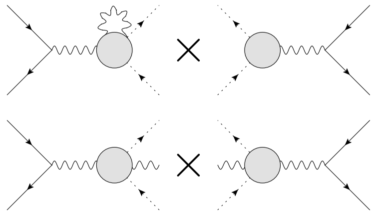
Direct scan experiments: LO



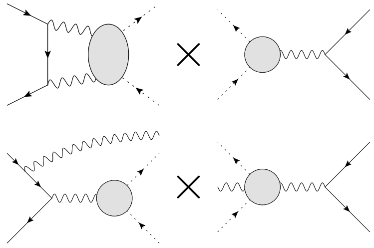
Direct scan experiments: NLO



pure QED



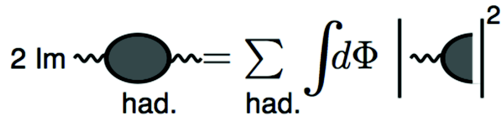
included in generators in terms of sQED
dispersive approach by Colangelo et al.



contributes only to asymmetry;

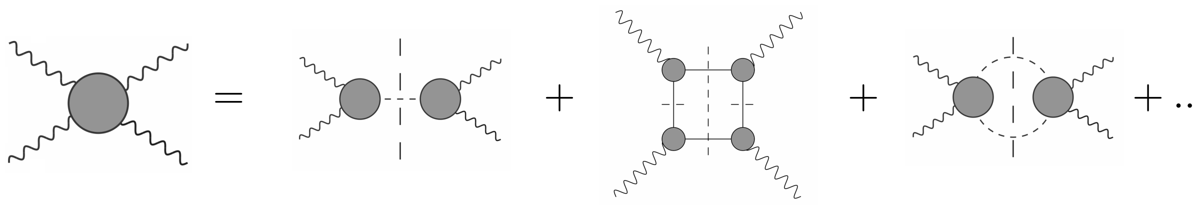
only pole terms:
→ Ignatov, Lee (2022)
→ Colangelo, Hoferichter, Monnard, Ruiz de Elvira (2022)

a_μ^{HLbL} : Hadronic Light-by-Light: Dispersive approach

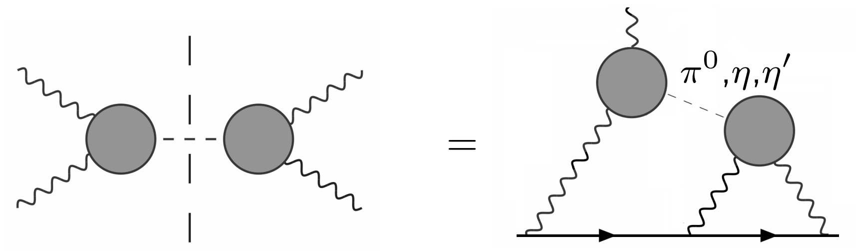
For HVP $\Rightarrow$ 

$\Rightarrow$ $\mathrm{Im}\Pi_{\mathrm{had}}(s) = \left(\frac{s}{4\pi\alpha}\right)\sigma_{\mathrm{had}}(s)$

For HLbL $\Rightarrow$ $\Pi_{\mu\nu\lambda\sigma} = \Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} + \Pi^{\mathrm{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \ldots$

$\Rightarrow$ 

$\Rightarrow$ Dominated by pole (pseudoscalar exchange) contributions

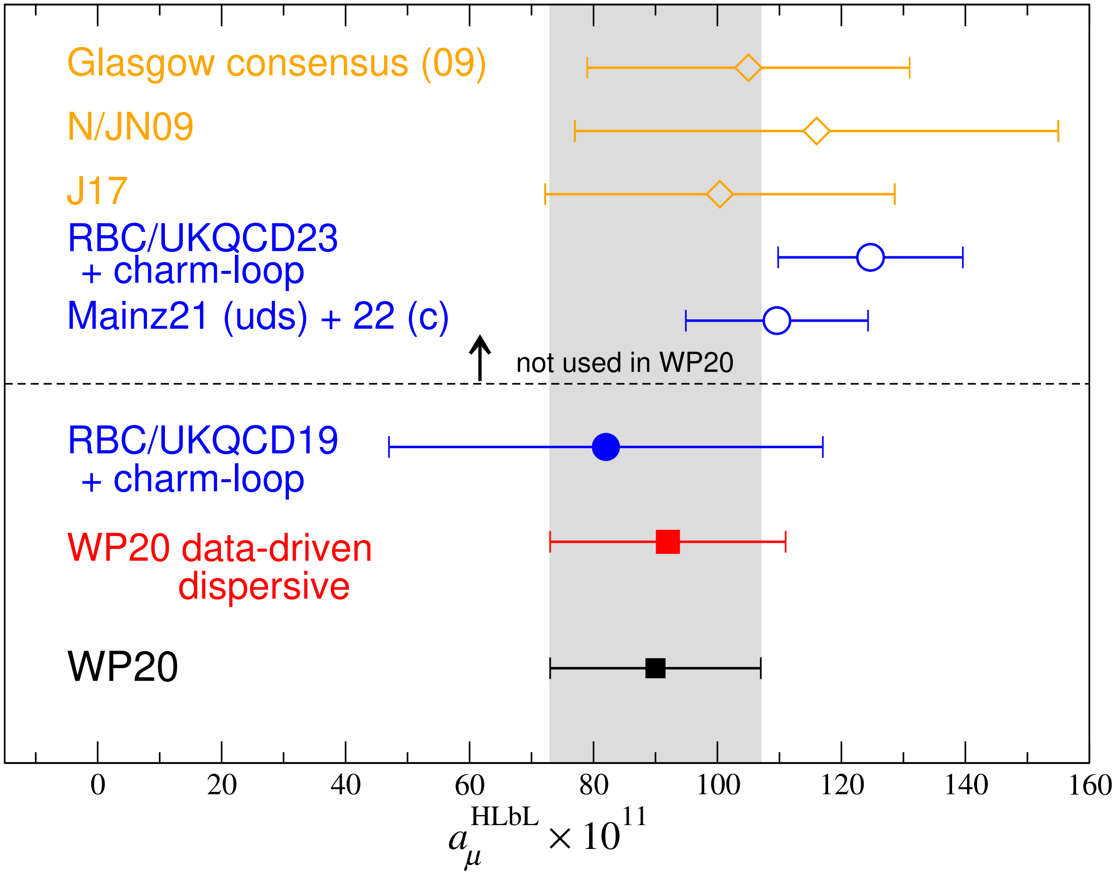
$\Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} =$ 

$\Rightarrow$ Sum all possible diagrams to get a_μ^{HLbL}

- See also review by Danilkin+Redmer+Vanderhaeghen using dispersive techniques estimates $(8.7 \pm 1.3) \times 10^{-10}$ [*Prog. Part. Nucl. Phys.* 107 (2019) 20]

- With new results & progress, L-by-L now more reliably predicted

a_μ^{HLbL} : WP Status/Summary of Hadronic Light-by-Light contributions



hadronic models + pQCD

lattice QCD + QED (after WP)

lattice QCD + QED

data-driven

TI White Paper 2020 value:

a_μ^{HLbL} = 92 (18) × 10^{-11} ✓

- data-driven dispersive & lattice results have confirmed the earlier model-based predictions
- **uncertainty better under control** and at 0.15ppm already **sub-leading compared to HVP**
- lattice predictions now competitive, good prospects for further error reduction needed for final expected FNAL g-2 precision

a_μ^{HVP} : Hadronic tau decay data

- Historically, hadronic tau decay data, e.g. $\tau^- \to \pi^0\pi^-\nu_\tau$, were used to improve precision of e^+e^- based evaluations

- However, with the increased precision of the e^+e^- data there is now limited merit in this (there are some conflicting evaluations, DHMZ have dropped it)

- The required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no ρ–ω mixing in τ decays)

- Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem."

- New contribution to the discussion by Masjuan, Miranda, Roig: arXiv:2305.20005
  \`τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization'

a_μ^{HVP}: Hadronic tau decay data

Mattia Bruno: Summary slide from TI talk on tau (Sep. 2023, Bern)

Windows very powerful quantities: intermediate window a_μ^W
- hadronic τ-decays can shed light on tension lattice vs e^+e^-

τ data very competitive on intermediate window
- historic tension w/ ee data and in IB τ effects
- preliminary analysis Aleph $< 0.5\%$ accuracy on a_μ^W
- (old) LQCD IB effects precision $O(1.5) \cdot 10^{-10}$ [MB Edinburgh '22]
  - new EuroHPC allocation, blinding

Work in progress to finalize full formalism [MB et al, in prep]
- W-regularization and short-distance corrections
- (re-)calculation of initial state rad.cor.
- initial-final rad.cor: proof for analytic continuation
- numerical calculation of final state IB corrections
  - relevant also for QED correction to HVP

Thanks for your attention

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